Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.7-Miscellaneous/140-4.7.6-f^-a+b-x+c-x^2-trig-d+e-x+f-x^2-^n

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December 8, 2023 Compiled on December 8, 2023 at 10:10pm

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [142]. This is test number [140].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (142)	0.00 (0)
Rubi	98.59 (140)	1.41 (2)
Fricas	80.99 (115)	19.01 (27)
Maple	80.28 (114)	19.72 (28)
Maxima	80.28 (114)	19.72 (28)
Giac	44.37 (63)	55.63 (79)
Mupad	35.21 (50)	64.79 (92)
Sympy	30.28 (43)	69.72 (99)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

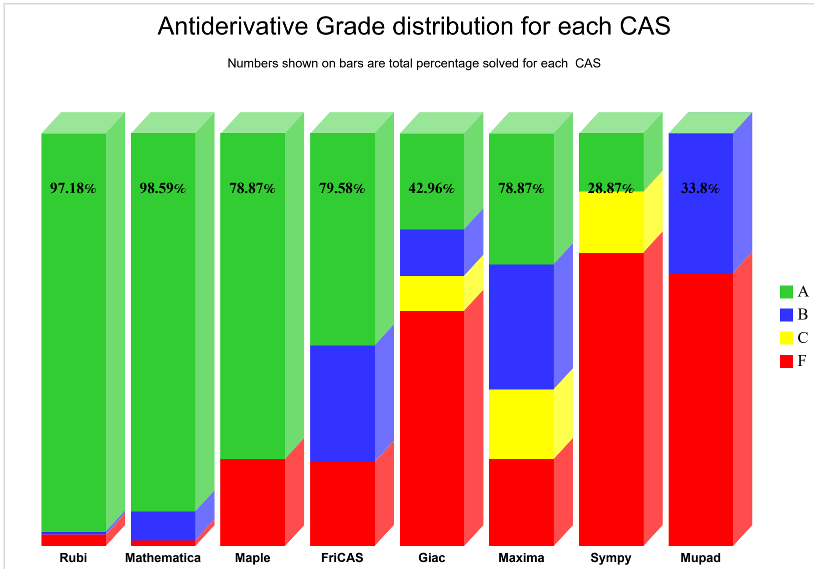
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

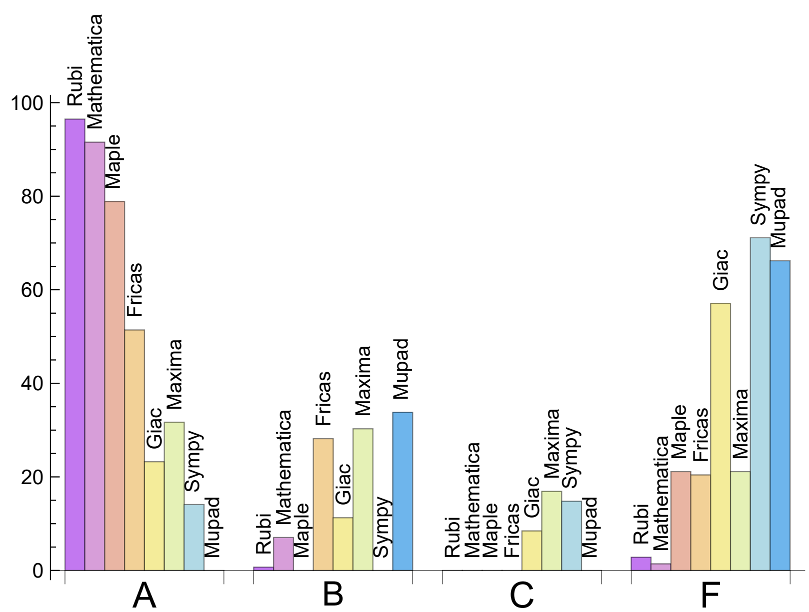
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.479	0.704	0.000	2.817
Mathematica	91.549	7.042	0.000	1.408
Maple	78.873	0.000	0.000	21.127
Fricas	51.408	28.169	0.000	20.423
Maxima	31.690	30.282	16.901	21.127
Giac	23.239	11.268	8.451	57.042
Sympy	14.085	0.000	14.789	71.127
Mupad	0.000	33.803	0.000	66.197

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	2	100.00	0.00	0.00
Fricas	27	100.00	0.00	0.00
Maple	28	100.00	0.00	0.00
Maxima	28	100.00	0.00	0.00
Giac	79	100.00	0.00	0.00
Mupad	92	0.00	100.00	0.00
Sympy	99	96.97	3.03	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.25
Maxima	0.27
Giac	0.31
Rubi	0.45
Maple	0.86
Mathematica	1.25
Sympy	4.29
Mupad	20.67

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	68.50	1.12	25.50	0.90
Maple	125.89	0.86	108.00	0.84
Rubi	141.22	1.14	129.00	1.00
Fricas	201.50	1.26	156.00	1.05
Mathematica	246.56	1.32	117.50	1.00
Maxima	499.46	3.97	238.00	1.66
Giac	522.17	12.25	127.00	1.17
Sympy	598.74	4.57	70.00	2.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

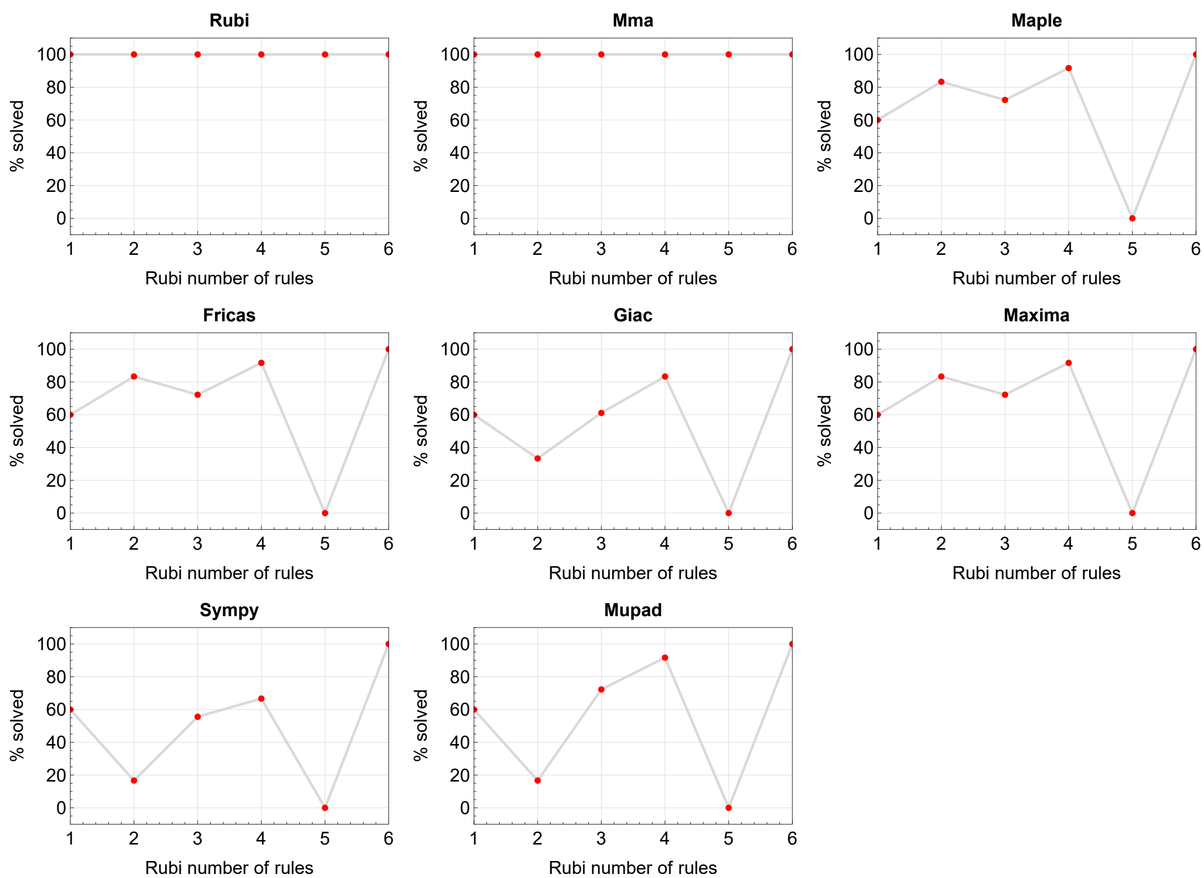


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

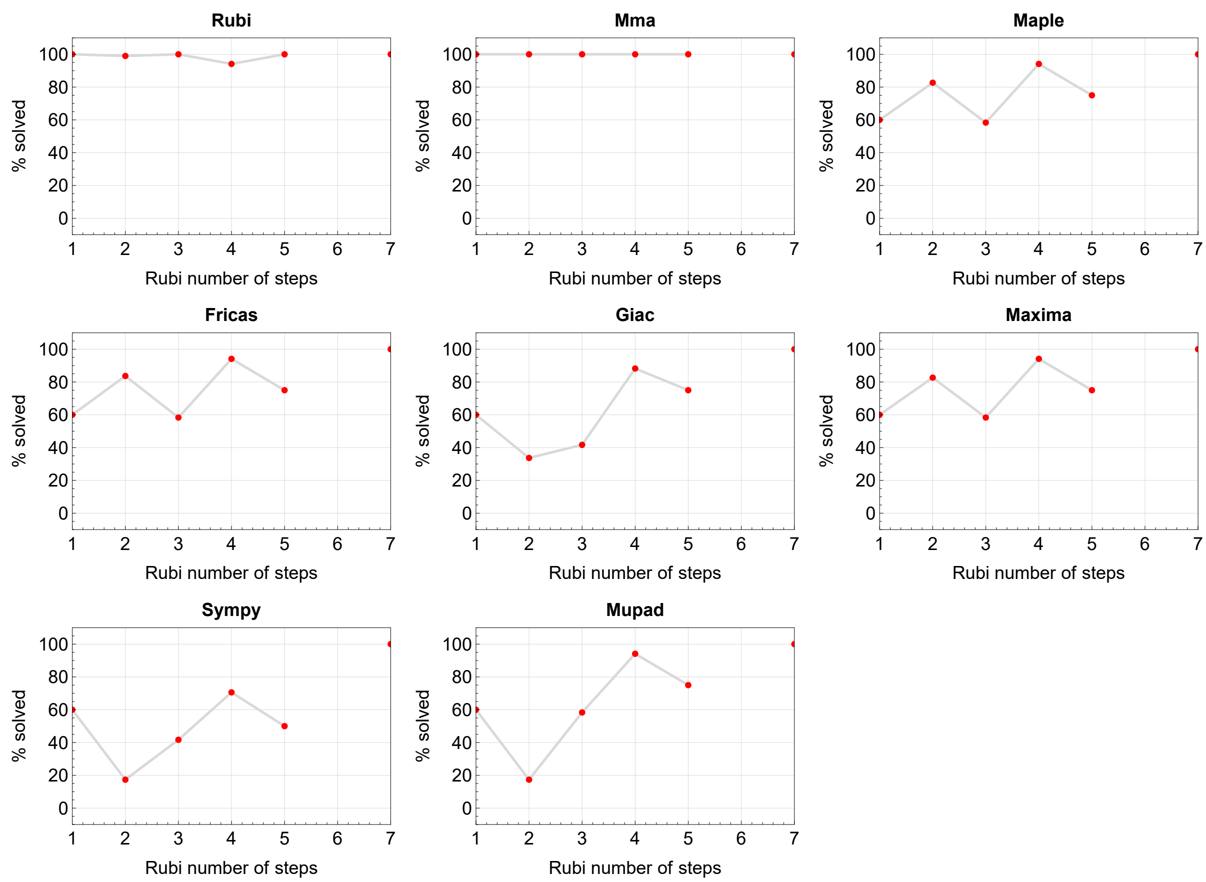


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

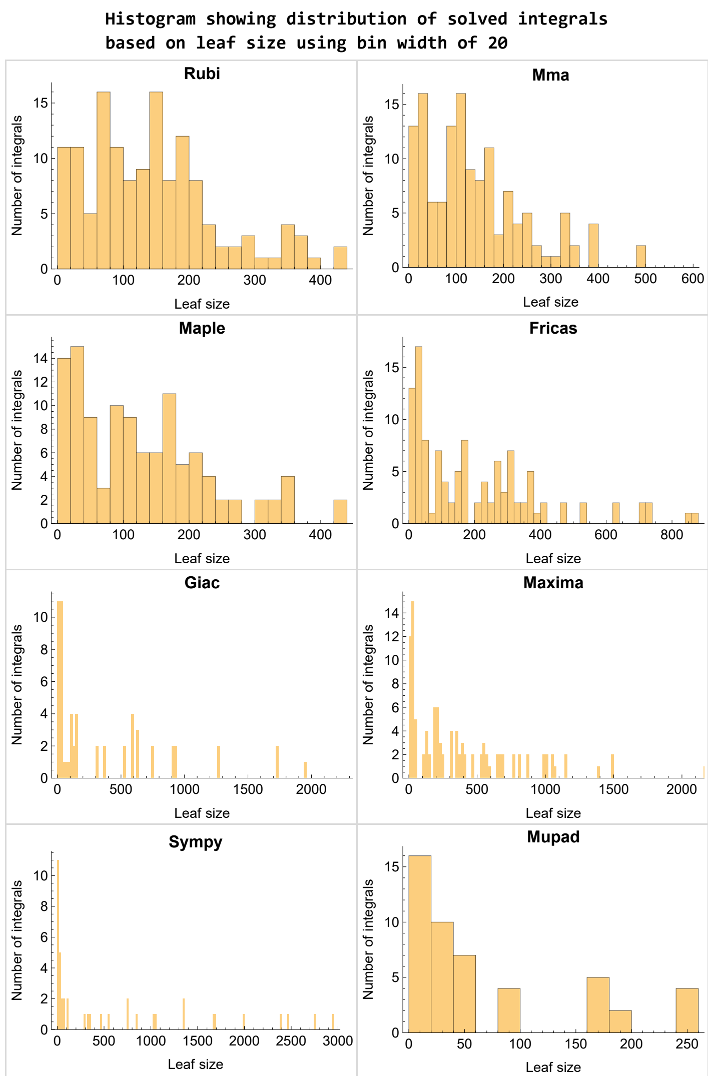


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

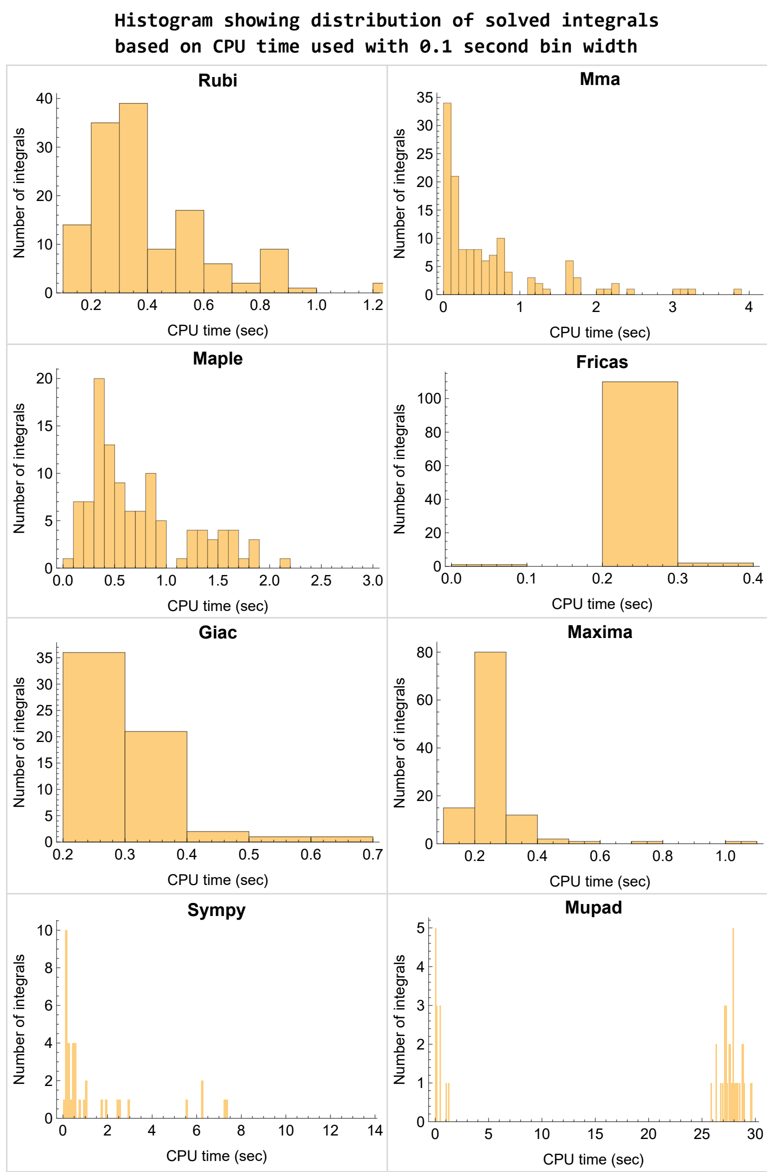


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

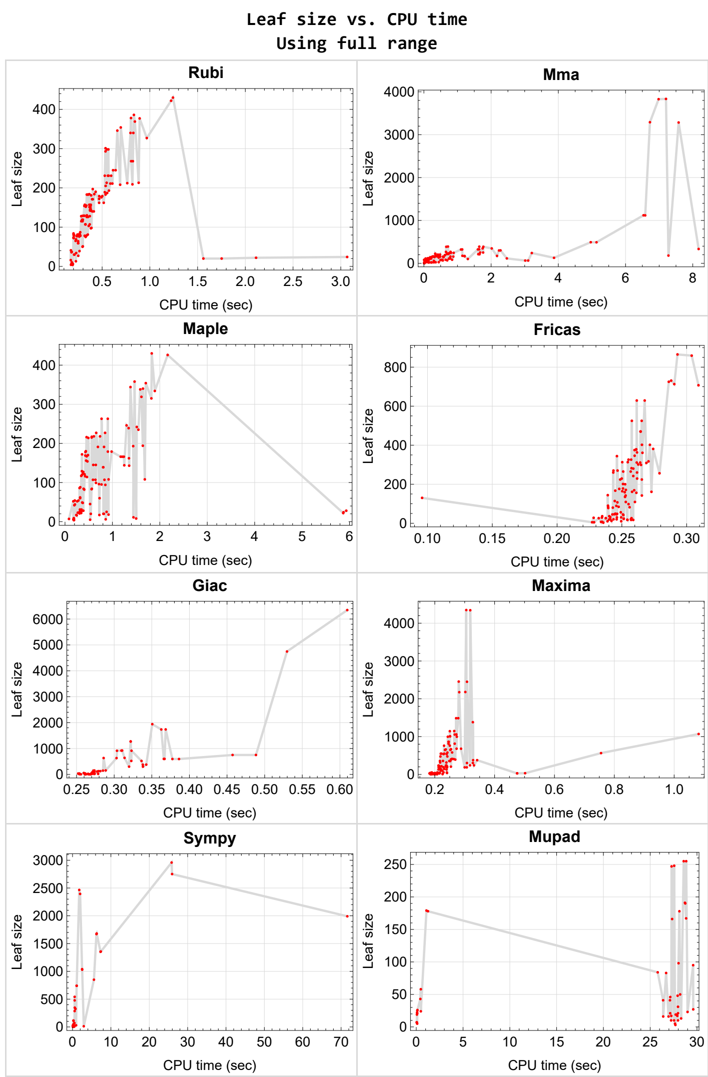


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{29, 30\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{\}$

Mathematica $\{98, 100, 101, 102, 103, 129, 131, 132, 133, 134\}$

Maple $\{\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

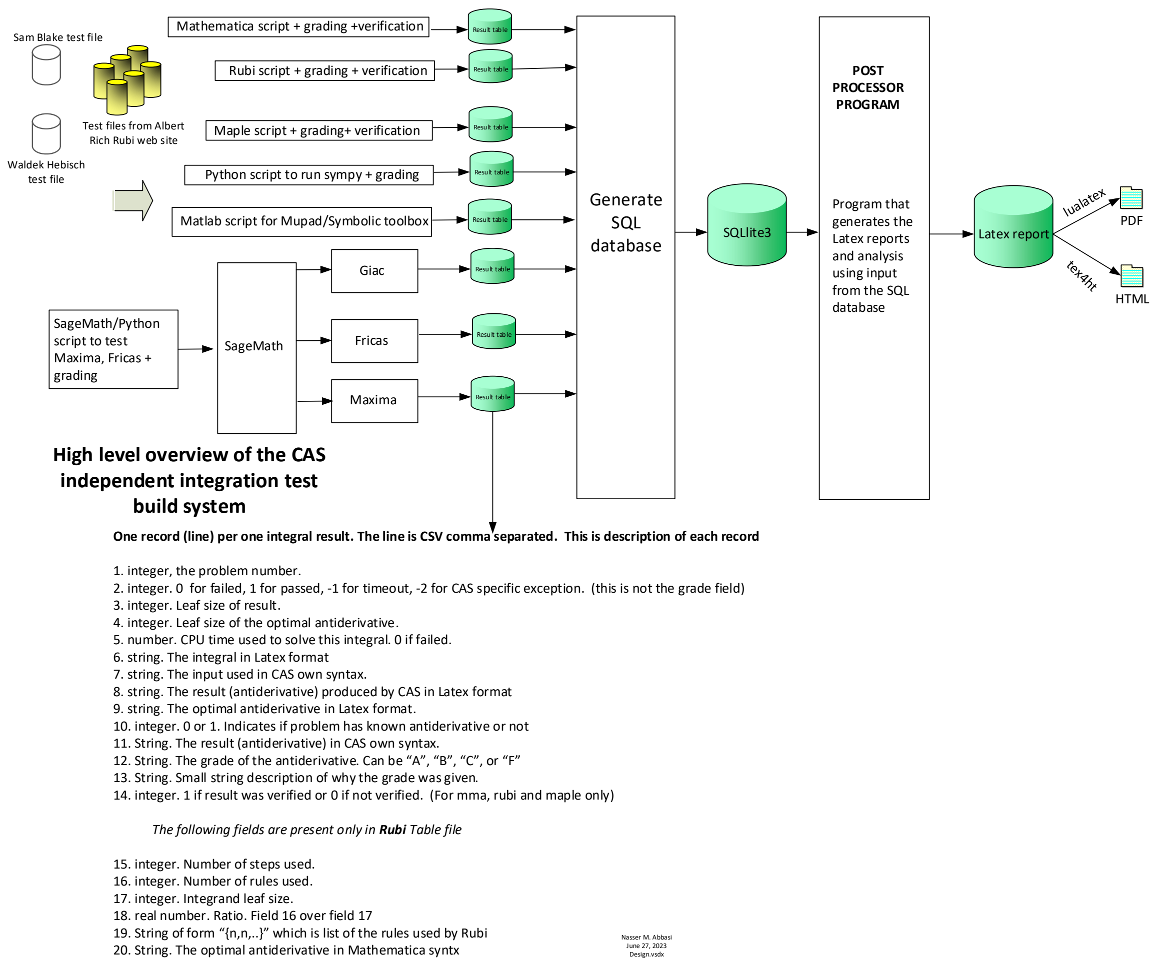
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142 }

B grade { 34 }

C grade { }

F normal fail { 28, 32 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142 }

B grade { 7, 21, 22, 63, 99, 101, 102, 130, 132, 133 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 2, 3, 4, 9, 11, 12, 13, 18, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140 }

B grade { }

C grade { }

F normal fail { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 53, 54, 55, 56, 137, 138, 141, 142 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 2, 3, 4, 9, 11, 12, 13, 18, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 71, 72, 73, 76, 77, 78, 85, 86, 87, 88, 89, 94, 95, 96, 104, 107, 108, 109, 116, 117, 118, 119, 120, 125, 126, 127, 135, 136, 139, 140 }

B grade { 63, 70, 74, 75, 79, 80, 81, 82, 83, 84, 90, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 105, 106, 110, 111, 112, 113, 114, 115, 121, 122, 123, 124, 128, 129, 130, 131, 132, 133, 134 }

C grade { }

F normal fail { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 53, 54, 55, 56, 137, 138, 141, 142 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 9, 18, 31, 32, 33, 38, 47, 48, 49, 50, 51, 52, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 79, 80, 81, 82, 84, 104, 105, 106, 107, 110, 111, 112, 113, 115 }

B grade { 2, 3, 4, 11, 12, 13, 34, 35, 39, 40, 41, 42, 43, 44, 45, 46, 63, 77, 78, 83, 88, 90, 91, 93, 97, 99, 100, 102, 108, 109, 114, 119, 121, 122, 124, 128, 130, 131, 133, 135, 136, 139, 140 }

C grade { 36, 37, 85, 86, 87, 89, 92, 94, 95, 96, 98, 101, 103, 116, 117, 118, 120, 123, 125, 126, 127, 129, 132, 134 }

F normal fail { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 53, 54, 55, 56, 137, 138, 141, 142 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { 9, 18, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 61, 62, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 104, 105, 106 }

B grade { 31, 32, 63, 70, 79, 80, 81, 82, 83, 84, 110, 111, 112, 113, 114, 115 }

C grade { 2, 3, 4, 11, 12, 13, 34, 35, 135, 136, 139, 140 }

F normal fail { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 33, 36, 37, 53, 54, 55, 56, 57, 58, 59, 60, 76, 77, 78, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 141, 142 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 2, 3, 4, 9, 11, 12, 13, 18, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 104, 135, 136, 139, 140 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 53, 54, 55, 56, 57, 58, 59, 60, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 141, 142 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 9, 18, 34, 35, 47, 48, 49, 50, 51, 52, 61, 62, 64, 65, 66, 67, 69, 70, 71, 72 }

B grade { }

C grade { 2, 3, 4, 11, 12, 13, 38, 39, 40, 41, 42, 43, 44, 45, 46, 73, 104, 135, 136, 139, 140 }

F normal fail { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 31, 32, 33, 36, 37, 53, 54, 55, 56, 57, 58, 59, 60, 63, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 137, 138, 141, 142 }

F(-1) timedout fail { 68, 102, 133 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	110	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.324	0.057	0.000	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	184	154	143	813	171	1681	1275	190
N.S.	1	0.92	0.77	0.72	4.09	0.86	8.45	6.41	0.95
time (sec)	N/A	0.356	0.569	1.353	0.267	0.253	6.297	0.322	28.751

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	86	95	356	89	741	915	95
N.S.	1	1.00	0.67	0.74	2.78	0.70	5.79	7.15	0.74
time (sec)	N/A	0.276	0.145	0.770	0.217	0.246	1.065	0.323	29.591

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	194	49	347	634	50
N.S.	1	1.00	0.66	0.67	2.66	0.67	4.75	8.68	0.68
time (sec)	N/A	0.195	0.084	0.363	0.215	0.245	0.581	0.286	28.202

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	114	0	0	0	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	2.469	0.000	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	101	0	0	0	0	0	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	1.308	0.000	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	152	334	0	0	0	0	0	0
N.S.	1	1.11	2.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	8.174	0.000	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	155	173	0	0	0	0	0	0
N.S.	1	1.10	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	2.176	0.000	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	59	33	31	37	36	70	35	41
N.S.	1	1.09	0.61	0.57	0.69	0.67	1.30	0.65	0.76
time (sec)	N/A	0.243	0.041	0.315	0.241	0.240	0.482	0.261	26.381

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	110	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	0.050	0.000	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	183	155	144	813	142	1671	1271	191
N.S.	1	0.92	0.78	0.72	4.09	0.71	8.40	6.39	0.96
time (sec)	N/A	0.355	0.486	1.250	0.246	0.265	6.231	0.322	28.714

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	85	94	356	78	741	915	98
N.S.	1	1.00	0.66	0.73	2.78	0.61	5.79	7.15	0.77
time (sec)	N/A	0.279	0.149	0.851	0.227	0.240	1.082	0.304	28.030

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	47	48	192	48	286	631	48
N.S.	1	1.00	0.65	0.67	2.67	0.67	3.97	8.76	0.67
time (sec)	N/A	0.201	0.076	0.375	0.213	0.246	0.545	0.303	27.928

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	0.018	0.000	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	154	112	0	0	0	0	0	0
N.S.	1	1.09	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.307	0.219	0.000	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	157	111	0	0	0	0	0	0
N.S.	1	1.10	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.310	0.195	0.000	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	59	33	31	37	36	70	35	41
N.S.	1	1.09	0.61	0.57	0.69	0.67	1.30	0.65	0.76
time (sec)	N/A	0.240	0.057	0.348	0.217	0.235	0.481	0.262	27.113

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	190	212	0	0	0	0	0	0
N.S.	1	0.98	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.422	1.619	0.000	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	175	0	0	0	0	0	0
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.330	1.146	0.000	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	166	0	0	0	0	0	0
N.S.	1	1.00	2.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.257	0.333	0.000	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	163	0	0	0	0	0	0
N.S.	1	1.00	2.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	1.212	0.000	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	126	171	0	0	0	0	0	0
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	1.211	0.000	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	188	184	210	0	0	0	0	0	0
N.S.	1	0.98	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.403	1.655	0.000	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	78	133	0	0	0	0	0	0
N.S.	1	1.03	1.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.309	0.441	0.000	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	126	102	0	0	0	0	0	0
N.S.	1	1.26	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	0.102	0.000	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	128	102	0	0	0	0	0	0
N.S.	1	1.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.359	0.109	0.000	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	0	143	0	0	130	0	0	0
N.S.	1	0.00	1.03	0.00	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.000	0.588	0.000	0.000	0.096	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	23	25	26	20	25	25
N.S.	1	1.00	1.10	1.10	1.19	1.24	0.95	1.19	1.19
time (sec)	N/A	0.658	16.773	0.162	0.511	0.239	2.979	0.288	27.463

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	31	22	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.35	0.96	1.09	1.09
time (sec)	N/A	0.826	14.701	0.214	0.634	0.240	2.834	0.295	28.569

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	26	28	32	26	0	6346	27
N.S.	1	1.00	1.08	1.17	1.33	1.08	0.00	264.42	1.12
time (sec)	N/A	2.961	0.443	5.932	0.502	0.257	0.000	0.610	29.600

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	0	24	24	30	24	0	4746	23
N.S.	1	0.00	1.04	1.04	1.30	1.04	0.00	206.35	1.00
time (sec)	N/A	0.000	0.352	5.873	0.475	0.251	0.000	0.530	28.992

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	22	27	22	0	0	21
N.S.	1	1.00	1.00	1.00	1.23	1.00	0.00	0.00	0.95
time (sec)	N/A	2.039	0.296	5.868	0.475	0.251	0.000	0.000	27.977

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	B	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	327	18	18	1382	18	17	1941	17
N.S.	1	19.24	1.06	1.06	81.29	1.06	1.00	114.18	1.00
time (sec)	N/A	0.948	0.137	0.918	0.327	0.254	0.317	0.351	27.257

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	392	17	15	639	16
N.S.	1	1.00	1.00	1.06	24.50	1.06	0.94	39.94	1.00
time (sec)	N/A	0.168	0.026	0.725	0.269	0.254	0.147	0.314	26.961

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	20	564	20	0	0	19
N.S.	1	1.00	1.00	1.00	28.20	1.00	0.00	0.00	0.95
time (sec)	N/A	1.512	0.234	0.820	0.756	0.244	0.000	0.000	27.720

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	20	1069	20	0	0	19
N.S.	1	1.00	1.00	1.00	53.45	1.00	0.00	0.00	0.95
time (sec)	N/A	1.701	0.219	0.809	1.082	0.246	0.000	0.000	27.838

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	64	44	45	44	56	325	55	46
N.S.	1	1.02	0.70	0.71	0.70	0.89	5.16	0.87	0.73
time (sec)	N/A	0.244	0.116	0.529	0.215	0.249	0.768	0.260	27.145

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	74	96	538	109	1030	98	166
N.S.	1	1.00	0.62	0.81	4.52	0.92	8.66	0.82	1.39
time (sec)	N/A	0.283	0.548	0.716	0.240	0.253	2.510	0.271	27.344

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	82	108	550	135	1353	111	178
N.S.	1	1.00	0.64	0.84	4.26	1.05	10.49	0.86	1.38
time (sec)	N/A	0.292	0.649	0.919	0.222	0.252	7.290	0.272	28.122

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	74	99	538	98	1040	100	167
N.S.	1	1.00	0.62	0.83	4.52	0.82	8.74	0.84	1.40
time (sec)	N/A	0.285	0.494	0.653	0.229	0.255	2.494	0.274	28.840

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	57	60	236	90	850	66	58
N.S.	1	1.00	0.72	0.76	2.99	1.14	10.76	0.84	0.73
time (sec)	N/A	0.255	0.298	0.729	0.212	0.239	5.577	0.270	0.488

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	110	166	1148	201	2958	155	255
N.S.	1	1.00	0.60	0.91	6.27	1.10	16.16	0.85	1.39
time (sec)	N/A	0.339	0.754	1.207	0.265	0.251	25.823	0.289	28.581

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	81	107	550	114	1357	111	179
N.S.	1	1.00	0.63	0.83	4.26	0.88	10.52	0.86	1.39
time (sec)	N/A	0.285	0.552	0.832	0.232	0.245	7.378	0.273	1.092

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	110	166	1144	200	2751	152	255
N.S.	1	1.00	0.60	0.91	6.25	1.09	15.03	0.83	1.39
time (sec)	N/A	0.334	0.744	1.174	0.250	0.253	25.944	0.274	28.849

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	111	108	550	156	1991	111	178
N.S.	1	1.00	0.86	0.84	4.26	1.21	15.43	0.86	1.38
time (sec)	N/A	0.302	0.786	1.683	0.220	0.261	71.619	0.277	1.255

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	19	17	17	17	27	16	16
N.S.	1	1.00	0.63	0.57	0.57	0.57	0.90	0.53	0.53
time (sec)	N/A	0.200	0.031	0.239	0.193	0.259	0.145	0.262	26.392

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	25	23	26	26	48	25	21
N.S.	1	1.00	0.50	0.46	0.52	0.52	0.96	0.50	0.42
time (sec)	N/A	0.290	0.035	0.270	0.182	0.232	0.260	0.265	27.137

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	18	16	17	17	27	15	17
N.S.	1	1.00	0.60	0.53	0.57	0.57	0.90	0.50	0.57
time (sec)	N/A	0.204	0.024	0.246	0.196	0.257	0.144	0.269	27.901

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	23	21	26	26	48	24	22
N.S.	1	1.00	0.45	0.41	0.51	0.51	0.94	0.47	0.43
time (sec)	N/A	0.285	0.027	0.306	0.198	0.238	0.274	0.274	0.085

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	39	21	27	39	19
N.S.	1	1.00	0.81	0.74	1.44	0.78	1.00	1.44	0.70
time (sec)	N/A	0.244	0.167	0.544	0.186	0.241	0.096	0.273	0.073

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	33	28	23	26	32	23	31
N.S.	1	1.00	0.80	0.68	0.56	0.63	0.78	0.56	0.76
time (sec)	N/A	0.171	0.056	0.387	0.201	0.238	0.175	0.271	27.973

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	81	64	0	0	0	0	0	0
N.S.	1	0.99	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.342	3.103	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	79	64	0	0	0	0	0	0
N.S.	1	0.96	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.330	3.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	77	68	0	0	0	0	0	0
N.S.	1	0.96	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.316	0.695	0.000	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	75	66	0	0	0	0	0	0
N.S.	1	0.96	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.322	0.564	0.000	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	43	42	37	30	0	0	0
N.S.	1	1.00	0.62	0.61	0.54	0.43	0.00	0.00	0.00
time (sec)	N/A	0.220	0.025	0.197	0.184	0.246	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	47	44	38	32	0	0	0
N.S.	1	1.00	0.72	0.68	0.58	0.49	0.00	0.00	0.00
time (sec)	N/A	0.217	0.023	0.209	0.201	0.241	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	52	51	45	0	0	0
N.S.	1	1.00	1.00	0.64	0.63	0.56	0.00	0.00	0.00
time (sec)	N/A	0.242	0.072	0.257	0.240	0.251	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	82	54	52	46	0	0	0
N.S.	1	1.00	1.06	0.70	0.68	0.60	0.00	0.00	0.00
time (sec)	N/A	0.228	0.074	0.283	0.190	0.244	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	39	24	22	29	29	29	21	21
N.S.	1	1.11	0.69	0.63	0.83	0.83	0.83	0.60	0.60
time (sec)	N/A	0.255	0.050	0.341	0.197	0.235	0.928	0.279	0.110

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	5	5	5
N.S.	1	1.00	1.00	1.00	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.170	0.013	0.176	0.198	0.227	0.119	0.261	27.651

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	21	5	19	21	0	17	43
N.S.	1	1.00	4.20	1.00	3.80	4.20	0.00	3.40	8.60
time (sec)	N/A	0.185	0.022	0.530	0.190	0.257	0.000	0.269	0.439

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	3	3	3	3
N.S.	1	1.00	1.00	1.00	0.75	0.75	0.75	0.75	0.75
time (sec)	N/A	0.165	0.012	0.184	0.186	0.236	0.102	0.267	27.695

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	7	7	7
N.S.	1	1.00	1.00	0.80	0.70	0.70	0.70	0.70	0.70
time (sec)	N/A	0.177	0.012	0.188	0.186	0.235	0.102	0.268	0.066

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	10	7	7
N.S.	1	1.00	1.00	0.80	0.70	0.70	1.00	0.70	0.70
time (sec)	N/A	0.176	0.011	1.501	0.192	0.234	0.187	0.266	0.048

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	11	10	10	12	10	10
N.S.	1	1.00	1.00	0.85	0.77	0.77	0.92	0.77	0.77
time (sec)	N/A	0.218	0.016	1.444	0.190	0.239	2.924	0.254	27.234

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	24	42	0	24	24
N.S.	1	1.13	1.00	0.83	0.80	1.40	0.00	0.80	0.80
time (sec)	N/A	0.217	0.108	0.877	0.186	0.251	0.000	0.269	0.491

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	4	12	10	7	10
N.S.	1	1.00	1.00	1.00	0.57	1.71	1.43	1.00	1.43
time (sec)	N/A	0.180	0.012	0.084	0.200	0.242	0.101	0.261	27.551

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	9	8	19	10	29	10
N.S.	1	1.00	1.00	1.80	1.60	3.80	2.00	5.80	2.00
time (sec)	N/A	0.176	0.006	0.185	0.205	0.240	0.478	0.272	27.972

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	5	5	3	5	5
N.S.	1	1.00	1.00	1.00	1.25	1.25	0.75	1.25	1.25
time (sec)	N/A	0.185	0.010	0.191	0.208	0.229	0.167	0.256	0.100

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	7	10	5	7	13
N.S.	1	1.00	1.00	1.00	1.17	1.67	0.83	1.17	2.17
time (sec)	N/A	0.189	0.017	0.858	0.226	0.250	0.400	0.263	28.306

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	27	27	28	30	114	35	26
N.S.	1	1.00	0.73	0.73	0.76	0.81	3.08	0.95	0.70
time (sec)	N/A	0.179	0.053	0.349	0.207	0.249	0.246	0.254	0.128

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	108	88	100	260	0	127	0
N.S.	1	1.00	0.94	0.77	0.87	2.26	0.00	1.10	0.00
time (sec)	N/A	0.286	0.147	0.322	0.225	0.243	0.000	0.279	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	134	119	131	229	0	147	0
N.S.	1	1.00	0.93	0.83	0.91	1.59	0.00	1.02	0.00
time (sec)	N/A	0.377	0.187	0.355	0.213	0.250	0.000	0.272	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	52	51	45	0	0	0
N.S.	1	1.00	1.00	0.64	0.63	0.56	0.00	0.00	0.00
time (sec)	N/A	0.232	0.007	0.175	0.208	0.243	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	129	62	137	82	0	0	0
N.S.	1	1.00	1.48	0.71	1.57	0.94	0.00	0.00	0.00
time (sec)	N/A	0.263	0.197	0.319	0.218	0.256	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	165	129	475	161	0	0	0
N.S.	1	1.00	1.06	0.83	3.06	1.04	0.00	0.00	0.00
time (sec)	N/A	0.370	0.444	0.356	0.242	0.258	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	132	116	147	265	0	300	0
N.S.	1	1.00	0.93	0.82	1.04	1.87	0.00	2.11	0.00
time (sec)	N/A	0.371	0.187	0.425	0.232	0.251	0.000	0.338	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	156	139	186	270	0	521	0
N.S.	1	1.00	0.99	0.89	1.18	1.72	0.00	3.32	0.00
time (sec)	N/A	0.367	0.875	0.808	0.300	0.247	0.000	0.323	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	268	239	302	525	0	595	0
N.S.	1	1.00	0.90	0.80	1.01	1.76	0.00	2.00	0.00
time (sec)	N/A	0.544	0.794	1.342	0.308	0.258	0.000	0.386	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	162	156	190	313	0	378	0
N.S.	1	1.00	1.00	0.96	1.17	1.93	0.00	2.33	0.00
time (sec)	N/A	0.474	0.394	0.445	0.241	0.250	0.000	0.338	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	244	179	240	326	0	599	0
N.S.	1	1.00	1.36	1.00	1.34	1.82	0.00	3.35	0.00
time (sec)	N/A	0.483	0.887	0.904	0.331	0.255	0.000	0.366	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	323	319	377	629	0	751	0
N.S.	1	1.00	0.95	0.94	1.11	1.85	0.00	2.21	0.00
time (sec)	N/A	0.794	1.141	1.612	0.328	0.262	0.000	0.457	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	119	123	206	144	0	0	0
N.S.	1	1.00	0.79	0.81	1.36	0.95	0.00	0.00	0.00
time (sec)	N/A	0.372	0.120	0.378	0.237	0.239	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	132	145	236	161	0	0	0
N.S.	1	1.00	0.77	0.85	1.38	0.94	0.00	0.00	0.00
time (sec)	N/A	0.397	0.195	0.596	0.239	0.273	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	224	246	412	282	0	0	0
N.S.	1	1.00	0.74	0.82	1.37	0.94	0.00	0.00	0.00
time (sec)	N/A	0.530	0.338	1.298	0.258	0.257	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	170	84	209	107	0	0	0
N.S.	1	1.00	1.59	0.79	1.95	1.00	0.00	0.00	0.00
time (sec)	N/A	0.358	0.373	0.392	0.241	0.247	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	188	107	315	169	0	0	0
N.S.	1	1.00	1.34	0.76	2.25	1.21	0.00	0.00	0.00
time (sec)	N/A	0.398	0.626	0.590	0.218	0.260	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	386	166	661	317	0	0	0
N.S.	1	1.00	1.81	0.78	3.10	1.49	0.00	0.00	0.00
time (sec)	N/A	0.540	1.770	1.241	0.237	0.271	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	216	169	760	299	0	0	0
N.S.	1	1.00	1.16	0.90	4.06	1.60	0.00	0.00	0.00
time (sec)	N/A	0.544	0.720	0.428	0.231	0.265	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	251	191	863	363	0	0	0
N.S.	1	1.00	1.19	0.91	4.09	1.72	0.00	0.00	0.00
time (sec)	N/A	0.583	1.763	0.694	0.246	0.262	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	490	338	2175	713	0	0	0
N.S.	1	1.00	1.30	0.90	5.77	1.89	0.00	0.00	0.00
time (sec)	N/A	0.854	5.136	1.587	0.282	0.290	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	155	172	354	178	0	0	0
N.S.	1	1.00	0.88	0.98	2.01	1.01	0.00	0.00	0.00
time (sec)	N/A	0.487	0.271	0.359	0.248	0.251	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	204	218	399	224	0	0	0
N.S.	1	1.00	0.88	0.94	1.73	0.97	0.00	0.00	0.00
time (sec)	N/A	0.539	0.482	0.609	0.254	0.265	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	391	344	680	346	0	0	0
N.S.	1	1.00	1.10	0.97	1.92	0.98	0.00	0.00	0.00
time (sec)	N/A	0.681	0.716	1.381	0.273	0.260	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	230	180	647	309	0	0	0
N.S.	1	1.00	1.19	0.93	3.35	1.60	0.00	0.00	0.00
time (sec)	N/A	0.549	0.755	0.439	0.234	0.269	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	299	227	997	402	0	0	0
N.S.	1	1.00	1.22	0.93	4.07	1.64	0.00	0.00	0.00
time (sec)	N/A	0.625	2.235	0.658	0.244	0.265	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	3291	358	2451	731	0	0	0
N.S.	1	1.00	8.53	0.93	6.35	1.89	0.00	0.00	0.00
time (sec)	N/A	0.822	6.726	1.467	0.308	0.288	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	347	216	1007	375	0	0	0
N.S.	1	1.00	1.64	1.02	4.75	1.77	0.00	0.00	0.00
time (sec)	N/A	0.717	2.019	0.455	0.251	0.260	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	C	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	1120	263	1487	470	0	0	0
N.S.	1	1.00	4.18	0.98	5.55	1.75	0.00	0.00	0.00
time (sec)	N/A	0.791	6.535	0.771	0.271	0.265	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	430	430	3835	430	4343	865	0	0	0
N.S.	1	1.00	8.92	1.00	10.10	2.01	0.00	0.00	0.00
time (sec)	N/A	1.126	7.203	1.829	0.319	0.293	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	324	217	1054	379	0	0	0
N.S.	1	1.00	1.52	1.02	4.95	1.78	0.00	0.00	0.00
time (sec)	N/A	0.859	1.607	0.564	0.267	0.261	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	26	26	25	28	114	33	25
N.S.	1	1.00	0.72	0.72	0.69	0.78	3.17	0.92	0.69
time (sec)	N/A	0.175	0.060	0.313	0.216	0.228	0.235	0.252	0.113

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	109	86	100	260	0	127	0
N.S.	1	1.00	0.95	0.75	0.87	2.26	0.00	1.10	0.00
time (sec)	N/A	0.270	0.153	0.307	0.220	0.262	0.000	0.281	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	135	117	131	229	0	147	0
N.S.	1	1.00	0.94	0.81	0.91	1.59	0.00	1.02	0.00
time (sec)	N/A	0.359	0.184	0.342	0.239	0.262	0.000	0.285	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	82	54	52	46	0	0	0
N.S.	1	1.00	1.06	0.70	0.68	0.60	0.00	0.00	0.00
time (sec)	N/A	0.235	0.057	0.207	0.229	0.246	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	107	60	133	84	0	0	0
N.S.	1	1.00	1.29	0.72	1.60	1.01	0.00	0.00	0.00
time (sec)	N/A	0.254	0.177	0.319	0.220	0.248	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	166	127	474	164	0	0	0
N.S.	1	1.00	1.10	0.84	3.14	1.09	0.00	0.00	0.00
time (sec)	N/A	0.355	0.423	0.386	0.219	0.250	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	133	114	147	265	0	300	0
N.S.	1	1.00	0.94	0.80	1.04	1.87	0.00	2.11	0.00
time (sec)	N/A	0.357	0.186	0.463	0.220	0.257	0.000	0.320	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	158	139	186	270	0	521	0
N.S.	1	1.00	1.01	0.89	1.18	1.72	0.00	3.32	0.00
time (sec)	N/A	0.355	0.790	0.804	0.307	0.244	0.000	0.336	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	267	235	302	525	0	595	0
N.S.	1	1.00	0.90	0.79	1.01	1.76	0.00	2.00	0.00
time (sec)	N/A	0.535	0.657	1.548	0.327	0.265	0.000	0.377	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	163	154	190	313	0	378	0
N.S.	1	1.00	1.01	0.95	1.17	1.93	0.00	2.33	0.00
time (sec)	N/A	0.444	0.273	0.473	0.229	0.255	0.000	0.343	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	245	179	240	326	0	599	0
N.S.	1	1.00	1.37	1.00	1.34	1.82	0.00	3.35	0.00
time (sec)	N/A	0.453	0.807	0.983	0.318	0.257	0.000	0.367	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	322	315	377	629	0	751	0
N.S.	1	1.00	0.95	0.93	1.11	1.85	0.00	2.21	0.00
time (sec)	N/A	0.777	1.105	1.822	0.342	0.268	0.000	0.488	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	116	121	204	142	0	0	0
N.S.	1	1.00	0.79	0.82	1.39	0.97	0.00	0.00	0.00
time (sec)	N/A	0.369	0.122	0.413	0.238	0.253	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	131	145	236	159	0	0	0
N.S.	1	1.00	0.77	0.85	1.38	0.93	0.00	0.00	0.00
time (sec)	N/A	0.380	0.193	0.650	0.237	0.252	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	218	242	406	280	0	0	0
N.S.	1	1.00	0.74	0.83	1.39	0.96	0.00	0.00	0.00
time (sec)	N/A	0.512	0.343	1.514	0.264	0.259	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	170	82	205	109	0	0	0
N.S.	1	1.00	1.65	0.80	1.99	1.06	0.00	0.00	0.00
time (sec)	N/A	0.352	0.363	0.406	0.254	0.260	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	189	107	315	167	0	0	0
N.S.	1	1.00	1.35	0.76	2.25	1.19	0.00	0.00	0.00
time (sec)	N/A	0.378	0.624	0.641	0.239	0.245	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	389	162	667	311	0	0	0
N.S.	1	1.00	1.90	0.79	3.25	1.52	0.00	0.00	0.00
time (sec)	N/A	0.513	1.648	1.349	0.254	0.261	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	217	167	761	301	0	0	0
N.S.	1	1.00	1.19	0.91	4.16	1.64	0.00	0.00	0.00
time (sec)	N/A	0.530	0.682	0.457	0.250	0.263	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	252	191	863	361	0	0	0
N.S.	1	1.00	1.19	0.91	4.09	1.71	0.00	0.00	0.00
time (sec)	N/A	0.576	1.674	0.823	0.247	0.266	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	492	334	2180	707	0	0	0
N.S.	1	1.00	1.33	0.91	5.91	1.92	0.00	0.00	0.00
time (sec)	N/A	0.817	4.965	1.894	0.302	0.309	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	151	170	354	176	0	0	0
N.S.	1	1.00	0.88	0.99	2.06	1.02	0.00	0.00	0.00
time (sec)	N/A	0.466	0.236	0.484	0.252	0.248	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	204	218	399	224	0	0	0
N.S.	1	1.00	0.88	0.94	1.73	0.97	0.00	0.00	0.00
time (sec)	N/A	0.527	0.417	0.739	0.256	0.249	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	386	340	680	344	0	0	0
N.S.	1	1.00	1.12	0.98	1.97	0.99	0.00	0.00	0.00
time (sec)	N/A	0.647	0.666	1.644	0.288	0.246	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	231	178	648	311	0	0	0
N.S.	1	1.00	1.22	0.94	3.43	1.65	0.00	0.00	0.00
time (sec)	N/A	0.512	0.733	0.431	0.252	0.264	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	301	227	997	402	0	0	0
N.S.	1	1.00	1.23	0.93	4.07	1.64	0.00	0.00	0.00
time (sec)	N/A	0.593	2.280	0.845	0.274	0.272	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	3285	354	2456	725	0	0	0
N.S.	1	1.00	8.69	0.94	6.50	1.92	0.00	0.00	0.00
time (sec)	N/A	0.797	7.579	1.704	0.280	0.286	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	348	214	1008	377	0	0	0
N.S.	1	1.00	1.67	1.03	4.85	1.81	0.00	0.00	0.00
time (sec)	N/A	0.599	1.749	0.492	0.251	0.261	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	C	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	1118	263	1487	470	0	0	0
N.S.	1	1.00	4.17	0.98	5.55	1.75	0.00	0.00	0.00
time (sec)	N/A	0.780	6.585	0.902	0.278	0.264	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	422	422	3829	426	4348	859	0	0	0
N.S.	1	1.00	9.07	1.01	10.30	2.04	0.00	0.00	0.00
time (sec)	N/A	1.109	6.983	2.165	0.305	0.304	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	325	215	1058	381	0	0	0
N.S.	1	1.00	1.56	1.03	5.06	1.82	0.00	0.00	0.00
time (sec)	N/A	0.756	1.643	0.562	0.272	0.274	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	231	180	194	581	256	2465	1738	248
N.S.	1	0.94	0.73	0.79	2.37	1.04	10.06	7.09	1.01
time (sec)	N/A	0.580	7.276	1.641	0.258	0.279	1.749	0.368	27.566

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	98	83	84	218	83	542	923	84
N.S.	1	0.99	0.84	0.85	2.20	0.84	5.47	9.32	0.85
time (sec)	N/A	0.381	0.840	0.566	0.216	0.239	0.548	0.311	25.804

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	128	0	0	0	0	0	0
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	3.872	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	184	197	240	0	0	0	0	0	0
N.S.	1	1.07	1.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.396	3.211	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	231	228	193	578	242	2394	1736	247
N.S.	1	0.94	0.93	0.79	2.36	0.99	9.77	7.09	1.01
time (sec)	N/A	0.551	0.706	1.438	0.242	0.259	1.970	0.363	27.284

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	97	82	83	216	80	478	920	83
N.S.	1	0.99	0.84	0.85	2.20	0.82	4.88	9.39	0.85
time (sec)	N/A	0.374	0.517	0.560	0.223	0.243	0.553	0.310	26.702

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	80	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	0.060	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	181	145	0	0	0	0	0	0
N.S.	1	1.07	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.385	0.317	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [67] had the largest ratio of [.599999999999999978]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	18	0.111
2	A	2	2	0.92	18	0.111
3	A	2	2	1.00	18	0.111
4	A	1	1	1.00	16	0.062
5	A	1	1	1.00	16	0.062
6	A	1	1	1.00	18	0.056
7	A	2	2	1.11	18	0.111
8	A	2	2	1.10	18	0.111
9	A	3	3	1.09	8	0.375
10	A	2	2	1.00	18	0.111
11	A	2	2	0.92	18	0.111
12	A	2	2	1.00	18	0.111
13	A	1	1	1.00	16	0.062
14	A	1	1	1.00	16	0.062
15	A	1	1	1.00	18	0.056
16	A	2	2	1.09	18	0.111
17	A	2	2	1.10	18	0.111
18	A	3	3	1.09	8	0.375
19	A	2	2	0.98	18	0.111
20	A	2	2	1.00	18	0.111
21	A	2	2	1.00	16	0.125
22	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	2	2	1.00	18	0.111
24	A	2	2	0.98	18	0.111
25	A	5	5	1.03	27	0.185
26	A	2	2	1.26	18	0.111
27	A	2	2	1.25	18	0.111
28	F	0	0	N/A	0.000	N/A
29	N/A	2	0	1.00	21	0.000
30	N/A	2	0	1.00	23	0.000
31	A	4	4	1.00	44	0.091
32	F	0	0	N/A	0.000	N/A
33	A	4	4	1.00	43	0.093
34	B	3	3	19.24	35	0.086
35	A	1	1	1.00	30	0.033
36	A	3	3	1.00	38	0.079
37	A	3	3	1.00	38	0.079
38	A	3	3	1.02	20	0.150
39	A	2	2	1.00	22	0.091
40	A	2	2	1.00	22	0.091
41	A	2	2	1.00	22	0.091
42	A	2	2	1.00	24	0.083
43	A	2	2	1.00	24	0.083
44	A	2	2	1.00	22	0.091
45	A	2	2	1.00	24	0.083
46	A	2	2	1.00	24	0.083
47	A	2	2	1.00	7	0.286
48	A	4	4	1.00	9	0.444
49	A	2	2	1.00	7	0.286
50	A	4	4	1.00	9	0.444
51	A	2	2	1.00	19	0.105
52	A	1	1	1.00	15	0.067
53	A	3	3	0.99	26	0.115
54	A	4	4	0.96	27	0.148
55	A	3	3	0.96	26	0.115

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	3	3	0.96	27	0.111
57	A	2	2	1.00	10	0.200
58	A	2	2	1.00	10	0.200
59	A	2	2	1.00	12	0.167
60	A	2	2	1.00	12	0.167
61	A	3	2	1.11	15	0.133
62	A	4	3	1.00	8	0.375
63	A	5	4	1.00	12	0.333
64	A	4	3	1.00	8	0.375
65	A	4	3	1.00	12	0.250
66	A	4	3	1.00	12	0.250
67	A	7	6	1.00	10	0.600
68	A	4	3	1.13	26	0.115
69	A	4	3	1.00	8	0.375
70	A	4	3	1.00	8	0.375
71	A	5	4	1.00	12	0.333
72	A	5	4	1.00	10	0.400
73	A	1	1	1.00	10	0.100
74	A	2	2	1.00	12	0.167
75	A	2	2	1.00	15	0.133
76	A	2	2	1.00	12	0.167
77	A	2	2	1.00	14	0.143
78	A	2	2	1.00	17	0.118
79	A	2	2	1.00	16	0.125
80	A	2	2	1.00	18	0.111
81	A	2	2	1.00	18	0.111
82	A	2	2	1.00	19	0.105
83	A	2	2	1.00	21	0.095
84	A	2	2	1.00	21	0.095
85	A	2	2	1.00	16	0.125
86	A	2	2	1.00	18	0.111
87	A	2	2	1.00	18	0.111
88	A	2	2	1.00	18	0.111
89	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
90	A	2	2	1.00	20	0.100
91	A	2	2	1.00	21	0.095
92	A	2	2	1.00	23	0.087
93	A	2	2	1.00	23	0.087
94	A	2	2	1.00	19	0.105
95	A	2	2	1.00	21	0.095
96	A	2	2	1.00	21	0.095
97	A	2	2	1.00	21	0.095
98	A	2	2	1.00	23	0.087
99	A	2	2	1.00	23	0.087
100	A	2	2	1.00	24	0.083
101	A	2	2	1.00	26	0.077
102	A	2	2	1.00	26	0.077
103	A	2	2	1.00	24	0.083
104	A	1	1	1.00	10	0.100
105	A	2	2	1.00	12	0.167
106	A	2	2	1.00	15	0.133
107	A	2	2	1.00	12	0.167
108	A	2	2	1.00	14	0.143
109	A	2	2	1.00	17	0.118
110	A	2	2	1.00	16	0.125
111	A	2	2	1.00	18	0.111
112	A	2	2	1.00	18	0.111
113	A	2	2	1.00	19	0.105
114	A	2	2	1.00	21	0.095
115	A	2	2	1.00	21	0.095
116	A	2	2	1.00	16	0.125
117	A	2	2	1.00	18	0.111
118	A	2	2	1.00	18	0.111
119	A	2	2	1.00	18	0.111
120	A	2	2	1.00	20	0.100
121	A	2	2	1.00	20	0.100
122	A	2	2	1.00	21	0.095
123	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
124	A	2	2	1.00	23	0.087
125	A	2	2	1.00	19	0.105
126	A	2	2	1.00	21	0.095
127	A	2	2	1.00	21	0.095
128	A	2	2	1.00	21	0.095
129	A	2	2	1.00	23	0.087
130	A	2	2	1.00	23	0.087
131	A	2	2	1.00	24	0.083
132	A	2	2	1.00	26	0.077
133	A	2	2	1.00	26	0.077
134	A	2	2	1.00	24	0.083
135	A	4	4	0.94	22	0.182
136	A	4	4	0.99	20	0.200
137	A	2	2	1.00	22	0.091
138	A	3	3	1.07	22	0.136
139	A	4	4	0.94	22	0.182
140	A	4	4	0.99	20	0.200
141	A	2	2	1.00	22	0.091
142	A	3	3	1.07	22	0.136

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int F^{c(a+bx)} \sin^n(d+ex)\,dx$

3.1.1 Optimal result

Integrand size = 18, antiderivative size = 107

$$\int F^{c(a+bx)} \sin^n(d+ex)\,dx =$$
$$-\frac{\left(1-e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)} \text{Hypergeometric2F1}\left(-n,-\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(2-n-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)\sin^n(}{ien-bc\log(F)}$$

output
```
-F^(c*(b*x+a))*hypergeom([-n, 1/2*(-e*n-I*b*c*ln(F))/e],[1-1/2*n-1/2*I*b*c
*ln(F)/e],exp(2*I*(e*x+d)))*sin(e*x+d)^n/((1-exp(2*I*(e*x+d)))^n)/(I*e*n-b
*c*ln(F))
```

3.1.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)} \sin^n(d+ex)\,dx$$
$$=\frac{\left(1-e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)} \text{Hypergeometric2F1}\left(-n,-\frac{i(-ien+bc\log(F))}{2e},1-\frac{i(-ien+bc\log(F))}{2e},e^{2i(d+ex)}\right)\sin^n(d+}{-ien+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sin[d + e*x]^n,x]
```

output
```
(F^(c*(a + b*x))*Hypergeometric2F1[-n, ((-1/2*I)*((-I)*e*n + b*c*Log[F]))/
e, 1 - ((I/2)*((-I)*e*n + b*c*Log[F]))/e, E^((2*I)*(d + e*x))]*Sin[d + e*x
]^n)/((1 - E^((2*I)*(d + e*x)))^n*((-I)*e*n + b*c*Log[F]))
```

3.1.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4940, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)} \sin^n(d+ex)\,dx$$

$$\Big\downarrow \text{4940}$$

$$e^{in(d+ex)}\left(-1+e^{2i(d+ex)}\right)^{-n}\sin^n(d+ex)\int e^{-in(d+ex)}\left(-1+e^{2i(d+ex)}\right)^n F^{c(a+bx)}dx$$

$$\Big\downarrow \text{2689}$$

$$-\frac{\left(1-e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}\sin^n(d+ex)\,\text{Hypergeometric2F1}\left(-n,-\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(-n-\frac{ibc\log(F)}{e}+2\right),e^{2i(d+ex}\right.}{-bc\log(F)+ien}$$

input
```
Int[F^(c*(a + b*x))*Sin[d + e*x]^n,x]
```

output
```
-((F^(c*(a + b*x))*Hypergeometric2F1[-n, -1/2*(e*n + I*b*c*Log[F])/e, (2 -
 n - (I*b*c*Log[F])/e)/2, E^((2*I)*(d + e*x))]*Sin[d + e*x]^n)/((1 - E^((2
*I)*(d + e*x)))^n*(I*e*n - b*c*Log[F])))
```

3.1.3.1 Defintions of rubi rules used

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 4940
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[E^(I*n*(d + e*x))*(Sin[d + e*x]^n/(-1 + E^(2*I*(d + e*x)))^n)
Int[F^(c*(a + b*x))*((-1 + E^(2*I*(d + e*x)))^n/E^(I*n*(d + e*x))), x], x]
/; FreeQ[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

3.1.4 Maple [F]

$$\int F^{c(xb+a)} \sin(ex+d)^n \, dx$$

input
```
int(F^(c*(b*x+a))*sin(e*x+d)^n,x)
```

output
```
int(F^(c*(b*x+a))*sin(e*x+d)^n,x)
```

3.1.5 Fricas [F]

$$\int F^{c(a+bx)} \sin^n(d+ex) \, dx = \int F^{(bx+a)c} \sin(ex+d)^n \, dx$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^n,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sin(e*x + d)^n, x)
```

3.1.6 Sympy [F]

$$\int F^{c(a+bx)} \sin^n(d+ex)\,dx = \int F^{c(a+bx)} \sin^n{(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*sin(e*x+d)**n,x)
```

output
```
Integral(F**(c*(a + b*x))*sin(d + e*x)**n, x)
```

3.1.7 Maxima [F]

$$\int F^{c(a+bx)} \sin^n(d+ex)\,dx = \int F^{(bx+a)c} \sin{(ex+d)}^n\,dx$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^n,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)*c)*sin(e*x + d)^n, x)
```

3.1.8 Giac [F]

$$\int F^{c(a+bx)} \sin^n(d+ex)\,dx = \int F^{(bx+a)c} \sin{(ex+d)}^n\,dx$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^n,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sin(e*x + d)^n, x)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)} \sin^n(d+ex)\,dx = \int F^{c\,(a+b\,x)} \sin\left(d+e\,x\right)^n\,dx$$

input
```
int(F^(c*(a + b*x))*sin(d + e*x)^n,x)
```

output
```
int(F^(c*(a + b*x))*sin(d + e*x)^n, x)
```

3.2 $\int F^{c(a+bx)} \sin^3(d+ex)\,dx$

3.2.1 Optimal result

Integrand size = 18, antiderivative size = 199

$$\int F^{c(a+bx)} \sin^3(d+ex)\,dx = -\frac{6e^3F^{c(a+bx)}\cos(d+ex)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{6bce^2F^{c(a+bx)}\log(F)\sin(d+ex)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} - \frac{3eF^{c(a+bx)}\cos(d+ex)\sin^2(d+ex)}{9e^2+b^2c^2\log^2(F)} + \frac{bcF^{c(a+bx)}\log(F)\sin^3(d+ex)}{9e^2+b^2c^2\log^2(F)}$$

output
```
-6*e^3*F^(c*(b*x+a))*cos(e*x+d)/(9*e^4+10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F
)^4)+6*b*c*e^2*F^(c*(b*x+a))*ln(F)*sin(e*x+d)/(9*e^4+10*b^2*c^2*e^2*ln(F)^
2+b^4*c^4*ln(F)^4)-3*e*F^(c*(b*x+a))*cos(e*x+d)*sin(e*x+d)^2/(9*e^2+b^2*c^
2*ln(F)^2)+b*c*F^(c*(b*x+a))*ln(F)*sin(e*x+d)^3/(9*e^2+b^2*c^2*ln(F)^2)
```

3.2.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.77

$$\int F^{c(a+bx)}\sin^3(d+ex)\,dx$$
$$=\frac{F^{c(a+bx)}\left(-3e\cos(d+ex)\left(9e^2+b^2c^2\log^2(F)\right)+3\cos(3(d+ex))\left(e^3+b^2c^2e\log^2(F)\right)-2bc\log(F)\left(-13\right.\right.}{4\left(9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F\right.}$$

input
```
Integrate[F^(c*(a + b*x))*Sin[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(-3*e*Cos[d + e*x]*(9*e^2 + b^2*c^2*Log[F]^2) + 3*Cos[3*(
d + e*x)]*(e^3 + b^2*c^2*e*Log[F]^2) - 2*b*c*Log[F]*(-13*e^2 - b^2*c^2*Log
[F]^2 + Cos[2*(d + e*x)]*(e^2 + b^2*c^2*Log[F]^2))*Sin[d + e*x]))/(4*(9*e^
4 + 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

3.2.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4934, 4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \; 4934$$
$$\frac{6e^2\int F^{c(a+bx)}\sin(d+ex)dx}{b^2c^2\log^2(F)+9e^2}+\frac{bc\log(F)\sin^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}-\frac{3e\sin^2(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}$$
$$\Big\downarrow \; 4932$$
$$\frac{bc\log(F)\sin^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}-\frac{3e\sin^2(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}+\frac{6e^2\left(\frac{bc\log(F)\sin(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}-\frac{e\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}\right)}{b^2c^2\log^2(F)+9e^2}$$

input
```
Int[F^(c*(a + b*x))*Sin[d + e*x]^3,x]
```

output
```
(-3*e*F^(c*(a + b*x))*Cos[d + e*x]*Sin[d + e*x]^2)/(9*e^2 + b^2*c^2*Log[F]
^2) + (b*c*F^(c*(a + b*x))*Log[F]*Sin[d + e*x]^3)/(9*e^2 + b^2*c^2*Log[F]^
2) + (6*e^2*(-((e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2))
+ (b*c*F^(c*(a + b*x))*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)))/(9*
e^2 + b^2*c^2*Log[F]^2)
```

3.2.3.1 Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

3.2.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{3\left(\left(\ln(F)^2 b^2 c^2 e+e^3\right)\cos(3ex+3d)-\frac{bc\ln(F)\left(e^2+b^2c^2\ln(F)^2\right)\sin(3ex+3d)}{3}+\left(9e^2+b^2c^2\ln(F)^2\right)(bc\ln(F)\sin(ex+d)-e\cos(ex+\right.}{4b^4c^4\ln(F)^4+40b^2c^2e^2\ln(F)^2+36e^4}$
risch	$-\frac{3e\,F^{c(xb+a)}\cos(ex+d)}{4\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{3bc\,F^{c(xb+a)}\ln(F)\sin(ex+d)}{4\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{3e\,F^{c(xb+a)}\cos(3ex+3d)}{4\left(9e^2+b^2c^2\ln(F)^2\right)}-\frac{cb\ln(F)F^{c(xb+a)}\sin(3ex+3d)}{4\left(9e^2+b^2c^2\ln(F)^2\right)}$
norman	$-\frac{6e^3 e^{c(xb+a)\ln(F)}}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}+\frac{6e^3 e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^6}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}-\frac{6e\left(2b^2c^2\ln(F)^2+3e^2\right)e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}+\frac{6e(2b^2}{}$
default	$F^{ac}\left(\frac{-\frac{4e\,e^{bcx\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{4e\,e^{bcx\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}+\frac{8bc\ln(F)e^{bcx\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}+\frac{4e\left(b^2c^2\ln(F)^2+3e^2\right)e^{bcx\ln(F)}}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}-\frac{4e(11b^2c}{9e}\right.$

input
```
int(F^(c*(b*x+a))*sin(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
3*((ln(F)^2*b^2*c^2*e+e^3)*cos(3*e*x+3*d)-1/3*b*c*ln(F)*(e^2+b^2*c^2*ln(F)
^2)*sin(3*e*x+3*d)+(9*e^2+b^2*c^2*ln(F)^2)*(b*c*ln(F)*sin(e*x+d)-e*cos(e*x
+d)))*F^(c*(b*x+a))/(4*b^4*c^4*ln(F)^4+40*b^2*c^2*e^2*ln(F)^2+36*e^4)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.86

$$\int F^{c(a+bx)}\sin^3(d+ex)\,dx$$
$$=\frac{\left(3\,e^3\cos\left(ex+d\right)^3-9\,e^3\cos\left(ex+d\right)+3\,\left(b^2c^2e\cos\left(ex+d\right)^3-b^2c^2e\cos\left(ex+d\right)\right)\log\left(F\right)^2-\left(\left(b^3c^3\cos\left(\right.\right.\right.}{b^4c^4\log\left(F\right)^4+10\,b^2c^2e^2\log\left(\right.}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^3,x, algorithm="fricas")
```

output
```
(3*e^3*cos(e*x + d)^3 - 9*e^3*cos(e*x + d) + 3*(b^2*c^2*e*cos(e*x + d)^3 -
 b^2*c^2*e*cos(e*x + d))*log(F)^2 - ((b^3*c^3*cos(e*x + d)^2 - b^3*c^3)*lo
g(F)^3 + (b*c*e^2*cos(e*x + d)^2 - 7*b*c*e^2)*log(F))*sin(e*x + d))*F^(b*c
*x + a*c)/(b^4*c^4*log(F)^4 + 10*b^2*c^2*e^2*log(F)^2 + 9*e^4)
```

3.2.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.30 (sec) , antiderivative size = 1681, normalized size of antiderivative = 8.45

$$\int F^{c(a+bx)}\sin^3(d+ex)\,dx=\text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*sin(e*x+d)**3,x)
```

output
```
Piecewise((x*sin(d)**3, Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sin(d)**3, Eq(b,
 0) & Eq(e, 0)), (x*sin(d)**3, Eq(c, 0) & Eq(e, 0)), (-3*F**(a*c + b*c*x)*
x*sin(I*b*c*x*log(F) - d)**3/8 + 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)
 - d)**2*cos(I*b*c*x*log(F) - d)/8 - 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(
F) - d)*cos(I*b*c*x*log(F) - d)**2/8 + 3*I*F**(a*c + b*c*x)*x*cos(I*b*c*x*
log(F) - d)**3/8 + F**(a*c + b*c*x)*sin(I*b*c*x*log(F) - d)**3/(8*b*c*log(
F)) - 3*I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F) - d)**2*cos(I*b*c*x*log(F) -
 d)/(4*b*c*log(F)) - 3*I*F**(a*c + b*c*x)*cos(I*b*c*x*log(F) - d)**3/(8*b*
c*log(F)), Eq(e, -I*b*c*log(F))), (-F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/
3 - d)**3/8 + 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 - d)**2*cos(I*b*
c*x*log(F)/3 - d)/8 + 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 - d)*cos(I
*b*c*x*log(F)/3 - d)**2/8 - I*F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/3 - d)
**3/8 + F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)) - 3*I
*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 - d)**2*cos(I*b*c*x*log(F)/3 - d)/(
b*c*log(F)) - 15*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 - d)*cos(I*b*c*x*lo
g(F)/3 - d)**2/(4*b*c*log(F)) + 11*I*F**(a*c + b*c*x)*cos(I*b*c*x*log(F)/3
 - d)**3/(8*b*c*log(F)), Eq(e, -I*b*c*log(F)/3)), (F**(a*c + b*c*x)*x*sin(
I*b*c*x*log(F)/3 + d)**3/8 - 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 +
 d)**2*cos(I*b*c*x*log(F)/3 + d)/8 - 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(
F)/3 + d)*cos(I*b*c*x*log(F)/3 + d)**2/8 + I*F**(a*c + b*c*x)*x*cos(I*b...
```

3.2.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(199) = 398$.

Time = 0.27 (sec) , antiderivative size = 813, normalized size of antiderivative = 4.09

$$\int F^{c(a+bx)}\sin^3(d+ex)\,dx = -\frac{\left(F^{ac}b^3c^3\log(F)^3\sin(3d) - 3F^{ac}b^2c^2e\cos(3d)\log(F)^2 + F^{ac}bce^2\log(F)\sin(3d) - 3F^{ac}e^3\cos(3d)\right)\ldots}{}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^3,x, algorithm="maxima")
```

output
```
-1/8*((F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) - 3*F^(a*c)*b^2*c^2*e*cos(3*d)*lo
g(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(3*d) - 3*F^(a*c)*e^3*cos(3*d))*F^(b*c*
x)*cos(3*e*x) - (F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) + 3*F^(a*c)*b^2*c^2*e*c
os(3*d)*log(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(3*d) + 3*F^(a*c)*e^3*cos(3*d
))*F^(b*c*x)*cos(3*e*x + 6*d) + 3*(F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) + F^(
a*c)*b^2*c^2*e*cos(3*d)*log(F)^2 + 9*F^(a*c)*b*c*e^2*log(F)*sin(3*d) + 9*F
^(a*c)*e^3*cos(3*d))*F^(b*c*x)*cos(e*x + 4*d) - 3*(F^(a*c)*b^3*c^3*log(F)^
3*sin(3*d) - F^(a*c)*b^2*c^2*e*cos(3*d)*log(F)^2 + 9*F^(a*c)*b*c*e^2*log(F
)*sin(3*d) - 9*F^(a*c)*e^3*cos(3*d))*F^(b*c*x)*cos(e*x - 2*d) + (F^(a*c)*b
^3*c^3*cos(3*d)*log(F)^3 + 3*F^(a*c)*b^2*c^2*e*log(F)^2*sin(3*d) + F^(a*c)
*b*c*e^2*cos(3*d)*log(F) + 3*F^(a*c)*e^3*sin(3*d))*F^(b*c*x)*sin(3*e*x) +
(F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 - 3*F^(a*c)*b^2*c^2*e*log(F)^2*sin(3*d)
 + F^(a*c)*b*c*e^2*cos(3*d)*log(F) - 3*F^(a*c)*e^3*sin(3*d))*F^(b*c*x)*sin
(3*e*x + 6*d) - 3*(F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 - F^(a*c)*b^2*c^2*e*l
og(F)^2*sin(3*d) + 9*F^(a*c)*b*c*e^2*cos(3*d)*log(F) - 9*F^(a*c)*e^3*sin(3
*d))*F^(b*c*x)*sin(e*x + 4*d) - 3*(F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 + F^(
a*c)*b^2*c^2*e*log(F)^2*sin(3*d) + 9*F^(a*c)*b*c*e^2*cos(3*d)*log(F) + 9*F
^(a*c)*e^3*sin(3*d))*F^(b*c*x)*sin(e*x - 2*d))/(b^4*c^4*cos(3*d)^2*log(F)^
4 + b^4*c^4*log(F)^4*sin(3*d)^2 + 9*(cos(3*d)^2 + sin(3*d)^2)*e^4 + 10*(b^
2*c^2*cos(3*d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(3*d)^2)*e^2)
```

3.2.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1275, normalized size of antiderivative = 6.41

$$\int F^{c(a+bx)} \sin^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^3,x, algorithm="giac")
```

output
```
-1/4*(2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*
c*sgn(F) - 1/2*pi*a*c + 3*e*x + 3*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sg
n(F) - pi*b*c + 6*e)^2) - (pi*b*c*sgn(F) - pi*b*c + 6*e)*cos(1/2*pi*b*c*x*
sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 3*e*x + 3*d)/(4*b
^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 6*e)^2))*e^(b*c*x*log(abs
(F)) + a*c*log(abs(F))) + 3/4*(2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) -
 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(a
bs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c + 2
*e)*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*
c + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))
*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 3/4*(2*b*c*log(abs(F))*sin(1/2*
pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)
/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2) - (pi*b*c*sg
n(F) - pi*b*c - 2*e)*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*s
gn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) -
pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/4*(2*b*c*log
(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*
pi*a*c - 3*e*x - 3*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c -
 6*e)^2) - (pi*b*c*sgn(F) - pi*b*c - 6*e)*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi
*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 3*e*x - 3*d)/(4*b^2*c^2*log(a...
```

3.2.9 Mupad [B] (verification not implemented)

Time = 28.75 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.95

$$\int F^{c(a+bx)}\sin^3(d+ex)\,dx$$
$$= -\frac{3\,F^{c\,(a+b\,x)}\,\left(\cos\left(e\,x\right)-\sin\left(e\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(d\right)-\sin\left(d\right)\,1\mathrm{i}\right)}{8\,\left(e+b\,c\,\ln\left(F\right)\,1\mathrm{i}\right)}$$
$$+\frac{F^{c\,(a+b\,x)}\,\left(\cos\left(3\,e\,x\right)+\sin\left(3\,e\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(3\,d\right)+\sin\left(3\,d\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{8\,\left(b\,c\,\ln\left(F\right)+e\,3\mathrm{i}\right)}$$
$$+\frac{F^{c\,(a+b\,x)}\,\left(\cos\left(3\,e\,x\right)-\sin\left(3\,e\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(3\,d\right)-\sin\left(3\,d\right)\,1\mathrm{i}\right)}{8\,\left(3\,e+b\,c\,\ln\left(F\right)\,1\mathrm{i}\right)}$$
$$-\frac{F^{c\,(a+b\,x)}\,\left(\cos\left(e\,x\right)+\sin\left(e\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(d\right)+\sin\left(d\right)\,1\mathrm{i}\right)\,3\mathrm{i}}{8\,\left(b\,c\,\ln\left(F\right)+e\,1\mathrm{i}\right)}$$

input
```
int(F^(c*(a + b*x))*sin(d + e*x)^3,x)
```

output
```
(F^(c*(a + b*x))*(cos(3*e*x) + sin(3*e*x)*1i)*(cos(3*d) + sin(3*d)*1i)*1i)
/(8*(e*3i + b*c*log(F))) - (3*F^(c*(a + b*x))*(cos(e*x) - sin(e*x)*1i)*(co
s(d) - sin(d)*1i))/(8*(e + b*c*log(F)*1i)) + (F^(c*(a + b*x))*(cos(3*e*x)
- sin(3*e*x)*1i)*(cos(3*d) - sin(3*d)*1i))/(8*(3*e + b*c*log(F)*1i)) - (F^
(c*(a + b*x))*(cos(e*x) + sin(e*x)*1i)*(cos(d) + sin(d)*1i)*3i)/(8*(e*1i +
 b*c*log(F)))
```

3.2. $\int F^{c(a+bx)}\sin^3(d+ex)\,dx$

3.3 $\int F^{c(a+bx)}\sin^2(d+ex)\,dx$

3.3.1 Optimal result

Integrand size = 18, antiderivative size = 128

$$\int F^{c(a+bx)}\sin^2(d+ex)\,dx = \frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)} - \frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)} + \frac{bcF^{c(a+bx)}\log(F)\sin^2(d+ex)}{4e^2+b^2c^2\log^2(F)}$$

output
```
2*e^2*F^(c*(b*x+a))/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)-2*e*F^(c*(b*x+a))*co
s(e*x+d)*sin(e*x+d)/(4*e^2+b^2*c^2*ln(F)^2)+b*c*F^(c*(b*x+a))*ln(F)*sin(e*
x+d)^2/(4*e^2+b^2*c^2*ln(F)^2)
```

3.3.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.67

$$\int F^{c(a+bx)}\sin^2(d+ex)\,dx = \frac{F^{c(a+bx)}\left(4e^2+b^2c^2\log^2(F)-b^2c^2\cos(2(d+ex))\log^2(F)-2bce\log(F)\sin(2(d+ex))\right)}{8bce^2\log(F)+2b^3c^3\log^3(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sin[d + e*x]^2,x]
```

output
```
(F^(c*(a + b*x))*(4*e^2 + b^2*c^2*Log[F]^2 - b^2*c^2*Cos[2*(d + e*x)]*Log[
F]^2 - 2*b*c*e*Log[F]*Sin[2*(d + e*x)]))/(8*b*c*e^2*Log[F] + 2*b^3*c^3*Log
[F]^3)
```

3.3.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4934, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \; 4934$$

$$\frac{2e^2\int F^{c(a+bx)}dx}{b^2c^2\log^2(F)+4e^2}+\frac{bc\log(F)\sin^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}-\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}$$

$$\Big\downarrow \; 2624$$

$$\frac{bc\log(F)\sin^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}-\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(b^2c^2\log^2(F)+4e^2\right)}$$

input
```
Int[F^(c*(a + b*x))*Sin[d + e*x]^2,x]
```

output
```
(2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 + b^2*c^2*Log[F]^2)) - (2*e*F^(
c*(a + b*x))*Cos[d + e*x]*Sin[d + e*x])/(4*e^2 + b^2*c^2*Log[F]^2) + (b*c*
F^(c*(a + b*x))*Log[F]*Sin[d + e*x]^2)/(4*e^2 + b^2*c^2*Log[F]^2)
```

3.3.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

3.3.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

method	result
parallelrisch	$-\frac{F^{c(xb+a)}\left(\frac{b^{2}c^{2}\ln(F)^{2}\cos(2ex+2d)}{2}-\frac{b^{2}c^{2}\ln(F)^{2}}{2}+e\sin(2ex+2d)bc\ln(F)-2e^{2}\right)}{bc\ln(F)\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}$
risch	$\frac{F^{c(xb+a)}}{2bc\ln(F)}-\frac{F^{c(xb+a)}bc\ln(F)\cos(2ex+2d)}{2\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}-\frac{e\,F^{c(xb+a)}\sin(2ex+2d)}{4e^{2}+b^{2}c^{2}\ln(F)^{2}}$
norman	$\frac{-\frac{4e\,e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{4e^{2}+b^{2}c^{2}\ln(F)^{2}}+\frac{4e\,e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{3}}{4e^{2}+b^{2}c^{2}\ln(F)^{2}}+\frac{2e^{2}e^{c(xb+a)\ln(F)}}{bc\ln(F)\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}+\frac{2e^{2}e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{4}}{bc\ln(F)\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}+\frac{4(e}{}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}\right)^{2}}$

input
```
int(F^(c*(b*x+a))*sin(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-F^(c*(b*x+a))*(1/2*b^2*c^2*ln(F)^2*cos(2*e*x+2*d)-1/2*b^2*c^2*ln(F)^2+e*s
in(2*e*x+2*d)*b*c*ln(F)-2*e^2)/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

$$\int F^{c(a+bx)}\sin^2(d+ex)\,dx = -\frac{\left(2\,bce\cos\left(ex+d\right)\log\left(F\right)\sin\left(ex+d\right)+\left(b^2c^2\cos\left(ex+d\right)^2-b^2c^2\right)\log\left(F\right)^2-2\,e^2\right)F^{bcx+ac}}{b^3c^3\log\left(F\right)^3+4\,bce^2\log\left(F\right)}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^2,x, algorithm="fricas")
```

output
```
-(2*b*c*e*cos(e*x + d)*log(F)*sin(e*x + d) + (b^2*c^2*cos(e*x + d)^2 - b^2
*c^2)*log(F)^2 - 2*e^2)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + 4*b*c*e^2*log(
F))
```

3.3.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 741, normalized size of antiderivative = 5.79

$$\int F^{c(a+bx)}\sin^2(d+ex)\,dx$$

$$= \begin{cases} x\sin^2\left(d\right) \\ \frac{x\sin^2\left(d+ex\right)}{2}+\frac{x\cos^2\left(d+ex\right)}{2}-\frac{\sin\left(d+ex\right)\cos\left(d+ex\right)}{2e} \\ F^{ac}\left(\frac{x\sin^2\left(d+ex\right)}{2}+\frac{x\cos^2\left(d+ex\right)}{2}-\frac{\sin\left(d+ex\right)\cos\left(d+ex\right)}{2e}\right) \\ \frac{x\sin^2\left(d+ex\right)}{2}+\frac{x\cos^2\left(d+ex\right)}{2}-\frac{\sin\left(d+ex\right)\cos\left(d+ex\right)}{2e} \\ \frac{F^{ac+bcx}x\sin^2\left(\frac{ibcx\log\left(F\right)}{2}-d\right)}{4}-\frac{iF^{ac+bcx}x\sin\left(\frac{ibcx\log\left(F\right)}{2}-d\right)\cos\left(\frac{ibcx\log\left(F\right)}{2}-d\right)}{2}-\frac{F^{ac+bcx}x\cos^2\left(\frac{ibcx\log\left(F\right)}{2}-d\right)}{4}+\frac{3iF^{ac+bcx}\sin\cdots}{\cdots} \\ \frac{F^{ac+bcx}x\sin^2\left(\frac{ibcx\log\left(F\right)}{2}+d\right)}{4}-\frac{iF^{ac+bcx}x\sin\left(\frac{ibcx\log\left(F\right)}{2}+d\right)\cos\left(\frac{ibcx\log\left(F\right)}{2}+d\right)}{2}-\frac{F^{ac+bcx}x\cos^2\left(\frac{ibcx\log\left(F\right)}{2}+d\right)}{4}+\frac{F^{ac+bcx}\sin^2\cdots}{bc\cdots} \\ \frac{F^{ac+bcx}b^2c^2\log\left(F\right)^2\sin^2\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+4bce^2\log\left(F\right)}-\frac{2F^{ac+bcx}bce\log\left(F\right)\sin\left(d+ex\right)\cos\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+4bce^2\log\left(F\right)}+\frac{2F^{ac+bcx}e^2\sin^2\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+4bce^2\log\left(F\right)}+\frac{2F^{ac+bcx}e^2\cos^2\left(d+e\cdots\right.}{b^3c^3\log\left(F\right)^3+4bce^2\log\cdots} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*sin(e*x+d)**2,x)
```

output
```
Piecewise((x*sin(d)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (x*sin
(d + e*x)**2/2 + x*cos(d + e*x)**2/2 - sin(d + e*x)*cos(d + e*x)/(2*e), Eq
(F, 1)), (F**(a*c)*(x*sin(d + e*x)**2/2 + x*cos(d + e*x)**2/2 - sin(d + e*
x)*cos(d + e*x)/(2*e)), Eq(b, 0)), (x*sin(d + e*x)**2/2 + x*cos(d + e*x)**
2/2 - sin(d + e*x)*cos(d + e*x)/(2*e), Eq(c, 0)), (F**(a*c + b*c*x)*x*sin(
I*b*c*x*log(F)/2 - d)**2/4 - I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 - d
)*cos(I*b*c*x*log(F)/2 - d)/2 - F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/2 -
d)**2/4 + 3*I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/2 - d)*cos(I*b*c*x*log(F
)/2 - d)/(2*b*c*log(F)) + F**(a*c + b*c*x)*cos(I*b*c*x*log(F)/2 - d)**2/(b
*c*log(F)), Eq(e, -I*b*c*log(F)/2)), (F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F
)/2 + d)**2/4 - I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 + d)*cos(I*b*c*x
*log(F)/2 + d)/2 - F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/2 + d)**2/4 + F**
(a*c + b*c*x)*sin(I*b*c*x*log(F)/2 + d)**2/(b*c*log(F)) - I*F**(a*c + b*c*
x)*sin(I*b*c*x*log(F)/2 + d)*cos(I*b*c*x*log(F)/2 + d)/(2*b*c*log(F)), Eq(
e, I*b*c*log(F)/2)), (F**(a*c + b*c*x)*b**2*c**2*log(F)**2*sin(d + e*x)**2
/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) - 2*F**(a*c + b*c*x)*b*c*e*log(
F)*sin(d + e*x)*cos(d + e*x)/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) + 2
*F**(a*c + b*c*x)*e**2*sin(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*c*e**2*l
og(F)) + 2*F**(a*c + b*c*x)*e**2*cos(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*
b*c*e**2*log(F)), True))
```

3.3.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(128) = 256$.

Time = 0.22 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.78

$$\int F^{c(a+bx)}\sin^2(d+ex)\,dx =$$
$$-\frac{\left(F^{ac}b^2c^2\cos(2d)\log(F)^2+2F^{ac}bce\log(F)\sin(2d)\right)F^{bcx}\cos(2ex)+\left(F^{ac}b^2c^2\cos(2d)\log(F)^2-2F\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^2,x, algorithm="maxima")
```

output
```
-1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d)
)*F^(b*c*x)*cos(2*e*x) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*
c*e*log(F)*sin(2*d))*F^(b*c*x)*cos(2*e*x + 4*d) - (F^(a*c)*b^2*c^2*log(F)^
2*sin(2*d) - 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2*e*x) + (F^(a
*c)*b^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)
*sin(2*e*x + 4*d) - 2*(F^(a*c)*b^2*c^2*cos(2*d)^2*log(F)^2 + F^(a*c)*b^2*c
^2*log(F)^2*sin(2*d)^2 + 4*(F^(a*c)*cos(2*d)^2 + F^(a*c)*sin(2*d)^2)*e^2)*
F^(b*c*x))/(b^3*c^3*cos(2*d)^2*log(F)^3 + b^3*c^3*log(F)^3*sin(2*d)^2 + 4*
(b*c*cos(2*d)^2*log(F) + b*c*log(F)*sin(2*d)^2)*e^2)
```

3.3.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 915, normalized size of antiderivative = 7.15

$$\int F^{c(a+bx)} \sin^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d)^2,x, algorithm="giac")
```

output
```
-1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1
/2*pi*a*c + 2*e*x + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sg
n(F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*sin(1/2*pi*b*c*x*
sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*e*x + 2*d)/(4*b
^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*log(abs
(F)) + a*c*log(abs(F))) - 1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*
x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)*log(abs(F))/(4*b^2*c^2*l
og(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c
 - 4*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*p
i*a*c - 2*e*x - 2*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c -
4*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + (2*b*c*cos(-1/2*pi*b*c*
x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b
^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b
*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a
*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(a
bs(F)) + a*c*log(abs(F))) + I*(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*
x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(4*I*pi*b*c*sgn(
F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*I*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F
) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - 2*I*e*x - 2*I*d)
/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e))*e^(b*c...
```

3.3.9 Mupad [B] (verification not implemented)

Time = 29.59 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

$$\int F^{c(a+bx)}\sin^2(d+ex)\,dx$$
$$=\frac{F^{a\,c+b\,c\,x}\left(2\,e^2+\frac{b^2\,c^2\,\ln(F)^2}{2}-\frac{b^2\,c^2\,\ln(F)^2\,\cos(2\,d+2\,e\,x)}{2}-b\,c\,e\,\ln\left(F\right)\,\sin\left(2\,d+2\,e\,x\right)\right)}{b\,c\,\ln\left(F\right)\,\left(b^2\,c^2\ln\left(F\right)^2+4\,e^2\right)}$$

input
```
int(F^(c*(a + b*x))*sin(d + e*x)^2,x)
```

output
```
(F^(a*c + b*c*x)*(2*e^2 + (b^2*c^2*log(F)^2)/2 - (b^2*c^2*log(F)^2*cos(2*d
 + 2*e*x))/2 - b*c*e*log(F)*sin(2*d + 2*e*x)))/(b*c*log(F)*(4*e^2 + b^2*c^
2*log(F)^2))
```

3.4 $\int F^{c(a+bx)} \sin(d+ex)\,dx$

3.4.1 Optimal result

Integrand size = 16, antiderivative size = 73

$$\int F^{c(a+bx)} \sin(d+ex)\,dx = -\frac{eF^{c(a+bx)}\cos(d+ex)}{e^2+b^2c^2\log^2(F)} + \frac{bcF^{c(a+bx)}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

output
```
-e*F^(c*(b*x+a))*cos(e*x+d)/(e^2+b^2*c^2*ln(F)^2)+b*c*F^(c*(b*x+a))*ln(F)*
sin(e*x+d)/(e^2+b^2*c^2*ln(F)^2)
```

3.4.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

$$\int F^{c(a+bx)} \sin(d+ex)\,dx = \frac{F^{c(a+bx)}(-e\cos(d+ex)+bc\log(F)\sin(d+ex))}{e^2+b^2c^2\log^2(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sin[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(-(e*Cos[d + e*x]) + b*c*Log[F]*Sin[d + e*x]))/(e^2 + b^2
*c^2*Log[F]^2)
```

3.4.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 4932}$$

$$\frac{bc\log(F)\sin(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}-\frac{e\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}$$

input
```
Int[F^(c*(a + b*x))*Sin[d + e*x],x]
```

output
```
-((e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)) + (b*c*F^(c*(
a + b*x))*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)
```

3.4.3.1 Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.4.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{F^{c(xb+a)}(bc\ln(F)\sin(ex+d)-e\cos(ex+d))}{e^2+b^2c^2\ln(F)^2}$	49
risch	$-\frac{e\,F^{c(xb+a)}\cos(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{bc\,F^{c(xb+a)}\ln(F)\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}$	74
norman	$\frac{\frac{e\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}-\frac{e\,\mathrm{e}^{c(xb+a)\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{2bc\ln(F)\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$	130

input
```
int(F^(c*(b*x+a))*sin(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
F^(c*(b*x+a))*(b*c*ln(F)*sin(e*x+d)-e*cos(e*x+d))/(e^2+b^2*c^2*ln(F)^2)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int F^{c(a+bx)}\sin(d+ex)\,dx = \frac{(bc\log(F)\sin(ex+d) - e\cos(ex+d))F^{bcx+ac}}{b^2c^2\log(F)^2+e^2}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d),x, algorithm="fricas")
```

output
```
(b*c*log(F)*sin(e*x + d) - e*cos(e*x + d))*F^(b*c*x + a*c)/(b^2*c^2*log(F)
^2 + e^2)
```

3.4.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 347, normalized size of antiderivative = 4.75

$$\int F^{c(a+bx)}\sin(d+ex)\,dx$$
$$= \begin{cases} x\sin(d) & \text{fo} \\ F^{ac}x\sin(d) & \text{fo} \\ x\sin(d) & \text{fo} \\ -\frac{F^{ac+bcx}x\sin(ibcx\log(F)-d)}{2} + \frac{iF^{ac+bcx}x\cos(ibcx\log(F)-d)}{2} + \frac{F^{ac+bcx}\sin(ibcx\log(F)-d)}{2bc\log(F)} - \frac{iF^{ac+bcx}\cos(ibcx\log(F)-d)}{bc\log(F)} & \text{fo} \\ \frac{F^{ac+bcx}x\sin(ibcx\log(F)+d)}{2} - \frac{iF^{ac+bcx}x\cos(ibcx\log(F)+d)}{2} - \frac{F^{ac+bcx}\sin(ibcx\log(F)+d)}{2bc\log(F)} + \frac{iF^{ac+bcx}\cos(ibcx\log(F)+d)}{bc\log(F)} & \text{fo} \\ \frac{F^{ac+bcx}bc\log(F)\sin(d+ex)}{b^2c^2\log(F)^2+e^2} - \frac{F^{ac+bcx}e\cos(d+ex)}{b^2c^2\log(F)^2+e^2} & \text{o} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*sin(e*x+d),x)
```

output
```
Piecewise((x*sin(d), Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sin(d), Eq(b, 0) &
Eq(e, 0)), (x*sin(d), Eq(c, 0) & Eq(e, 0)), (-F**(a*c + b*c*x)*x*sin(I*b*c
*x*log(F) - d)/2 + I*F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F) - d)/2 + F**(a*
c + b*c*x)*sin(I*b*c*x*log(F) - d)/(2*b*c*log(F)) - I*F**(a*c + b*c*x)*cos
(I*b*c*x*log(F) - d)/(b*c*log(F)), Eq(e, -I*b*c*log(F))), (F**(a*c + b*c*x
)*x*sin(I*b*c*x*log(F) + d)/2 - I*F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F) +
d)/2 - F**(a*c + b*c*x)*sin(I*b*c*x*log(F) + d)/(2*b*c*log(F)) + I*F**(a*c
 + b*c*x)*cos(I*b*c*x*log(F) + d)/(b*c*log(F)), Eq(e, I*b*c*log(F))), (F**
(a*c + b*c*x)*b*c*log(F)*sin(d + e*x)/(b**2*c**2*log(F)**2 + e**2) - F**(a
*c + b*c*x)*e*cos(d + e*x)/(b**2*c**2*log(F)**2 + e**2), True))
```

3.4.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(73) = 146$.

Time = 0.22 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.66

$$\int F^{c(a+bx)} \sin(d+ex)\,dx = -\frac{(F^{ac}bc\log(F)\sin(d) + F^{ac}e\cos(d))F^{bcx}\cos(ex+2d) - (F^{ac}bc\log(F)\sin(d) - F^{ac}e\cos(d))F^{bcx}\cos}{2\left(b^2c^2\cos(d)^2\log(F)^2 + b^2c^2\right.}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d),x, algorithm="maxima")
```

output
```
-1/2*((F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x + 2
*d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x) -
(F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x + 2*d) -
(F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x))/(b^2*c^
2*cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^
2)
```

3.4.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 634, normalized size of antiderivative = 8.68

$$\int F^{c(a+bx)} \sin(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sin(e*x+d),x, algorithm="giac")
```

output
```
(2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn
(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi
*b*c + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c + 2*e)*cos(1/2*pi*b*c*x*sgn(F) -
1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(ab
s(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*lo
g(abs(F))) - (2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1
/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*
c*sgn(F) - pi*b*c - 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c - 2*e)*cos(1/2*pi*b*
c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b
^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs
(F)) + a*c*log(abs(F))) - (-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x +
1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*I
*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I
*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-2*I*pi*b*c
*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) +
a*c*log(abs(F))) - (I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi
*a*c*sgn(F) - 1/2*I*pi*a*c - I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c
+ 4*b*c*log(abs(F)) - 4*I*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*
x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*sgn(F)
+ 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*l...
```

3.4.9 Mupad [B] (verification not implemented)

Time = 28.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int F^{c(a+bx)}\sin(d+ex)\,dx = -\frac{F^{a\,c+b\,c\,x}\,(e\,\cos\left(d+e\,x\right)-b\,c\,\sin\left(d+e\,x\right)\,\ln\left(F\right))}{b^2\,c^2\,\ln\left(F\right)^2+e^2}$$

input
```
int(F^(c*(a + b*x))*sin(d + e*x),x)
```

output
```
-(F^(a*c + b*c*x)*(e*cos(d + e*x) - b*c*sin(d + e*x)*log(F)))/(e^2 + b^2*c
^2*log(F)^2)
```

3.5 $\int F^{c(a+bx)}\csc(d+ex)\,dx$

3.5.1 Optimal result

Integrand size = 16, antiderivative size = 81

$$\int F^{c(a+bx)}\csc(d+ex)\,dx = -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)}{e-ibc\log(F)}$$

output
```
-2*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([1, 1/2*(e-I*b*c*ln(F))/e],[3/2-
1/2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))/(e-I*b*c*ln(F))
```

3.5.2 Mathematica [A] (verified)

Time = 2.47 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.41

$$\int F^{c(a+bx)}\csc(d+ex)\,dx = \frac{iF^{c(a+bx)}\left(\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{e},1-\frac{ibc\log(F)}{e},-\cos(d+ex)-i\sin(d+ex)\right)-\text{Hypergeomet}\right.}{bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Csc[d + e*x],x]
```

output
```
(I*F^(c*(a + b*x))*(Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b*c*L
og[F])/e, -Cos[d + e*x] - I*Sin[d + e*x]] - Hypergeometric2F1[1, ((-I)*b*c
*Log[F])/e, 1 - (I*b*c*Log[F])/e, Cos[d + e*x] + I*Sin[d + e*x]]))/(b*c*Lo
g[F])
```

3.5.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \; 4953$$

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)}{e-ibc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Csc[d + e*x],x]
```

output
```
(-2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F]
)/(2*e), (3 - (I*b*c*Log[F])/e)/2, E^((2*I)*(d + e*x))])/(e - I*b*c*Log[F]
)
```

3.5.3.1 Defintions of rubi rules used

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

3.5.4 Maple [F]

$$\int F^{c(xb+a)} \csc(ex+d)\, dx$$

input
```
int(F^(c*(b*x+a))*csc(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*csc(e*x+d),x)
```

3.5.5 Fricas [F]

$$\int F^{c(a+bx)} \csc(d+ex)\, dx = \int F^{(bx+a)c} \csc(ex+d)\, dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csc(e*x + d), x)
```

3.5.6 Sympy [F]

$$\int F^{c(a+bx)} \csc(d+ex)\, dx = \int F^{c(a+bx)} \csc(d+ex)\, dx$$

input
```
integrate(F**(c*(b*x+a))*csc(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*csc(d + e*x), x)
```

3.5.7 Maxima [F]

$$\int F^{c(a+bx)}\csc(d+ex)\,dx=\int F^{(bx+a)c}\csc{(ex+d)}\;dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d),x, algorithm="maxima")
```

output
```
2*(F^(b*c*x)*F^(a*c)*b*c*log(F)*sin(e*x + d) + F^(b*c*x)*F^(a*c)*e*cos(e*x
 + d) - (F^(b*c*x)*F^(a*c)*b*c*log(F)*sin(e*x + d) + F^(b*c*x)*F^(a*c)*e*c
os(e*x + d))*cos(2*e*x + 2*d) - 2*(F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^
3 + (F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*cos(2*e*x + 2*d)^2 + (F^(a*
c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*sin(2*e*x + 2*d)^2 - 2*(F^(a*c)*b^2*c
^2*e*log(F)^2 + F^(a*c)*e^3)*cos(2*e*x + 2*d))*integrate((F^(b*c*x)*b*c*co
s(e*x + d)*log(F) - F^(b*c*x)*e*sin(e*x + d) + (F^(b*c*x)*b*c*cos(e*x + d)
*log(F) - F^(b*c*x)*e*sin(e*x + d))*cos(4*e*x + 4*d) - 2*(F^(b*c*x)*b*c*co
s(e*x + d)*log(F) - F^(b*c*x)*e*sin(e*x + d))*cos(2*e*x + 2*d) + (F^(b*c*x
)*b*c*log(F)*sin(e*x + d) + F^(b*c*x)*e*cos(e*x + d))*sin(4*e*x + 4*d) - 2
*(F^(b*c*x)*b*c*log(F)*sin(e*x + d) + F^(b*c*x)*e*cos(e*x + d))*sin(2*e*x
+ 2*d))/(b^2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + e^2)*cos(4*e*x + 4*d)^2 +
4*(b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d)^2 + (b^2*c^2*log(F)^2 + e^2)*s
in(4*e*x + 4*d)^2 - 4*(b^2*c^2*log(F)^2 + e^2)*sin(4*e*x + 4*d)*sin(2*e*x
+ 2*d) + 4*(b^2*c^2*log(F)^2 + e^2)*sin(2*e*x + 2*d)^2 + e^2 + 2*(b^2*c^2*
log(F)^2 + e^2 - 2*(b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d))*cos(4*e*x +
4*d) - 4*(b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d)), x) + (F^(b*c*x)*F^(a*
c)*b*c*cos(e*x + d)*log(F) - F^(b*c*x)*F^(a*c)*e*sin(e*x + d))*sin(2*e*x +
 2*d))/(b^2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d)^2 + (
b^2*c^2*log(F)^2 + e^2)*sin(2*e*x + 2*d)^2 + e^2 - 2*(b^2*c^2*log(F)^2 ...
```

3.5.8 Giac [F]

$$\int F^{c(a+bx)}\csc(d+ex)\,dx=\int F^{(bx+a)c}\csc{(ex+d)}\;dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csc(e*x + d), x)
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\csc(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sin\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))/sin(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))/sin(d + e*x), x)
```

3.6 $\int F^{c(a+bx)}\csc^2(d+ex)\,dx$

3.6.1 Optimal result

Integrand size = 18, antiderivative size = 78

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx = -\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

output
```
-4*exp(2*I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-1/2*I*b*c*ln(F)/e],[2-1/
2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))/(2*I*e+b*c*ln(F))
```

3.6.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.29

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx = -\frac{2iF^{c(a+bx)}\left(\left(-1+e^{2id}\right)\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)+\csc(d+ex)\sin(d)(}{e\left(-1+e^{2id}\right)}$$

input
```
Integrate[F^(c*(a + b*x))*Csc[d + e*x]^2,x]
```

output
```
((-2*I)*F^(c*(a + b*x))*((-1 + E^((2*I)*d))*Hypergeometric2F1[1, ((-1/2*I)
*b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))] + Csc[d + e
*x]*Sin[d]*(Cos[e*x] - I*Sin[e*x])))/(e*(-1 + E^((2*I)*d)))
```

3.6.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \; 4953$$

$$-\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{bc\log(F)+2ie}$$

input
```
Int[F^(c*(a + b*x))*Csc[d + e*x]^2,x]
```

output
```
(-4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*
c*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])/((2*I)*e + b*
c*Log[F])
```

3.6.3.1 Defintions of rubi rules used

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

3.6.4 Maple [F]

$$\int F^{c(xb+a)}\csc\left(ex+d\right)^2 dx$$

input
```
int(F^(c*(b*x+a))*csc(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*csc(e*x+d)^2,x)
```

3.6.5 Fricas [F]

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx = \int F^{(bx+a)c}\csc\left(ex+d\right)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csc(e*x + d)^2, x)
```

3.6.6 Sympy [F]

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx = \int F^{c(a+bx)}\csc^2\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*csc(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*csc(d + e*x)**2, x)
```

3.6.7 Maxima [F]

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx = \int F^{(bx+a)c}\csc\left(ex+d\right)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^2,x, algorithm="maxima")
```

output
```
4*(24*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F) + 2*(F^(a*c)*b^3*c^3*log(F)^3 + 16*
F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d)^2 + 2*(F^(a*c)*b^3*c^3*
log(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(2*e*x + 2*d)^2 - (F^(a
*c)*b^3*c^3*log(F)^3 + 64*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*
d) + 2*(5*F^(a*c)*b^2*c^2*e*log(F)^2 - 16*F^(a*c)*e^3)*F^(b*c*x)*sin(2*e*x
 + 2*d) + (24*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F) - (F^(a*c)*b^3*c^3*log(F)^3
 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) + 2*(F^(a*c)*b^2*
c^2*e*log(F)^2 + 16*F^(a*c)*e^3)*F^(b*c*x)*sin(2*e*x + 2*d))*cos(4*e*x + 4
*d) + 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64
*F^(a*c)*b*c*e^5*log(F) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3
*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*cos(4*e*x + 4*d)^2 + 4*(F^(a*c)
*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5
*log(F))*cos(2*e*x + 2*d)^2 + (F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3
*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*sin(4*e*x + 4*d)^2 - 4*(F^(
a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c
*e^5*log(F))*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*(F^(a*c)*b^5*c^5*e*log(
F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*sin(2*
e*x + 2*d)^2 + 2*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(
F)^3 + 64*F^(a*c)*b*c*e^5*log(F) - 2*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a
*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*cos(2*e*x + 2*d))...
```

3.6.8 Giac [F]

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx=\int F^{(bx+a)c}\csc{(ex+d)}^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csc(e*x + d)^2, x)
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sin\left(d+e\,x\right)^2}\,dx$$

input
```
int(F^(c*(a + b*x))/sin(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))/sin(d + e*x)^2, x)
```

3.7 $\int F^{c(a+bx)}\csc^3(d+ex)\,dx$

3.7.1 Optimal result

Integrand size = 18, antiderivative size = 137

$$\int F^{c(a+bx)}\csc^3(d+ex)\,dx = -\frac{F^{c(a+bx)}\cot(d+ex)\csc(d+ex)}{2e} - \frac{bcF^{c(a+bx)}\csc(d+ex)\log(F)}{2e^2} - \frac{e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)(e+ibc\log(F))}{e^2}$$

output
```
-1/2*F^(c*(b*x+a))*cot(e*x+d)*csc(e*x+d)/e-1/2*b*c*F^(c*(b*x+a))*csc(e*x+d
)*ln(F)/e^2-exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([1, 1/2*(e-I*b*c*ln(F))
/e],[3/2-1/2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))*(e+I*b*c*ln(F))/e^2
```

3.7.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 334 vs. 2(137) = 274.

Time = 8.17 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.44

$$\int F^{c(a+bx)}\csc^3(d+ex)\,dx = \frac{F^{c(a+bx)}\left(-e\csc^2\left(\frac{1}{2}(d+ex)\right)-4bc\csc(d)\log(F)+\csc(d)\left(\frac{4e^2}{bc\log(F)}+4bc\log(F)\right)+e\sec^2\left(\frac{1}{2}(d+ex)\right)-\right.}{}$$

input
```
Integrate[F^(c*(a + b*x))*Csc[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(-(e*Csc[(d + e*x)/2]^2) - 4*b*c*Csc[d]*Log[F] + Csc[d]*(
(4*e^2)/(b*c*Log[F]) + 4*b*c*Log[F]) + e*Sec[(d + e*x)/2]^2 - ((4*I)*(e^2
+ b^2*c^2*Log[F]^2)*(1 + Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*
b*c*Log[F])/e, Cos[d + e*x] + I*Sin[d + e*x]]*(-1 + Cos[d] + I*Sin[d])))/(
b*c*Log[F]*(-1 + Cos[d] + I*Sin[d])) - ((4*I)*(e^2 + b^2*c^2*Log[F]^2)*(1
- Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b*c*Log[F])/e, -Cos[d +
 e*x] - I*Sin[d + e*x]]*(1 + Cos[d] + I*Sin[d])))/(b*c*Log[F]*(1 + Cos[d]
+ I*Sin[d])) + 2*b*c*Csc[d/2]*Csc[(d + e*x)/2]*Log[F]*Sin[(e*x)/2] - 2*b*c
*Log[F]*Sec[d/2]*Sec[(d + e*x)/2]*Sin[(e*x)/2]))/(8*e^2)
```

3.7.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4949, 4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{4949}$$

$$\frac{1}{2}\left(\frac{b^2c^2\log^2(F)}{e^2}+1\right)\int F^{c(a+bx)}\csc(d+ex)dx-\frac{bc\log(F)\csc(d+ex)F^{c(a+bx)}}{2e^2}-\frac{\cot(d+ex)\csc(d+ex)F^{c(a+bx)}}{2e}$$

$$\Big\downarrow \text{4953}$$

$$-\frac{e^{i(d+ex)}F^{c(a+bx)}\left(\frac{b^2c^2\log^2(F)}{e^2}+1\right)\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)}{e-ibc\log(F)}-\frac{bc\log(F)\csc(d+ex)F^{c(a+bx)}}{2e^2}-\frac{\cot(d+ex)\csc(d+ex)F^{c(a+bx)}}{2e}$$

input
```
Int[F^(c*(a + b*x))*Csc[d + e*x]^3,x]
```

output
```
-1/2*(F^(c*(a + b*x))*Cot[d + e*x]*Csc[d + e*x])/e - (b*c*F^(c*(a + b*x))*
Csc[d + e*x]*Log[F])/(2*e^2) - (E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeome
tric2F1[1, (e - I*b*c*Log[F])/(2*e), (3 - (I*b*c*Log[F])/e)/2, E^((2*I)*(d
 + e*x))]*(1 + (b^2*c^2*Log[F]^2)/e^2))/(e - I*b*c*Log[F])
```

3.7.3.1 Defintions of rubi rules used

rule 4949
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (-Simp[F^(c*(a + b*x))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

3.7.4 Maple [F]

$$\int F^{c(xb+a)} \csc(ex+d)^3 \, dx$$

input
```
int(F^(c*(b*x+a))*csc(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*csc(e*x+d)^3,x)
```

3.7.5 Fricas [F]

$$\int F^{c(a+bx)} \csc^3(d+ex)\,dx = \int F^{(bx+a)c} \csc\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csc(e*x + d)^3, x)
```

3.7.6 Sympy [F]

$$\int F^{c(a+bx)} \csc^3(d+ex)\,dx = \int F^{c(a+bx)} \csc^3\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*csc(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*csc(d + e*x)**3, x)
```

3.7.7 Maxima [F]

$$\int F^{c(a+bx)} \csc^3(d+ex)\,dx = \int F^{(bx+a)c} \csc\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^3,x, algorithm="maxima")
```

output
```
8*(48*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F)*sin(e*x + d) - 6*(F^(a*c)*b^2*c^2*e
*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*cos(e*x + d) - (48*F^(b*c*x)*F^(a*c)
*b*c*e^2*log(F)*sin(e*x + d) - 3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*
e^3)*F^(b*c*x)*cos(3*e*x + 3*d) - 6*(F^(a*c)*b^2*c^2*e*log(F)^2 - 15*F^(a*
c)*e^3)*F^(b*c*x)*cos(e*x + d) - (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*
c*e^2*log(F))*F^(b*c*x)*sin(3*e*x + 3*d))*cos(6*e*x + 6*d) + 3*(48*F^(b*c*
x)*F^(a*c)*b*c*e^2*log(F)*sin(e*x + d) - 3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2
5*F^(a*c)*e^3)*F^(b*c*x)*cos(3*e*x + 3*d) - 6*(F^(a*c)*b^2*c^2*e*log(F)^2
 - 15*F^(a*c)*e^3)*F^(b*c*x)*cos(e*x + d) - (F^(a*c)*b^3*c^3*log(F)^3 + 25*
F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(3*e*x + 3*d))*cos(4*e*x + 4*d) + 3*(
3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x)*cos(2*e*x + 2*d)
 - (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(2*
e*x + 2*d) - (F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x))*cos(
3*e*x + 3*d) - 18*(8*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F)*sin(e*x + d) - (F^(a
*c)*b^2*c^2*e*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*cos(e*x + d))*cos(2*e*x
 + 2*d) - 6*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d
)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e
^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*c
os(d) + (F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*lo
g(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^...
```

3.7.8 Giac [F]

$$\int F^{c(a+bx)}\csc^3(d+ex)\,dx = \int F^{(bx+a)c}\csc{(ex+d)}^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csc(e*x + d)^3, x)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\csc^3(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sin\left(d+e\,x\right)^3}\,dx$$

input
```
int(F^(c*(a + b*x))/sin(d + e*x)^3,x)
```

output
```
int(F^(c*(a + b*x))/sin(d + e*x)^3, x)
```

3.8 $\int F^{c(a+bx)} \csc^4(d+ex)\,dx$

3.8.1 Optimal result

Integrand size = 18, antiderivative size = 141

$$\int F^{c(a+bx)} \csc^4(d+ex)\,dx = -\frac{F^{c(a+bx)}\cot(d+ex)\csc^2(d+ex)}{3e} - \frac{bcF^{c(a+bx)}\csc^2(d+ex)\log(F)}{6e^2} + \frac{2e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)(2ie-bc\log(F))}{3e^2}$$

output
```
-1/3*F^(c*(b*x+a))*cot(e*x+d)*csc(e*x+d)^2/e-1/6*b*c*F^(c*(b*x+a))*csc(e*x
+d)^2*ln(F)/e^2+2/3*exp(2*I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-1/2*I*b
*c*ln(F)/e],[2-1/2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))*(2*I*e-b*c*ln(F))/e^2
```

3.8.2 Mathematica [A] (verified)

Time = 2.18 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.23

$$\int F^{c(a+bx)} \csc^4(d+ex)\,dx = \frac{F^{c(a+bx)}\left(-e\csc^2(d+ex)(2e\cot(d)+bc\log(F)) - \frac{2i\left(1+(-1+e^{2id})\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+e}\right.\right.}{-1+e^{2id}}\right.}{6e^3}$$

input
```
Integrate[F^(c*(a + b*x))*Csc[d + e*x]^4,x]
```

output
```
(F^(c*(a + b*x))*(-(e*Csc[d + e*x]^2*(2*e*Cot[d] + b*c*Log[F])) - ((2*I)*(
1 + (-1 + E^((2*I)*d))*Hypergeometric2F1[1, ((-1/2*I)*b*c*Log[F])/e, 1 - (
(I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])*(4*e^2 + b^2*c^2*Log[F]^2))/(-1
 + E^((2*I)*d)) + 2*e^2*Csc[d]*Csc[d + e*x]^3*Sin[e*x] + Csc[d]*Csc[d + e*
x]*(4*e^2 + b^2*c^2*Log[F]^2)*Sin[e*x]))/(6*e^3)
```

3.8.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4949, 4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \; 4949$$

$$\frac{1}{6}\left(\frac{b^2c^2\log^2(F)}{e^2}+4\right)\int F^{c(a+bx)}\csc^2(d+ex)dx-\frac{bc\log(F)\csc^2(d+ex)F^{c(a+bx)}}{6e^2}-\frac{\cot(d+ex)\csc^2(d+ex)F^{c(a+bx)}}{3e}$$

$$\Big\downarrow \; 4953$$

$$-\frac{2e^{2i(d+ex)}F^{c(a+bx)}\left(\frac{b^2c^2\log^2(F)}{e^2}+4\right)\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{3(bc\log(F)+2ie)}-\frac{bc\log(F)\csc^2(d+ex)F^{c(a+bx)}}{6e^2}-\frac{\cot(d+ex)\csc^2(d+ex)F^{c(a+bx)}}{3e}$$

input
```
Int[F^(c*(a + b*x))*Csc[d + e*x]^4,x]
```

output
```
-1/3*(F^(c*(a + b*x))*Cot[d + e*x]*Csc[d + e*x]^2)/e - (b*c*F^(c*(a + b*x)
)*Csc[d + e*x]^2*Log[F])/(6*e^2) - (2*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*
Hypergeometric2F1[2, 1 - ((I/2)*b*c*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, E
^((2*I)*(d + e*x))]*(4 + (b^2*c^2*Log[F]^2)/e^2))/(3*((2*I)*e + b*c*Log[F]
))
```

3.8.3.1 Defintions of rubi rules used

rule 4949
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (-Simp[F^(c*(a + b*x))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

3.8.4 Maple [F]

$$\int F^{c(xb+a)} \csc\left(ex+d\right)^{4} dx$$

input
```
int(F^(c*(b*x+a))*csc(e*x+d)^4,x)
```

output
```
int(F^(c*(b*x+a))*csc(e*x+d)^4,x)
```

3.8.5 Fricas [F]

$$\int F^{c(a+bx)} \csc^4(d+ex)\,dx = \int F^{(bx+a)c} \csc\left(ex+d\right)^{4}\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csc(e*x + d)^4, x)
```

3.8.6 Sympy [F]

$$\int F^{c(a+bx)}\csc^4(d+ex)\,dx=\int F^{c(a+bx)}\csc^4\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*csc(e*x+d)**4,x)
```

output
```
Integral(F**(c*(a + b*x))*csc(d + e*x)**4, x)
```

3.8.7 Maxima [F]

$$\int F^{c(a+bx)}\csc^4(d+ex)\,dx=\int F^{(bx+a)c}\csc\left(ex+d\right)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^4,x, algorithm="maxima")
```

output
```
16*(6*(F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*
F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(4*e*x + 4*d)^2 + 320*(F^(a*c)*b^3*c^
3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d)^2 +
 6*(F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F^(
a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(4*e*x + 4*d)^2 + 320*(F^(a*c)*b^3*c^3*e
^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(2*e*x + 2*d)^2 + 56
0*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 32*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos
(2*e*x + 2*d) - 40*(F^(a*c)*b^4*c^4*e*log(F)^4 - 104*F^(a*c)*b^2*c^2*e^3*l
og(F)^2)*F^(b*c*x)*sin(2*e*x + 2*d) - 160*(F^(a*c)*b^3*c^3*e^2*log(F)^3 -
20*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x) + ((F^(a*c)*b^5*c^5*log(F)^5 + 100*F^
(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(4*
e*x + 4*d) - 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))
*F^(b*c*x)*cos(2*e*x + 2*d) - 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 100*F^(a*c)*
b^2*c^2*e^3*log(F)^2 + 2304*F^(a*c)*e^5)*F^(b*c*x)*sin(4*e*x + 4*d) - 8*(F
^(a*c)*b^4*c^4*e*log(F)^4 + 40*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 1536*F^(a*c)
*e^5)*F^(b*c*x)*sin(2*e*x + 2*d) - 160*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 20*
F^(a*c)*b*c*e^4*log(F))*F^(b*c*x))*cos(8*e*x + 8*d) - 4*((F^(a*c)*b^5*c^5*
log(F)^5 + 100*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F^(a*c)*b*c*e^4*log(F))
*F^(b*c*x)*cos(4*e*x + 4*d) - 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c
)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) - 4*(F^(a*c)*b^4*c^4*e*log...
```

3.8.8 Giac [F]

$$\int F^{c(a+bx)}\csc^4(d+ex)\,dx=\int F^{(bx+a)c}\csc\left(ex+d\right)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csc(e*x + d)^4, x)
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\csc^4(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sin\left(d+e\,x\right)^4}\,dx$$

input
```
int(F^(c*(a + b*x))/sin(d + e*x)^4,x)
```

output
```
int(F^(c*(a + b*x))/sin(d + e*x)^4, x)
```

3.9 $\int e^x \sin^4(x)\,dx$

3.9.1 Optimal result

Integrand size = 8, antiderivative size = 54

$$\int e^x \sin^4(x)\,dx = \frac{24e^x}{85} - \frac{24}{85}e^x\cos(x)\sin(x) + \frac{12}{85}e^x\sin^2(x) - \frac{4}{17}e^x\cos(x)\sin^3(x) + \frac{1}{17}e^x\sin^4(x)$$

output
```
24/85*exp(x)-24/85*exp(x)*cos(x)*sin(x)+12/85*exp(x)*sin(x)^2-4/17*exp(x)*
cos(x)*sin(x)^3+1/17*exp(x)*sin(x)^4
```

3.9.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.61

$$\int e^x \sin^4(x)\,dx = \frac{1}{680}e^x(255 - 68\cos(2x) + 5\cos(4x) - 136\sin(2x) + 20\sin(4x))$$

input
```
Integrate[E^x*Sin[x]^4,x]
```

output
```
(E^x*(255 - 68*Cos[2*x] + 5*Cos[4*x] - 136*Sin[2*x] + 20*Sin[4*x]))/680
```

3.9.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4934, 4934, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sin^4(x)\,dx$$

$$\Big\downarrow \text{4934}$$

$$\frac{12}{17}\int e^x \sin^2(x)dx+\frac{1}{17}e^x \sin^4(x)-\frac{4}{17}e^x \sin^3(x)\cos(x)$$

$$\Big\downarrow \text{4934}$$

$$\frac{12}{17}\left(\frac{2\int e^x dx}{5}+\frac{1}{5}e^x \sin^2(x)-\frac{2}{5}e^x \sin(x)\cos(x)\right)+\frac{1}{17}e^x \sin^4(x)-\frac{4}{17}e^x \sin^3(x)\cos(x)$$

$$\Big\downarrow \text{2624}$$

$$\frac{1}{17}e^x \sin^4(x)-\frac{4}{17}e^x \sin^3(x)\cos(x)+\frac{12}{17}\left(\frac{2e^x}{5}+\frac{1}{5}e^x \sin^2(x)-\frac{2}{5}e^x \sin(x)\cos(x)\right)$$

input
```
Int[E^x*Sin[x]^4,x]
```

output
```
(-4*E^x*Cos[x]*Sin[x]^3)/17 + (E^x*Sin[x]^4)/17 + (12*((2*E^x)/5 - (2*E^x*
Cos[x]*Sin[x])/5 + (E^x*Sin[x]^2)/5))/17
```

3.9.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

3.9.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.57

method	result
parallelrisch	$-\frac{e^{x}(-255+136\sin(2x)-20\sin(4x)-5\cos(4x)+68\cos(2x))}{680}$
default	$\frac{(\sin(x)-4\cos(x))e^{x}\sin(x)^{3}}{17}+\frac{12(\sin(x)-2\cos(x))e^{x}\sin(x)}{85}+\frac{24\,e^{x}}{85}$
risch	$\frac{3\,e^{x}}{8}+\frac{e^{(1+4i)x}}{272}-\frac{i e^{(1+4i)x}}{68}-\frac{e^{(1+2i)x}}{20}+\frac{i e^{(1+2i)x}}{10}-\frac{e^{(1-2i)x}}{20}-\frac{i e^{(1-2i)x}}{10}+\frac{e^{(1-4i)x}}{272}+\frac{i e^{(1-4i)x}}{68}$
norman	$\frac{-\frac{48\,e^{x}\tan(\frac{x}{2})}{85}+\frac{144\,e^{x}\tan(\frac{x}{2})^{2}}{85}-\frac{208\,e^{x}\tan(\frac{x}{2})^{3}}{85}+\frac{64\,e^{x}\tan(\frac{x}{2})^{4}}{17}+\frac{208\,e^{x}\tan(\frac{x}{2})^{5}}{85}+\frac{144\,e^{x}\tan(\frac{x}{2})^{6}}{85}+\frac{48\,e^{x}\tan(\frac{x}{2})^{7}}{85}+\frac{24\,e^{x}\tan(\frac{x}{2}}{85}}{\left(1+\tan(\frac{x}{2})^{2}\right)^{4}}$

input
```
int(exp(x)*sin(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/680*exp(x)*(-255+136*sin(2*x)-20*sin(4*x)-5*cos(4*x)+68*cos(2*x))
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.67

$$\int e^{x}\sin^{4}(x)\,dx=\frac{4}{85}\left(5\cos(x)^{3}-11\cos(x)\right)e^{x}\sin(x)+\frac{1}{85}\left(5\cos(x)^{4}-22\cos(x)^{2}+41\right)e^{x}$$

input
```
integrate(exp(x)*sin(x)^4,x, algorithm="fricas")
```

output
```
4/85*(5*cos(x)^3 - 11*cos(x))*e^x*sin(x) + 1/85*(5*cos(x)^4 - 22*cos(x)^2
+ 41)*e^x
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int e^{x}\sin^{4}(x)\,dx=\frac{41e^{x}\sin^{4}(x)}{85}-\frac{44e^{x}\sin^{3}(x)\cos(x)}{85}+\frac{12e^{x}\sin^{2}(x)\cos^{2}(x)}{17}-\frac{24e^{x}\sin(x)\cos^{3}(x)}{85}+\frac{24e^{x}\cos^{4}(x)}{85}$$

input
```
integrate(exp(x)*sin(x)**4,x)
```

output
```
41*exp(x)*sin(x)**4/85 - 44*exp(x)*sin(x)**3*cos(x)/85 + 12*exp(x)*sin(x)*
*2*cos(x)**2/17 - 24*exp(x)*sin(x)*cos(x)**3/85 + 24*exp(x)*cos(x)**4/85
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int e^x \sin^4(x)\,dx = \frac{1}{136}\cos(4x)e^x - \frac{1}{10}\cos(2x)e^x + \frac{1}{34}e^x\sin(4x) - \frac{1}{5}e^x\sin(2x) + \frac{3}{8}e^x$$

input
```
integrate(exp(x)*sin(x)^4,x, algorithm="maxima")
```

output
```
1/136*cos(4*x)*e^x - 1/10*cos(2*x)*e^x + 1/34*e^x*sin(4*x) - 1/5*e^x*sin(2
*x) + 3/8*e^x
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.65

$$\int e^x \sin^4(x)\,dx = \frac{1}{136}(\cos(4x) + 4\sin(4x))e^x - \frac{1}{10}(\cos(2x) + 2\sin(2x))e^x + \frac{3}{8}e^x$$

input
```
integrate(exp(x)*sin(x)^4,x, algorithm="giac")
```

output
```
1/136*(cos(4*x) + 4*sin(4*x))*e^x - 1/10*(cos(2*x) + 2*sin(2*x))*e^x + 3/8
*e^x
```

3.9.9 Mupad [B] (verification not implemented)

Time = 26.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int e^x \sin^4(x)\,dx = \frac{3\,\mathrm{e}^x}{8} - \frac{\mathrm{e}^x\left(\frac{4\,\cos(2\,x)}{5} + \frac{8\,\sin(2\,x)}{5} - \frac{2\cos(2\,x)^2}{17} - \frac{8\,\cos(2\,x)\,\sin(2\,x)}{17} + \frac{1}{17}\right)}{8}$$

input
```
int(exp(x)*sin(x)^4,x)
```

output
```
(3*exp(x))/8 - (exp(x)*((4*cos(2*x))/5 + (8*sin(2*x))/5 - (2*cos(2*x)^2)/1
7 - (8*cos(2*x)*sin(2*x))/17 + 1/17))/8
```

3.10 $\int F^{c(a+bx)} \cos^n(d+ex)\,dx$

3.10.1 Optimal result

Integrand size = 18, antiderivative size = 107

$$\int F^{c(a+bx)} \cos^n(d+ex)\,dx = -\frac{\left(1+e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)} \cos^n(d+ex) \text{Hypergeometric2F1}\left(-n,-\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(2-n-\frac{ibc\log(F)}{e}\right),-\right.}{ien-bc\log(F)}$$

output
```
-F^(c*(b*x+a))*cos(e*x+d)^n*hypergeom([-n, 1/2*(-e*n-I*b*c*ln(F))/e],[1-1/
2*n-1/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))/((1+exp(2*I*(e*x+d)))^n)/(I*e*n-
b*c*ln(F))
```

3.10.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)} \cos^n(d+ex)\,dx$$
$$= \frac{\left(1+e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)} \cos^n(d+ex) \text{Hypergeometric2F1}\left(-n,-\frac{i(-ien+bc\log(F))}{2e},1-\frac{i(-ien+bc\log(F))}{2e},-e^{2}\right.}{-ien+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Cos[d + e*x]^n,x]
```

output
```
(F^(c*(a + b*x))*Cos[d + e*x]^n*Hypergeometric2F1[-n, ((-1/2*I)*((-I)*e*n
+ b*c*Log[F]))/e, 1 - ((I/2)*((-I)*e*n + b*c*Log[F]))/e, -E^((2*I)*(d + e*
x))])/((1 + E^((2*I)*(d + e*x)))^n*((-I)*e*n + b*c*Log[F]))
```

3.10.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4941, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\cos^n(d+ex)\,dx$$

$$\Big\downarrow \text{4941}$$

$$e^{in(d+ex)}\left(1+e^{2i(d+ex)}\right)^{-n}\cos^n(d+ex)\int e^{-in(d+ex)}\left(1+e^{2i(d+ex)}\right)^nF^{c(a+bx)}dx$$

$$\Big\downarrow \text{2689}$$

$$-\frac{\left(1+e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}\cos^n(d+ex)\,\text{Hypergeometric2F1}\left(-n,-\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(-n-\frac{ibc\log(F)}{e}+2\right),-e^{2i(d+}\right.}{-bc\log(F)+ien}$$

input
```
Int[F^(c*(a + b*x))*Cos[d + e*x]^n,x]
```

output
```
-((F^(c*(a + b*x))*Cos[d + e*x]^n*Hypergeometric2F1[-n, -1/2*(e*n + I*b*c*
Log[F])/e, (2 - n - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))])/((1 + E^((
2*I)*(d + e*x)))^n*(I*e*n - b*c*Log[F])))
```

3.10.3.1 Defintions of rubi rules used

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 4941
```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[E^(I*n*(d + e*x))*(Cos[d + e*x]^n/(1 + E^(2*I*(d + e*x)))^n)   I
nt[F^(c*(a + b*x))*((1 + E^(2*I*(d + e*x)))^n/E^(I*n*(d + e*x))), x], x] /;
 FreeQ[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

3.10.4 Maple [F]

$$\int F^{c(xb+a)} \cos(ex+d)^n\, dx$$

input
```
int(F^(c*(b*x+a))*cos(e*x+d)^n,x)
```

output
```
int(F^(c*(b*x+a))*cos(e*x+d)^n,x)
```

3.10.5 Fricas [F]

$$\int F^{c(a+bx)} \cos^n(d+ex)\, dx = \int F^{(bx+a)c} \cos(ex+d)^n\, dx$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^n,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*cos(e*x + d)^n, x)
```

3.10.6 Sympy [F]

$$\int F^{c(a+bx)}\cos^n(d+ex)\,dx=\int F^{c(a+bx)}\cos^n{(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*cos(e*x+d)**n,x)
```

output
```
Integral(F**(c*(a + b*x))*cos(d + e*x)**n, x)
```

3.10.7 Maxima [F]

$$\int F^{c(a+bx)}\cos^n(d+ex)\,dx=\int F^{(bx+a)c}\cos{(ex+d)}^n\,dx$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^n,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)*c)*cos(e*x + d)^n, x)
```

3.10.8 Giac [F]

$$\int F^{c(a+bx)}\cos^n(d+ex)\,dx=\int F^{(bx+a)c}\cos{(ex+d)}^n\,dx$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^n,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*cos(e*x + d)^n, x)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\cos^n(d+ex)\,dx = \int F^{c\,(a+b\,x)}\cos\left(d+e\,x\right)^n\,dx$$

input
```
int(F^(c*(a + b*x))*cos(d + e*x)^n,x)
```

output
```
int(F^(c*(a + b*x))*cos(d + e*x)^n, x)
```

3.11 $\int F^{c(a+bx)} \cos^3(d+ex)\,dx$

3.11.1 Optimal result

Integrand size = 18, antiderivative size = 199

$$\int F^{c(a+bx)} \cos^3(d+ex)\,dx = \frac{bcF^{c(a+bx)}\cos^3(d+ex)\log(F)}{9e^2+b^2c^2\log^2(F)} + \frac{6bce^2F^{c(a+bx)}\cos(d+ex)\log(F)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{3eF^{c(a+bx)}\cos^2(d+ex)\sin(d+ex)}{9e^2+b^2c^2\log^2(F)} + \frac{6e^3F^{c(a+bx)}\sin(d+ex)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)}$$

output
```
b*c*F^(c*(b*x+a))*cos(e*x+d)^3*ln(F)/(9*e^2+b^2*c^2*ln(F)^2)+6*b*c*e^2*F^(
c*(b*x+a))*cos(e*x+d)*ln(F)/(9*e^4+10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)^4)
+3*e*F^(c*(b*x+a))*cos(e*x+d)^2*sin(e*x+d)/(9*e^2+b^2*c^2*ln(F)^2)+6*e^3*F
^(c*(b*x+a))*sin(e*x+d)/(9*e^4+10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)^4)
```

3.11.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.78

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx$$
$$= \frac{F^{c(a+bx)}\left(bc\cos(3(d+ex))\log(F)\left(e^2+b^2c^2\log^2(F)\right)+3bc\cos(d+ex)\log(F)\left(9e^2+b^2c^2\log^2(F)\right)+6e(\right.}{4\left(9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F\right.}$$

input
```
Integrate[F^(c*(a + b*x))*Cos[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(b*c*Cos[3*(d + e*x)]*Log[F]*(e^2 + b^2*c^2*Log[F]^2) + 3
*b*c*Cos[d + e*x]*Log[F]*(9*e^2 + b^2*c^2*Log[F]^2) + 6*e*(5*e^2 + b^2*c^2
*Log[F]^2 + Cos[2*(d + e*x)]*(e^2 + b^2*c^2*Log[F]^2))*Sin[d + e*x]))/(4*(
9*e^4 + 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

3.11.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4935, 4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \ 4935$$
$$\frac{6e^2\int F^{c(a+bx)}\cos(d+ex)dx}{b^2c^2\log^2(F)+9e^2}+\frac{bc\log(F)\cos^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}+\frac{3e\sin(d+ex)\cos^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}$$
$$\Big\downarrow \ 4933$$
$$\frac{bc\log(F)\cos^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}+\frac{3e\sin(d+ex)\cos^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}+\frac{6e^2\left(\frac{e\sin(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}+\frac{bc\log(F)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}\right)}{b^2c^2\log^2(F)+9e^2}$$

input
```
Int[F^(c*(a + b*x))*Cos[d + e*x]^3,x]
```

output
```
(b*c*F^(c*(a + b*x))*Cos[d + e*x]^3*Log[F])/(9*e^2 + b^2*c^2*Log[F]^2) + (
3*e*F^(c*(a + b*x))*Cos[d + e*x]^2*Sin[d + e*x])/(9*e^2 + b^2*c^2*Log[F]^2
) + (6*e^2*((b*c*F^(c*(a + b*x))*Cos[d + e*x]*Log[F])/(e^2 + b^2*c^2*Log[F
]^2) + (e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)))/(9*e^2
+ b^2*c^2*Log[F]^2)
```

3.11.3.1 Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

3.11.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{F^{c(xb+a)} (bc\ln(F) (e^2+b^2c^2\ln(F)^2) \cos(3ex+3d)+ (3\ln(F)^2b^2c^2e+3e^3) \sin(3ex+3d)+3 (9e^2+b^2c^2\ln(F)^2) (\cos(ex+d)\ln(F}{4b^4c^4\ln(F)^4+40b^2c^2e^2\ln(F)^2+36e^4}$
risch	$\frac{3bc\,F^{c(xb+a)}\cos(ex+d)\ln(F)}{4\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{3e\,F^{c(xb+a)}\sin(ex+d)}{4\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{F^{c(xb+a)}bc\ln(F)\cos(3ex+3d)}{4b^2c^2\ln(F)^2+36e^2}+\frac{3e\,F^{c(xb+a)}\sin(3ex+3d)}{4\left(9e^2+b^2c^2\ln(F)^2\right)}$
norman	$\frac{\frac{\ln(F)bc\left(b^2c^2\ln(F)^2+7e^2\right)e^{c(xb+a)\ln(F)}}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}-\frac{12e\left(b^2c^2\ln(F)^2-e^2\right)e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}+\frac{6e\left(b^2c^2\ln(F)^2+3e^2\right)e^{c(xb+a)\ln(F)}\tan}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F}}{}$

input
```
int(F^(c*(b*x+a))*cos(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
F^(c*(b*x+a))*(b*c*ln(F)*(e^2+b^2*c^2*ln(F)^2)*cos(3*e*x+3*d)+(3*ln(F)^2*b
^2*c^2*e+3*e^3)*sin(3*e*x+3*d)+3*(9*e^2+b^2*c^2*ln(F)^2)*(cos(e*x+d)*ln(F)
*b*c+e*sin(e*x+d)))/(4*b^4*c^4*ln(F)^4+40*b^2*c^2*e^2*ln(F)^2+36*e^4)
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx = \frac{\left(b^3c^3\cos\left(ex+d\right)^3\log\left(F\right)^3+\left(bce^2\cos\left(ex+d\right)^3+6\,bce^2\cos\left(ex+d\right)\right)\log\left(F\right)+3\left(b^2c^2e\cos\left(ex+d\right)^2\log\right.}{b^4c^4\log\left(F\right)^4+10\,b^2c^2e^2\log\left(F\right)^2+9\,e^4}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^3,x, algorithm="fricas")
```

output
```
(b^3*c^3*cos(e*x + d)^3*log(F)^3 + (b*c*e^2*cos(e*x + d)^3 + 6*b*c*e^2*cos
(e*x + d))*log(F) + 3*(b^2*c^2*e*cos(e*x + d)^2*log(F)^2 + e^3*cos(e*x + d
)^2 + 2*e^3)*sin(e*x + d))*F^(b*c*x + a*c)/(b^4*c^4*log(F)^4 + 10*b^2*c^2*
e^2*log(F)^2 + 9*e^4)
```

3.11.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.23 (sec) , antiderivative size = 1671, normalized size of antiderivative = 8.40

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cos(e*x+d)**3,x)
```

output
```
Piecewise((x*cos(d)**3, Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*cos(d)**3, Eq(b,
 0) & Eq(e, 0)), (x*cos(d)**3, Eq(c, 0) & Eq(e, 0)), (3*I*F**(a*c + b*c*x)
*x*sin(I*b*c*x*log(F) - d)**3/8 + 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)
- d)**2*cos(I*b*c*x*log(F) - d)/8 + 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log
(F) - d)*cos(I*b*c*x*log(F) - d)**2/8 + 3*F**(a*c + b*c*x)*x*cos(I*b*c*x*l
og(F) - d)**3/8 - 5*I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F) - d)**3/(8*b*c*l
og(F)) - F**(a*c + b*c*x)*sin(I*b*c*x*log(F) - d)**2*cos(I*b*c*x*log(F) -
d)/(4*b*c*log(F)) - I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F) - d)*cos(I*b*c*x
*log(F) - d)**2/(b*c*log(F)) - 3*F**(a*c + b*c*x)*cos(I*b*c*x*log(F) - d)*
*3/(8*b*c*log(F)), Eq(e, -I*b*c*log(F))), (-I*F**(a*c + b*c*x)*x*sin(I*b*c
*x*log(F)/3 - d)**3/8 - 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 - d)**2*
cos(I*b*c*x*log(F)/3 - d)/8 + 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3
- d)*cos(I*b*c*x*log(F)/3 - d)**2/8 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F
)/3 - d)**3/8 + I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 - d)**3/(8*b*c*log
(F)) + 3*I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 - d)*cos(I*b*c*x*log(F)/3
 - d)**2/(4*b*c*log(F)) + 9*F**(a*c + b*c*x)*cos(I*b*c*x*log(F)/3 - d)**3/
(8*b*c*log(F)), Eq(e, -I*b*c*log(F)/3)), (-I*F**(a*c + b*c*x)*x*sin(I*b*c*
x*log(F)/3 + d)**3/8 - 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 + d)**2*c
os(I*b*c*x*log(F)/3 + d)/8 + 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 +
 d)*cos(I*b*c*x*log(F)/3 + d)**2/8 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log...
```

3.11.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(199) = 398$.

Time = 0.25 (sec) , antiderivative size = 813, normalized size of antiderivative = 4.09

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx$$
$$= \frac{\left(F^{ac}b^3c^3\cos(3\,d)\log(F)^3 + 3\,F^{ac}b^2c^2e\log(F)^2\sin(3\,d) + F^{ac}bce^2\cos(3\,d)\log(F) + 3\,F^{ac}e^3\sin(3\,d)\right)F^{bc}}{}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/8*((F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 + 3*F^(a*c)*b^2*c^2*e*log(F)^2*sin
(3*d) + F^(a*c)*b*c*e^2*cos(3*d)*log(F) + 3*F^(a*c)*e^3*sin(3*d))*F^(b*c*x
)*cos(3*e*x) + (F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 - 3*F^(a*c)*b^2*c^2*e*lo
g(F)^2*sin(3*d) + F^(a*c)*b*c*e^2*cos(3*d)*log(F) - 3*F^(a*c)*e^3*sin(3*d)
)*F^(b*c*x)*cos(3*e*x + 6*d) + 3*(F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 - F^(a
*c)*b^2*c^2*e*log(F)^2*sin(3*d) + 9*F^(a*c)*b*c*e^2*cos(3*d)*log(F) - 9*F^
(a*c)*e^3*sin(3*d))*F^(b*c*x)*cos(e*x + 4*d) + 3*(F^(a*c)*b^3*c^3*cos(3*d)
*log(F)^3 + F^(a*c)*b^2*c^2*e*log(F)^2*sin(3*d) + 9*F^(a*c)*b*c*e^2*cos(3*
d)*log(F) + 9*F^(a*c)*e^3*sin(3*d))*F^(b*c*x)*cos(e*x - 2*d) - (F^(a*c)*b^
3*c^3*log(F)^3*sin(3*d) - 3*F^(a*c)*b^2*c^2*e*cos(3*d)*log(F)^2 + F^(a*c)*
b*c*e^2*log(F)*sin(3*d) - 3*F^(a*c)*e^3*cos(3*d))*F^(b*c*x)*sin(3*e*x) + (
F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) + 3*F^(a*c)*b^2*c^2*e*cos(3*d)*log(F)^2
+ F^(a*c)*b*c*e^2*log(F)*sin(3*d) + 3*F^(a*c)*e^3*cos(3*d))*F^(b*c*x)*sin(
3*e*x + 6*d) + 3*(F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) + F^(a*c)*b^2*c^2*e*co
s(3*d)*log(F)^2 + 9*F^(a*c)*b*c*e^2*log(F)*sin(3*d) + 9*F^(a*c)*e^3*cos(3*
d))*F^(b*c*x)*sin(e*x + 4*d) - 3*(F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) - F^(a
*c)*b^2*c^2*e*cos(3*d)*log(F)^2 + 9*F^(a*c)*b*c*e^2*log(F)*sin(3*d) - 9*F^
(a*c)*e^3*cos(3*d))*F^(b*c*x)*sin(e*x - 2*d))/(b^4*c^4*cos(3*d)^2*log(F)^4
 + b^4*c^4*log(F)^4*sin(3*d)^2 + 9*(cos(3*d)^2 + sin(3*d)^2)*e^4 + 10*(b^2
*c^2*cos(3*d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(3*d)^2)*e^2)
```

3.11.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1271, normalized size of antiderivative = 6.39

$$\int F^{c(a+bx)} \cos^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^3,x, algorithm="giac")
```

output
```
1/4*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/
2*pi*a*c + 3*e*x + 3*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn
(F) - pi*b*c + 6*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 6*e)*sin(1/2*pi*b*c*x*s
gn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 3*e*x + 3*d)/(4*b^
2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 6*e)^2))*e^(b*c*x*log(abs(
F)) + a*c*log(abs(F))) + 3/4*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x
 + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)*log(abs(F))/(4*b^2*c^2*log(ab
s(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 2*
e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c
 + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*
e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 3/4*(2*b*c*cos(1/2*pi*b*c*x*sgn(
F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)*log(abs(F))/
(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2) + (pi*b*c*sgn
(F) - pi*b*c - 2*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sg
n(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - p
i*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/4*(2*b*c*cos(
1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 3*e*
x - 3*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c -
6*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 6*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*
b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 3*e*x - 3*d)/(4*b^2*c^2*log(ab...
```

3.11.9 Mupad [B] (verification not implemented)

Time = 28.71 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.96

$$\begin{aligned}
&\int F^{c(a+bx)}\cos^3(d+ex)\,dx \\
&= -\frac{F^{c\,(a+b\,x)}\,(\cos(e\,x)+\sin(e\,x)\;1\mathrm{i})\;(\cos(d)+\sin(d)\;1\mathrm{i})\;3\mathrm{i}}{8\;(e-b\,c\,\ln(F)\;1\mathrm{i})} \\
&\quad -\frac{F^{c\,(a+b\,x)}\,(\cos(3\,e\,x)-\sin(3\,e\,x)\;1\mathrm{i})\;(\cos(3\,d)-\sin(3\,d)\;1\mathrm{i})}{8\;(-b\,c\,\ln(F)+e\,3\mathrm{i})} \\
&\quad -\frac{F^{c\,(a+b\,x)}\,(\cos(3\,e\,x)+\sin(3\,e\,x)\;1\mathrm{i})\;(\cos(3\,d)+\sin(3\,d)\;1\mathrm{i})\;1\mathrm{i}}{8\;(3\,e-b\,c\,\ln(F)\;1\mathrm{i})} \\
&\quad -\frac{3\,F^{c\,(a+b\,x)}\,(\cos(e\,x)-\sin(e\,x)\;1\mathrm{i})\;(\cos(d)-\sin(d)\;1\mathrm{i})}{8\;(-b\,c\,\ln(F)+e\,1\mathrm{i})}
\end{aligned}$$

input
```
int(F^(c*(a + b*x))*cos(d + e*x)^3,x)
```

output
```
- (F^(c*(a + b*x))*(cos(e*x) + sin(e*x)*1i)*(cos(d) + sin(d)*1i)*3i)/(8*(e
 - b*c*log(F)*1i)) - (F^(c*(a + b*x))*(cos(3*e*x) - sin(3*e*x)*1i)*(cos(3*
d) - sin(3*d)*1i))/(8*(e*3i - b*c*log(F))) - (F^(c*(a + b*x))*(cos(3*e*x)
+ sin(3*e*x)*1i)*(cos(3*d) + sin(3*d)*1i)*1i)/(8*(3*e - b*c*log(F)*1i)) -
(3*F^(c*(a + b*x))*(cos(e*x) - sin(e*x)*1i)*(cos(d) - sin(d)*1i))/(8*(e*1i
 - b*c*log(F)))
```

3.11. $\int F^{c(a+bx)} \cos^3(d+ex)\, dx$

3.12 $\int F^{c(a+bx)}\cos^2(d+ex)\,dx$

3.12.1 Optimal result

Integrand size = 18, antiderivative size = 128

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx=\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)}+\frac{bcF^{c(a+bx)}\cos^2(d+ex)\log(F)}{4e^2+b^2c^2\log^2(F)}+\frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)}$$

output
```
2*e^2*F^(c*(b*x+a))/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)+b*c*F^(c*(b*x+a))*co
s(e*x+d)^2*ln(F)/(4*e^2+b^2*c^2*ln(F)^2)+2*e*F^(c*(b*x+a))*cos(e*x+d)*sin(
e*x+d)/(4*e^2+b^2*c^2*ln(F)^2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.66

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx=\frac{F^{c(a+bx)}\left(4e^2+b^2c^2\log^2(F)+b^2c^2\cos(2(d+ex))\log^2(F)+2bce\log(F)\sin(2(d+ex))\right)}{8bce^2\log(F)+2b^3c^3\log^3(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Cos[d + e*x]^2,x]
```

output
```
(F^(c*(a + b*x))*(4*e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cos[2*(d + e*x)]*Log[
F]^2 + 2*b*c*e*Log[F]*Sin[2*(d + e*x)]))/(8*b*c*e^2*Log[F] + 2*b^3*c^3*Log
[F]^3)
```

3.12.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4935, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \; 4935$$

$$\frac{2e^2\int F^{c(a+bx)}dx}{b^2c^2\log^2(F)+4e^2}+\frac{bc\log(F)\cos^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}$$

$$\Big\downarrow \; 2624$$

$$\frac{bc\log(F)\cos^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(b^2c^2\log^2(F)+4e^2\right)}$$

input
```
Int[F^(c*(a + b*x))*Cos[d + e*x]^2,x]
```

output
```
(2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 + b^2*c^2*Log[F]^2)) + (b*c*F^(
c*(a + b*x))*Cos[d + e*x]^2*Log[F])/(4*e^2 + b^2*c^2*Log[F]^2) + (2*e*F^(c
*(a + b*x))*Cos[d + e*x]*Sin[d + e*x])/(4*e^2 + b^2*c^2*Log[F]^2)
```

3.12.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

3.12.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{F^{c(xb+a)}\left(b^2c^2\ln(F)^2\cos(2ex+2d)+b^2c^2\ln(F)^2+2e\sin(2ex+2d)bc\ln(F)+4e^2\right)}{2bc\ln(F)\left(4e^2+b^2c^2\ln(F)^2\right)}$
risch	$\frac{F^{c(xb+a)}}{2bc\ln(F)}+\frac{F^{c(xb+a)}bc\ln(F)\cos(2ex+2d)}{2b^2c^2\ln(F)^2+8e^2}+\frac{e\,F^{c(xb+a)}\sin(2ex+2d)}{4e^2+b^2c^2\ln(F)^2}$
norman	$\frac{\frac{\left(b^2c^2\ln(F)^2+2e^2\right)e^{c(xb+a)\ln(F)}}{bc\ln(F)\left(4e^2+b^2c^2\ln(F)^2\right)}+\frac{\left(b^2c^2\ln(F)^2+2e^2\right)e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4}{bc\ln(F)\left(4e^2+b^2c^2\ln(F)^2\right)}+\frac{4e\,e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{4e^2+b^2c^2\ln(F)^2}-\frac{4e\,e^{c(xb+a)\ln(F}}{4e^2+b^2c^2}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)^2}$

input
```
int(F^(c*(b*x+a))*cos(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*F^(c*(b*x+a))*(b^2*c^2*ln(F)^2*cos(2*e*x+2*d)+b^2*c^2*ln(F)^2+2*e*sin(
2*e*x+2*d)*b*c*ln(F)+4*e^2)/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.61

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx = \frac{\left(b^2c^2\cos\left(ex+d\right)^2\log\left(F\right)^2+2\,bce\cos\left(ex+d\right)\log\left(F\right)\sin\left(ex+d\right)+2\,e^2\right)F^{bcx+ac}}{b^3c^3\log\left(F\right)^3+4\,bce^2\log\left(F\right)}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^2,x, algorithm="fricas")
```

output
```
(b^2*c^2*cos(e*x + d)^2*log(F)^2 + 2*b*c*e*cos(e*x + d)*log(F)*sin(e*x + d
) + 2*e^2)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + 4*b*c*e^2*log(F))
```

3.12.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 741, normalized size of antiderivative = 5.79

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx$$

$$= \begin{cases} x\cos^2(d) \\ \frac{x\sin^2(d+ex)}{2} + \frac{x\cos^2(d+ex)}{2} + \frac{\sin(d+ex)\cos(d+ex)}{2e} \\ F^{ac}\left(\frac{x\sin^2(d+ex)}{2} + \frac{x\cos^2(d+ex)}{2} + \frac{\sin(d+ex)\cos(d+ex)}{2e}\right) \\ \frac{x\sin^2(d+ex)}{2} + \frac{x\cos^2(d+ex)}{2} + \frac{\sin(d+ex)\cos(d+ex)}{2e} \\ -\frac{F^{ac+bcx}x\sin^2\left(\frac{ibcx\log(F)}{2}-d\right)}{4} + \frac{iF^{ac+bcx}x\sin\left(\frac{ibcx\log(F)}{2}-d\right)\cos\left(\frac{ibcx\log(F)}{2}-d\right)}{2} + \frac{F^{ac+bcx}x\cos^2\left(\frac{ibcx\log(F)}{2}-d\right)}{4} + \frac{iF^{ac+bcx}\,\mathrm{s}}{} \\ -\frac{F^{ac+bcx}x\sin^2\left(\frac{ibcx\log(F)}{2}+d\right)}{4} + \frac{iF^{ac+bcx}x\sin\left(\frac{ibcx\log(F)}{2}+d\right)\cos\left(\frac{ibcx\log(F)}{2}+d\right)}{2} + \frac{F^{ac+bcx}x\cos^2\left(\frac{ibcx\log(F)}{2}+d\right)}{4} + \frac{F^{ac+bcx}\,\mathrm{si}}{} \\ \frac{F^{ac+bcx}b^2c^2\log(F)^2\cos^2(d+ex)}{b^3c^3\log(F)^3+4bce^2\log(F)} + \frac{2F^{ac+bcx}bce\log(F)\sin(d+ex)\cos(d+ex)}{b^3c^3\log(F)^3+4bce^2\log(F)} + \frac{2F^{ac+bcx}e^2\sin^2(d+ex)}{b^3c^3\log(F)^3+4bce^2\log(F)} + \frac{2F^{ac+bcx}e^2\cos^2(d+e}{b^3c^3\log(F)^3+4bce^2\log} \end{cases}$$

input

```
integrate(F**(c*(b*x+a))*cos(e*x+d)**2,x)
```

output
```
Piecewise((x*cos(d)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (x*sin
(d + e*x)**2/2 + x*cos(d + e*x)**2/2 + sin(d + e*x)*cos(d + e*x)/(2*e), Eq
(F, 1)), (F**(a*c)*(x*sin(d + e*x)**2/2 + x*cos(d + e*x)**2/2 + sin(d + e*
x)*cos(d + e*x)/(2*e)), Eq(b, 0)), (x*sin(d + e*x)**2/2 + x*cos(d + e*x)**
2/2 + sin(d + e*x)*cos(d + e*x)/(2*e), Eq(c, 0)), (-F**(a*c + b*c*x)*x*sin
(I*b*c*x*log(F)/2 - d)**2/4 + I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 -
d)*cos(I*b*c*x*log(F)/2 - d)/2 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/2 -
 d)**2/4 + I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/2 - d)*cos(I*b*c*x*log(F)
/2 - d)/(2*b*c*log(F)) + F**(a*c + b*c*x)*cos(I*b*c*x*log(F)/2 - d)**2/(b*
c*log(F)), Eq(e, -I*b*c*log(F)/2)), (-F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F
)/2 + d)**2/4 + I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 + d)*cos(I*b*c*x
*log(F)/2 + d)/2 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/2 + d)**2/4 + F**
(a*c + b*c*x)*sin(I*b*c*x*log(F)/2 + d)**2/(b*c*log(F)) - 3*I*F**(a*c + b*
c*x)*sin(I*b*c*x*log(F)/2 + d)*cos(I*b*c*x*log(F)/2 + d)/(2*b*c*log(F)), E
q(e, I*b*c*log(F)/2)), (F**(a*c + b*c*x)*b**2*c**2*log(F)**2*cos(d + e*x)*
*2/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) + 2*F**(a*c + b*c*x)*b*c*e*lo
g(F)*sin(d + e*x)*cos(d + e*x)/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) +
 2*F**(a*c + b*c*x)*e**2*sin(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*c*e**2
*log(F)) + 2*F**(a*c + b*c*x)*e**2*cos(d + e*x)**2/(b**3*c**3*log(F)**3 +
4*b*c*e**2*log(F)), True))
```

3.12.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(128) = 256$.

Time = 0.23 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.78

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx$$
$$= \frac{\left(F^{ac}b^2c^2\cos(2\,d)\log(F)^2 + 2\,F^{ac}bce\log(F)\sin(2\,d)\right)F^{bcx}\cos(2\,ex) + \left(F^{ac}b^2c^2\cos(2\,d)\log(F)^2 - 2\,F^{ac}\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d))
*F^(b*c*x)*cos(2*e*x) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*c
*e*log(F)*sin(2*d))*F^(b*c*x)*cos(2*e*x + 4*d) - (F^(a*c)*b^2*c^2*log(F)^2
*sin(2*d) - 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2*e*x) + (F^(a*
c)*b^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*
sin(2*e*x + 4*d) + 2*(F^(a*c)*b^2*c^2*cos(2*d)^2*log(F)^2 + F^(a*c)*b^2*c^
2*log(F)^2*sin(2*d)^2 + 4*(F^(a*c)*cos(2*d)^2 + F^(a*c)*sin(2*d)^2)*e^2)*F
^(b*c*x))/(b^3*c^3*cos(2*d)^2*log(F)^3 + b^3*c^3*log(F)^3*sin(2*d)^2 + 4*(
b*c*cos(2*d)^2*log(F) + b*c*log(F)*sin(2*d)^2)*e^2)
```

3.12.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 915, normalized size of antiderivative = 7.15

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d)^2,x, algorithm="giac")
```

output
```
1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/
2*pi*a*c + 2*e*x + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn
(F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*sin(1/2*pi*b*c*x*s
gn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*e*x + 2*d)/(4*b^
2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*log(abs(
F)) + a*c*log(abs(F))) + 1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x
 + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)*log(abs(F))/(4*b^2*c^2*lo
g(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c
- 4*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi
*a*c - 2*e*x - 2*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4
*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + (2*b*c*cos(-1/2*pi*b*c*x
*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^
2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*
c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*
c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(ab
s(F)) + a*c*log(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x
+ 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(4*I*pi*b*c*sgn(F)
 - 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F)
+ 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - 2*I*e*x - 2*I*d)/(
-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e))*e^(b*c*x...
```

3.12.9 Mupad [B] (verification not implemented)

Time = 28.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.77

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx$$
$$=\frac{2\,F^{a\,c+b\,c\,x}\,e^2+F^{a\,c+b\,c\,x}\,b^2\,c^2\cos\left(d+e\,x\right)^2\ln\left(F\right)^2+2\,F^{a\,c+b\,c\,x}\,b\,c\,e\,\cos\left(d+e\,x\right)\,\sin\left(d+e\,x\right)\,\ln\left(F\right)}{b^3\,c^3\ln\left(F\right)^3+4\,b\,c\,e^2\,\ln\left(F\right)}$$

input
```
int(F^(c*(a + b*x))*cos(d + e*x)^2,x)
```

output
```
(2*F^(a*c + b*c*x)*e^2 + F^(a*c + b*c*x)*b^2*c^2*cos(d + e*x)^2*log(F)^2 +
 2*F^(a*c + b*c*x)*b*c*e*cos(d + e*x)*sin(d + e*x)*log(F))/(b^3*c^3*log(F)
^3 + 4*b*c*e^2*log(F))
```

3.13 $\int F^{c(a+bx)} \cos(d+ex)\,dx$

3.13.1 Optimal result

Integrand size = 16, antiderivative size = 72

$$\int F^{c(a+bx)} \cos(d+ex)\,dx = \frac{bcF^{c(a+bx)}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)} + \frac{eF^{c(a+bx)}\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

output
```
b*c*F^(c*(b*x+a))*cos(e*x+d)*ln(F)/(e^2+b^2*c^2*ln(F)^2)+e*F^(c*(b*x+a))*s
in(e*x+d)/(e^2+b^2*c^2*ln(F)^2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.65

$$\int F^{c(a+bx)} \cos(d+ex)\,dx = \frac{F^{c(a+bx)}(bc\cos(d+ex)\log(F)+e\sin(d+ex))}{e^2+b^2c^2\log^2(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Cos[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(b*c*Cos[d + e*x]*Log[F] + e*Sin[d + e*x]))/(e^2 + b^2*c^
2*Log[F]^2)
```

3.13.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 4933}$$

$$\frac{e\sin(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}+\frac{bc\log(F)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}$$

input
```
Int[F^(c*(a + b*x))*Cos[d + e*x],x]
```

output
```
(b*c*F^(c*(a + b*x))*Cos[d + e*x]*Log[F])/(e^2 + b^2*c^2*Log[F]^2) + (e*F^
(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)
```

3.13.3.1 Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.13.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{F^{c(xb+a)}(\cos(ex+d)\ln(F)bc+e\sin(ex+d))}{e^2+b^2c^2\ln(F)^2}$	48
risch	$\frac{bc\,F^{c(xb+a)}\cos(ex+d)\ln(F)}{e^2+b^2c^2\ln(F)^2}+\frac{e\,F^{c(xb+a)}\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}$	73
norman	$\frac{\frac{bc\ln(F)\mathrm{e}^{c(xb+a)\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{2e\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}-\frac{bc\ln(F)\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$	133

input
```
int(F^(c*(b*x+a))*cos(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
F^(c*(b*x+a))*(cos(e*x+d)*ln(F)*b*c+e*sin(e*x+d))/(e^2+b^2*c^2*ln(F)^2)
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

$$\int F^{c(a+bx)}\cos(d+ex)\,dx=\frac{(bc\cos{(ex+d)}\log{(F)}+e\sin{(ex+d)})F^{bcx+ac}}{b^2c^2\log{(F)}^2+e^2}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d),x, algorithm="fricas")
```

output
```
(b*c*cos(e*x + d)*log(F) + e*sin(e*x + d))*F^(b*c*x + a*c)/(b^2*c^2*log(F)
^2 + e^2)
```

3.13.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.97

$$\int F^{c(a+bx)}\cos(d+ex)\,dx$$

$$=\begin{cases} x\cos{(d)} & \text{for } F=1\wedge e=0\\ F^{ac}x\cos{(d)} & \text{for } b=0\wedge e=0\\ x\cos{(d)} & \text{for } c=0\wedge e=0\\ \frac{iF^{ac+bcx}x\sin{(ibcx\log{(F)}-d)}}{2}+\frac{F^{ac+bcx}x\cos{(ibcx\log{(F)}-d)}}{2}-\frac{iF^{ac+bcx}\sin{(ibcx\log{(F)}-d)}}{2bc\log{(F)}} & \text{for } e=-ibc\log{(F)}\\ \frac{iF^{ac+bcx}x\sin{(ibcx\log{(F)}+d)}}{2}+\frac{F^{ac+bcx}x\cos{(ibcx\log{(F)}+d)}}{2}-\frac{iF^{ac+bcx}\sin{(ibcx\log{(F)}+d)}}{2bc\log{(F)}} & \text{for } e=ibc\log{(F)}\\ \frac{F^{ac+bcx}bc\log{(F)}\cos{(d+ex)}}{b^2c^2\log{(F)}^2+e^2}+\frac{F^{ac+bcx}e\sin{(d+ex)}}{b^2c^2\log{(F)}^2+e^2} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*cos(e*x+d),x)
```

output
```
Piecewise((x*cos(d), Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*cos(d), Eq(b, 0) &
Eq(e, 0)), (x*cos(d), Eq(c, 0) & Eq(e, 0)), (I*F**(a*c + b*c*x)*x*sin(I*b*
c*x*log(F) - d)/2 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F) - d)/2 - I*F**(a
*c + b*c*x)*sin(I*b*c*x*log(F) - d)/(2*b*c*log(F)), Eq(e, -I*b*c*log(F))),
 (I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F) + d)/2 + F**(a*c + b*c*x)*x*cos(
I*b*c*x*log(F) + d)/2 - I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F) + d)/(2*b*c*
log(F)), Eq(e, I*b*c*log(F))), (F**(a*c + b*c*x)*b*c*log(F)*cos(d + e*x)/(
b**2*c**2*log(F)**2 + e**2) + F**(a*c + b*c*x)*e*sin(d + e*x)/(b**2*c**2*l
og(F)**2 + e**2), True))
```

3.13.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(72) = 144$.

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.67

$$\int F^{c(a+bx)}\cos(d+ex)\,dx$$
$$= \frac{(F^{ac}bc\cos(d)\log(F) - F^{ac}e\sin(d))F^{bcx}\cos(ex+2d) + (F^{ac}bc\cos(d)\log(F) + F^{ac}e\sin(d))F^{bcx}\cos(e}{2\left(b^2c^2\cos(d)^2\log(F)^2 + b^2c^2\log\right.}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d),x, algorithm="maxima")
```

output
```
1/2*((F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x + 2*
d) + (F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x) + (
F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x + 2*d) - (
F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x))/(b^2*c^2
*cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2
)
```

3.13.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 631, normalized size of antiderivative = 8.76

$$\int F^{c(a+bx)}\cos(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cos(e*x+d),x, algorithm="giac")
```

output
```
(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi
*a*c + e*x + d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi
*b*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 2*e)*sin(1/2*pi*b*c*x*sgn(F) -
1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(ab
s(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*lo
g(abs(F))) + (2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sg
n(F) - 1/2*pi*a*c - e*x - d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*
c*sgn(F) - pi*b*c - 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 2*e)*sin(1/2*pi*b*
c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b
^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs
(F)) + a*c*log(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x +
 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*
I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*
I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-2*I*pi*b*
c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) +
 a*c*log(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I
*pi*a*c*sgn(F) - 1/2*I*pi*a*c - I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b
*c + 4*b*c*log(abs(F)) - 4*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b
*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*sgn(
F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*...
```

3.13.9 Mupad [B] (verification not implemented)

Time = 27.93 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

$$\int F^{c(a+bx)}\cos(d+ex)\,dx = \frac{F^{a\,c+b\,c\,x}\,(e\,\sin(d+e\,x)+b\,c\,\cos(d+e\,x)\,\ln(F))}{b^2\,c^2\ln(F)^2+e^2}$$

input
```
int(F^(c*(a + b*x))*cos(d + e*x),x)
```

output
```
(F^(a*c + b*c*x)*(e*sin(d + e*x) + b*c*cos(d + e*x)*log(F)))/(e^2 + b^2*c^
2*log(F)^2)
```

3.14 $\int F^{c(a+bx)} \sec(d+ex)\,dx$

3.14.1 Optimal result

Integrand size = 16, antiderivative size = 84

$$\int F^{c(a+bx)} \sec(d+ex)\,dx = \frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)}{ie+bc\log(F)}$$

output
```
2*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([1, 1/2*(e-I*b*c*ln(F))/e],[3/2-1
/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))/(b*c*ln(F)+I*e)
```

3.14.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)} \sec(d+ex)\,dx = \frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{ibc\log(F)}{2e},\frac{3}{2}-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{ie+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sec[d + e*x],x]
```

output
```
(2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, 1/2 - ((I/2)*b*c*L
og[F])/e, 3/2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/(I*e + b*c*Lo
g[F])
```

3.14.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \ 4951$$

$$\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)}{bc\log(F)+ie}$$

input
```
Int[F^(c*(a + b*x))*Sec[d + e*x],x]
```

output
```
(2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])
/(2*e), (3 - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))])/(I*e + b*c*Log[F]
)
```

3.14.3.1 Defintions of rubi rules used

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.14.4 Maple [F]

$$\int F^{c(xb+a)}\sec(ex+d)\,dx$$

input
```
int(F^(c*(b*x+a))*sec(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*sec(e*x+d),x)
```

3.14.5 Fricas [F]

$$\int F^{c(a+bx)}\sec(d+ex)\,dx = \int F^{(bx+a)c}\sec\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sec(e*x + d), x)
```

3.14.6 Sympy [F]

$$\int F^{c(a+bx)}\sec(d+ex)\,dx = \int F^{c(a+bx)}\sec\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*sec(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*sec(d + e*x), x)
```

3.14.7 Maxima [F]

$$\int F^{c(a+bx)}\sec(d+ex)\,dx = \int F^{(bx+a)c}\sec\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d),x, algorithm="maxima")
```

output
```
2*(F^(b*c*x)*F^(a*c)*b*c*cos(e*x + d)*log(F) - F^(b*c*x)*F^(a*c)*e*sin(e*x
 + d) + (F^(b*c*x)*F^(a*c)*b*c*cos(e*x + d)*log(F) - F^(b*c*x)*F^(a*c)*e*s
in(e*x + d))*cos(2*e*x + 2*d) + 2*(F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^
3 + (F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*cos(2*e*x + 2*d)^2 + (F^(a*
c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*sin(2*e*x + 2*d)^2 + 2*(F^(a*c)*b^2*c
^2*e*log(F)^2 + F^(a*c)*e^3)*cos(2*e*x + 2*d))*integrate((F^(b*c*x)*b*c*lo
g(F)*sin(e*x + d) + F^(b*c*x)*e*cos(e*x + d) + (F^(b*c*x)*b*c*log(F)*sin(e
*x + d) + F^(b*c*x)*e*cos(e*x + d))*cos(4*e*x + 4*d) + 2*(F^(b*c*x)*b*c*lo
g(F)*sin(e*x + d) + F^(b*c*x)*e*cos(e*x + d))*cos(2*e*x + 2*d) - (F^(b*c*x
)*b*c*cos(e*x + d)*log(F) - F^(b*c*x)*e*sin(e*x + d))*sin(4*e*x + 4*d) - 2
*(F^(b*c*x)*b*c*cos(e*x + d)*log(F) - F^(b*c*x)*e*sin(e*x + d))*sin(2*e*x
+ 2*d))/(b^2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + e^2)*cos(4*e*x + 4*d)^2 +
4*(b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d)^2 + (b^2*c^2*log(F)^2 + e^2)*s
in(4*e*x + 4*d)^2 + 4*(b^2*c^2*log(F)^2 + e^2)*sin(4*e*x + 4*d)*sin(2*e*x
+ 2*d) + 4*(b^2*c^2*log(F)^2 + e^2)*sin(2*e*x + 2*d)^2 + e^2 + 2*(b^2*c^2*
log(F)^2 + e^2 + 2*(b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d))*cos(4*e*x +
4*d) + 4*(b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d)), x) + (F^(b*c*x)*F^(a*
c)*b*c*log(F)*sin(e*x + d) + F^(b*c*x)*F^(a*c)*e*cos(e*x + d))*sin(2*e*x +
 2*d))/(b^2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d)^2 + (
b^2*c^2*log(F)^2 + e^2)*sin(2*e*x + 2*d)^2 + e^2 + 2*(b^2*c^2*log(F)^2 ...
```

3.14.8 Giac [F]

$$\int F^{c(a+bx)}\sec(d+ex)\,dx = \int F^{(bx+a)c}\sec(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sec(e*x + d), x)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)} \sec(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\cos\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))/cos(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))/cos(d + e*x), x)
```

3.15 $\int F^{c(a+bx)} \sec^2(d+ex)\,dx$

3.15.1 Optimal result

Integrand size = 18, antiderivative size = 80

$$\int F^{c(a+bx)} \sec^2(d+ex)\,dx = \frac{4e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

output
```
4*exp(2*I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-1/2*I*b*c*ln(F)/e],[2-1/2
*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))/(2*I*e+b*c*ln(F))
```

3.15.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)} \sec^2(d+ex)\,dx = \frac{4e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sec[d + e*x]^2,x]
```

output
```
(4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c
*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/((2*I)*e + b*
c*Log[F])
```

3.15.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \; 4951$$

$$\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{bc\log(F)+2ie}$$

input
```
Int[F^(c*(a + b*x))*Sec[d + e*x]^2,x]
```

output
```
(4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c
*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/((2*I)*e + b*
c*Log[F])
```

3.15.3.1 Defintions of rubi rules used

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.15.4 Maple [F]

$$\int F^{c(xb+a)}\sec(ex+d)^2\,dx$$

input
```
int(F^(c*(b*x+a))*sec(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*sec(e*x+d)^2,x)
```

3.15.5 Fricas [F]

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx = \int F^{(bx+a)c}\sec(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sec(e*x + d)^2, x)
```

3.15.6 Sympy [F]

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx = \int F^{c(a+bx)}\sec^2(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*sec(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*sec(d + e*x)**2, x)
```

3.15.7 Maxima [F]

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx = \int F^{(bx+a)c}\sec(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^2,x, algorithm="maxima")
```

output
```
4*(24*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F) + 2*(F^(a*c)*b^3*c^3*log(F)^3 + 16*
F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d)^2 + 2*(F^(a*c)*b^3*c^3*
log(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(2*e*x + 2*d)^2 + (F^(a
*c)*b^3*c^3*log(F)^3 + 64*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*
d) - 2*(5*F^(a*c)*b^2*c^2*e*log(F)^2 - 16*F^(a*c)*e^3)*F^(b*c*x)*sin(2*e*x
 + 2*d) + (24*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F) + (F^(a*c)*b^3*c^3*log(F)^3
 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) - 2*(F^(a*c)*b^2*
c^2*e*log(F)^2 + 16*F^(a*c)*e^3)*F^(b*c*x)*sin(2*e*x + 2*d))*cos(4*e*x + 4
*d) - 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64
*F^(a*c)*b*c*e^5*log(F) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3
*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*cos(4*e*x + 4*d)^2 + 4*(F^(a*c)
*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5
*log(F))*cos(2*e*x + 2*d)^2 + (F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3
*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*sin(4*e*x + 4*d)^2 + 4*(F^(
a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c
*e^5*log(F))*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*(F^(a*c)*b^5*c^5*e*log(
F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*sin(2*
e*x + 2*d)^2 + 2*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(
F)^3 + 64*F^(a*c)*b*c*e^5*log(F) + 2*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a
*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*cos(2*e*x + 2*d))...
```

3.15.8 Giac [F]

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx = \int F^{(bx+a)c}\sec(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sec(e*x + d)^2, x)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\cos\left(d+e\,x\right)^2}\,dx$$

input
```
int(F^(c*(a + b*x))/cos(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))/cos(d + e*x)^2, x)
```

3.16 $\int F^{c(a+bx)} \sec^3(d+ex)\,dx$

3.16.1 Optimal result

Integrand size = 18, antiderivative size = 141

$$\int F^{c(a+bx)} \sec^3(d+ex)\,dx = -\frac{e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)(ie-bc\log(F))}{e^2} - \frac{bcF^{c(a+bx)}\log(F)\sec(d+ex)}{2e^2} + \frac{F^{c(a+bx)}\sec(d+ex)\tan(d+ex)}{2e}$$

output
```
-exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([1, 1/2*(e-I*b*c*ln(F))/e],[3/2-1/
2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))*(I*e-b*c*ln(F))/e^2-1/2*b*c*F^(c*(b*x+
a))*ln(F)*sec(e*x+d)/e^2+1/2*F^(c*(b*x+a))*sec(e*x+d)*tan(e*x+d)/e
```

3.16.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int F^{c(a+bx)} \sec^3(d+ex)\,dx = \frac{F^{c(a+bx)}\left(2e^{i(d+ex)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{3}{2}-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)(-ie+bc\log(F))+\sec(d -\right.}{2e^2}$$

input
```
Integrate[F^(c*(a + b*x))*Sec[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(2*E^(I*(d + e*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F]
)/(2*e), 3/2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))]*((-I)*e + b*c*L
og[F]) + Sec[d + e*x]*(-(b*c*Log[F]) + e*Tan[d + e*x])))/(2*e^2)
```

3.16.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4948, 4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{4948}$$

$$\frac{1}{2}\left(\frac{b^2c^2\log^2(F)}{e^2}+1\right)\int F^{c(a+bx)}\sec(d+ex)dx-\frac{bc\log(F)\sec(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tan(d+ex)\sec(d+ex)F^{c(a+bx)}}{2e}$$

$$\Big\downarrow \text{4951}$$

$$\frac{e^{i(d+ex)}F^{c(a+bx)}\left(\frac{b^2c^2\log^2(F)}{e^2}+1\right)\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)}{bc\log(F)+ie}-\frac{bc\log(F)\sec(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tan(d+ex)\sec(d+ex)F^{c(a+bx)}}{2e}$$

input
```
Int[F^(c*(a + b*x))*Sec[d + e*x]^3,x]
```

output
```
(E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])/(
2*e), (3 - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))]*(1 + (b^2*c^2*Log[F]
^2)/e^2))/(I*e + b*c*Log[F]) - (b*c*F^(c*(a + b*x))*Log[F]*Sec[d + e*x])/(
2*e^2) + (F^(c*(a + b*x))*Sec[d + e*x]*Tan[d + e*x])/(2*e)
```

3.16.3.1 Defintions of rubi rules used

rule 4948
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(
e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2))   Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b,
 c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && N
eQ[n, 2]
```

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.16.4 Maple [F]

$$\int F^{c(xb+a)} \sec(ex+d)^3 \, dx$$

input
```
int(F^(c*(b*x+a))*sec(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*sec(e*x+d)^3,x)
```

3.16.5 Fricas [F]

$$\int F^{c(a+bx)} \sec^3(d+ex) \, dx = \int F^{(bx+a)c} \sec(ex+d)^3 \, dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sec(e*x + d)^3, x)
```

3.16.6 Sympy [F]

$$\int F^{c(a+bx)} \sec^3(d+ex)\,dx = \int F^{c(a+bx)} \sec^3(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*sec(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*sec(d + e*x)**3, x)
```

3.16.7 Maxima [F]

$$\int F^{c(a+bx)} \sec^3(d+ex)\,dx = \int F^{(bx+a)c} \sec(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^3,x, algorithm="maxima")
```

output
```
8*(48*F^(b*c*x)*F^(a*c)*b*c*e^2*cos(e*x + d)*log(F) + 6*(F^(a*c)*b^2*c^2*e
*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*sin(e*x + d) + (48*F^(b*c*x)*F^(a*c)
*b*c*e^2*cos(e*x + d)*log(F) + (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*
e^2*log(F))*F^(b*c*x)*cos(3*e*x + 3*d) - 3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2
5*F^(a*c)*e^3)*F^(b*c*x)*sin(3*e*x + 3*d) + 6*(F^(a*c)*b^2*c^2*e*log(F)^2
- 15*F^(a*c)*e^3)*F^(b*c*x)*sin(e*x + d))*cos(6*e*x + 6*d) + 3*(48*F^(b*c*
x)*F^(a*c)*b*c*e^2*cos(e*x + d)*log(F) + (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^
(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(3*e*x + 3*d) - 3*(F^(a*c)*b^2*c^2*e*lo
g(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x)*sin(3*e*x + 3*d) + 6*(F^(a*c)*b^2*c^2*e
*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*sin(e*x + d))*cos(4*e*x + 4*d) + (3*
(F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x
 + 2*d) + 9*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x)*sin(2*
e*x + 2*d) + (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c
*x))*cos(3*e*x + 3*d) + 18*(8*F^(b*c*x)*F^(a*c)*b*c*e^2*cos(e*x + d)*log(F
) + (F^(a*c)*b^2*c^2*e*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*sin(e*x + d))*
cos(2*e*x + 2*d) - 6*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*
e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*
b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*F^(a
*c)*e^6*sin(d) + (F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*
log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b...
```

3.16.8 Giac [F]

$$\int F^{c(a+bx)}\sec^3(d+ex)\,dx = \int F^{(bx+a)c}\sec(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sec(e*x + d)^3, x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sec^3(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\cos(d+e\,x)^3}\,dx$$

input
```
int(F^(c*(a + b*x))/cos(d + e*x)^3,x)
```

output
```
int(F^(c*(a + b*x))/cos(d + e*x)^3, x)
```

3.17 $\int F^{c(a+bx)} \sec^4(d+ex)\,dx$

3.17.1 Optimal result

Integrand size = 18, antiderivative size = 143

$$\int F^{c(a+bx)} \sec^4(d+ex)\,dx = -\frac{2e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)(2ie-bc\log(F))}{3e^2} - \frac{bcF^{c(a+bx)}\log(F)\sec^2(d+ex)}{6e^2} + \frac{F^{c(a+bx)}\sec^2(d+ex)\tan(d+ex)}{3e}$$

output
```
-2/3*exp(2*I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-1/2*I*b*c*ln(F)/e],[2-
1/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))*(2*I*e-b*c*ln(F))/e^2-1/6*b*c*F^(c*(
b*x+a))*ln(F)*sec(e*x+d)^2/e^2+1/3*F^(c*(b*x+a))*sec(e*x+d)^2*tan(e*x+d)/e
```

3.17.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.78

$$\int F^{c(a+bx)} \sec^4(d+ex)\,dx = \frac{F^{c(a+bx)}\left(4e^{2i(d+ex)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)(-2ie+bc\log(F))+\sec\right.}{6e^2}$$

input
```
Integrate[F^(c*(a + b*x))*Sec[d + e*x]^4,x]
```

output
```
(F^(c*(a + b*x))*(4*E^((2*I)*(d + e*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*
c*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))]*((-2*I)*e + b
*c*Log[F]) + Sec[d + e*x]^2*(-(b*c*Log[F]) + 2*e*Tan[d + e*x])))/(6*e^2)
```

3.17.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4948, 4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{4948}$$

$$\frac{1}{6}\left(\frac{b^2c^2\log^2(F)}{e^2}+4\right)\int F^{c(a+bx)}\sec^2(d+ex)dx-\frac{bc\log(F)\sec^2(d+ex)F^{c(a+bx)}}{6e^2}+\frac{\tan(d+ex)\sec^2(d+ex)F^{c(a+bx)}}{3e}$$

$$\Big\downarrow \text{4951}$$

$$\frac{2e^{2i(d+ex)}F^{c(a+bx)}\left(\frac{b^2c^2\log^2(F)}{e^2}+4\right)\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{3(bc\log(F)+2ie)}-\frac{bc\log(F)\sec^2(d+ex)F^{c(a+bx)}}{6e^2}+\frac{\tan(d+ex)\sec^2(d+ex)F^{c(a+bx)}}{3e}$$

input
```
Int[F^(c*(a + b*x))*Sec[d + e*x]^4,x]
```

output
```
(2*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c
*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))]*(4 + (b^2*c^2*
Log[F]^2)/e^2))/(3*((2*I)*e + b*c*Log[F])) - (b*c*F^(c*(a + b*x))*Log[F]*S
ec[d + e*x]^2)/(6*e^2) + (F^(c*(a + b*x))*Sec[d + e*x]^2*Tan[d + e*x])/(3*
e)
```

3.17.3.1 Defintions of rubi rules used

rule 4948
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(
e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2))   Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b,
 c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && N
eQ[n, 2]
```

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.17.4 Maple [F]

$$\int F^{c(xb+a)} \sec(ex+d)^4 \, dx$$

input
```
int(F^(c*(b*x+a))*sec(e*x+d)^4,x)
```

output
```
int(F^(c*(b*x+a))*sec(e*x+d)^4,x)
```

3.17.5 Fricas [F]

$$\int F^{c(a+bx)} \sec^4(d+ex) \, dx = \int F^{(bx+a)c} \sec(ex+d)^4 \, dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sec(e*x + d)^4, x)
```

3.17.6 Sympy [F]

$$\int F^{c(a+bx)}\sec^4(d+ex)\,dx=\int F^{c(a+bx)}\sec^4\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*sec(e*x+d)**4,x)
```

output
```
Integral(F**(c*(a + b*x))*sec(d + e*x)**4, x)
```

3.17.7 Maxima [F]

$$\int F^{c(a+bx)}\sec^4(d+ex)\,dx=\int F^{(bx+a)c}\sec\left(ex+d\right)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^4,x, algorithm="maxima")
```

output
```
16*(6*(F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*
F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(4*e*x + 4*d)^2 + 320*(F^(a*c)*b^3*c^
3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d)^2 +
 6*(F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F^(
a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(4*e*x + 4*d)^2 + 320*(F^(a*c)*b^3*c^3*e
^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(2*e*x + 2*d)^2 - 56
0*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 32*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos
(2*e*x + 2*d) + 40*(F^(a*c)*b^4*c^4*e*log(F)^4 - 104*F^(a*c)*b^2*c^2*e^3*l
og(F)^2)*F^(b*c*x)*sin(2*e*x + 2*d) - 160*(F^(a*c)*b^3*c^3*e^2*log(F)^3 -
20*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x) + ((F^(a*c)*b^5*c^5*log(F)^5 + 100*F^
(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(4*
e*x + 4*d) + 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))
*F^(b*c*x)*cos(2*e*x + 2*d) - 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 100*F^(a*c)*
b^2*c^2*e^3*log(F)^2 + 2304*F^(a*c)*e^5)*F^(b*c*x)*sin(4*e*x + 4*d) + 8*(F
^(a*c)*b^4*c^4*e*log(F)^4 + 40*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 1536*F^(a*c)
*e^5)*F^(b*c*x)*sin(2*e*x + 2*d) - 160*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 20*
F^(a*c)*b*c*e^4*log(F))*F^(b*c*x))*cos(8*e*x + 8*d) + 4*((F^(a*c)*b^5*c^5*
log(F)^5 + 100*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F^(a*c)*b*c*e^4*log(F))
*F^(b*c*x)*cos(4*e*x + 4*d) + 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c
)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) - 4*(F^(a*c)*b^4*c^4*e*log...
```

3.17.8 Giac [F]

$$\int F^{c(a+bx)}\sec^4(d+ex)\,dx=\int F^{(bx+a)c}\sec\left(ex+d\right)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sec(e*x + d)^4, x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sec^4(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\cos\left(d+e\,x\right)^4}\,dx$$

input
```
int(F^(c*(a + b*x))/cos(d + e*x)^4,x)
```

output
```
int(F^(c*(a + b*x))/cos(d + e*x)^4, x)
```

3.18 $\int e^x \cos^4(x)\,dx$

3.18.1 Optimal result

Integrand size = 8, antiderivative size = 54

$$\int e^x \cos^4(x)\,dx = \frac{24e^x}{85} + \frac{12}{85}e^x\cos^2(x) + \frac{1}{17}e^x\cos^4(x) + \frac{24}{85}e^x\cos(x)\sin(x) + \frac{4}{17}e^x\cos^3(x)\sin(x)$$

output
```
24/85*exp(x)+12/85*exp(x)*cos(x)^2+1/17*exp(x)*cos(x)^4+24/85*exp(x)*cos(x
)*sin(x)+4/17*exp(x)*cos(x)^3*sin(x)
```

3.18.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.61

$$\int e^x \cos^4(x)\,dx = \frac{1}{680}e^x(255 + 68\cos(2x) + 5\cos(4x) + 136\sin(2x) + 20\sin(4x))$$

input
```
Integrate[E^x*Cos[x]^4,x]
```

output
```
(E^x*(255 + 68*Cos[2*x] + 5*Cos[4*x] + 136*Sin[2*x] + 20*Sin[4*x]))/680
```

3.18.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4935, 4935, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cos^4(x)\,dx$$

$$\Big\downarrow \text{4935}$$

$$\frac{12}{17}\int e^x \cos^2(x)dx + \frac{1}{17}e^x \cos^4(x) + \frac{4}{17}e^x \sin(x)\cos^3(x)$$

$$\Big\downarrow \text{4935}$$

$$\frac{12}{17}\left(\frac{2\int e^x dx}{5} + \frac{1}{5}e^x \cos^2(x) + \frac{2}{5}e^x \sin(x)\cos(x)\right) + \frac{1}{17}e^x \cos^4(x) + \frac{4}{17}e^x \sin(x)\cos^3(x)$$

$$\Big\downarrow \text{2624}$$

$$\frac{1}{17}e^x \cos^4(x) + \frac{4}{17}e^x \sin(x)\cos^3(x) + \frac{12}{17}\left(\frac{2e^x}{5} + \frac{1}{5}e^x \cos^2(x) + \frac{2}{5}e^x \sin(x)\cos(x)\right)$$

input
```
Int[E^x*Cos[x]^4,x]
```

output
```
(E^x*Cos[x]^4)/17 + (4*E^x*Cos[x]^3*Sin[x])/17 + (12*((2*E^x)/5 + (E^x*Cos
[x]^2)/5 + (2*E^x*Cos[x]*Sin[x])/5))/17
```

3.18.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

3.18.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.57

method	result
parallelrisch	$\frac{e^{x}\left(255+5\cos(4x)+68\cos(2x)+136\sin(2x)+20\sin(4x)\right)}{680}$
default	$\frac{(\cos(x)+4\sin(x))e^{x}\cos(x)^{3}}{17}+\frac{12(\cos(x)+2\sin(x))e^{x}\cos(x)}{85}+\frac{24\,e^{x}}{85}$
risch	$\frac{3\,e^{x}}{8}+\frac{e^{(1+4i)x}}{272}-\frac{i e^{(1+4i)x}}{68}+\frac{e^{(1+2i)x}}{20}-\frac{i e^{(1+2i)x}}{10}+\frac{e^{(1-2i)x}}{20}+\frac{i e^{(1-2i)x}}{10}+\frac{e^{(1-4i)x}}{272}+\frac{i e^{(1-4i)x}}{68}$
norman	$\frac{\frac{88\,e^{x}\tan\left(\frac{x}{2}\right)}{85}+\frac{76\,e^{x}\tan\left(\frac{x}{2}\right)^{2}}{85}-\frac{72\,e^{x}\tan\left(\frac{x}{2}\right)^{3}}{85}+\frac{30\,e^{x}\tan\left(\frac{x}{2}\right)^{4}}{17}+\frac{72\,e^{x}\tan\left(\frac{x}{2}\right)^{5}}{85}+\frac{76\,e^{x}\tan\left(\frac{x}{2}\right)^{6}}{85}-\frac{88\,e^{x}\tan\left(\frac{x}{2}\right)^{7}}{85}+\frac{41\,e^{x}\tan\left(\frac{x}{2}\right)^{8}}{85}+4}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{4}}$

input
```
int(exp(x)*cos(x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/680*exp(x)*(255+5*cos(4*x)+68*cos(2*x)+136*sin(2*x)+20*sin(4*x))
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.67

$$\int e^{x}\cos^{4}(x)\,dx=\frac{4}{85}\left(5\cos(x)^{3}+6\cos(x)\right)e^{x}\sin(x)+\frac{1}{85}\left(5\cos(x)^{4}+12\cos(x)^{2}+24\right)e^{x}$$

input
```
integrate(exp(x)*cos(x)^4,x, algorithm="fricas")
```

output
```
4/85*(5*cos(x)^3 + 6*cos(x))*e^x*sin(x) + 1/85*(5*cos(x)^4 + 12*cos(x)^2 +
 24)*e^x
```

3.18.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int e^{x}\cos^{4}(x)\,dx=\frac{24e^{x}\sin^{4}(x)}{85}+\frac{24e^{x}\sin^{3}(x)\cos(x)}{85}+\frac{12e^{x}\sin^{2}(x)\cos^{2}(x)}{17}+\frac{44e^{x}\sin(x)\cos^{3}(x)}{85}+\frac{41e^{x}\cos^{4}(x)}{85}$$

input
```
integrate(exp(x)*cos(x)**4,x)
```

output
```
24*exp(x)*sin(x)**4/85 + 24*exp(x)*sin(x)**3*cos(x)/85 + 12*exp(x)*sin(x)*
*2*cos(x)**2/17 + 44*exp(x)*sin(x)*cos(x)**3/85 + 41*exp(x)*cos(x)**4/85
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int e^x \cos^4(x)\,dx = \frac{1}{136}\cos(4x)e^x + \frac{1}{10}\cos(2x)e^x + \frac{1}{34}e^x\sin(4x) + \frac{1}{5}e^x\sin(2x) + \frac{3}{8}e^x$$

input
```
integrate(exp(x)*cos(x)^4,x, algorithm="maxima")
```

output
```
1/136*cos(4*x)*e^x + 1/10*cos(2*x)*e^x + 1/34*e^x*sin(4*x) + 1/5*e^x*sin(2
*x) + 3/8*e^x
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.65

$$\int e^x \cos^4(x)\,dx = \frac{1}{136}(\cos(4x) + 4\sin(4x))e^x + \frac{1}{10}(\cos(2x) + 2\sin(2x))e^x + \frac{3}{8}e^x$$

input
```
integrate(exp(x)*cos(x)^4,x, algorithm="giac")
```

output
```
1/136*(cos(4*x) + 4*sin(4*x))*e^x + 1/10*(cos(2*x) + 2*sin(2*x))*e^x + 3/8
*e^x
```

3.18.9 Mupad [B] (verification not implemented)

Time = 27.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int e^x \cos^4(x)\,dx = \frac{3\,\mathrm{e}^x}{8} + \frac{\mathrm{e}^x\left(\frac{4\,\cos(2\,x)}{5} + \frac{8\,\sin(2\,x)}{5} + \frac{2\cos(2\,x)^2}{17} + \frac{8\,\cos(2\,x)\,\sin(2\,x)}{17} - \frac{1}{17}\right)}{8}$$

input
```
int(exp(x)*cos(x)^4,x)
```

output
```
(3*exp(x))/8 + (exp(x)*((4*cos(2*x))/5 + (8*sin(2*x))/5 + (2*cos(2*x)^2)/1
7 + (8*cos(2*x)*sin(2*x))/17 - 1/17))/8
```

3.19 $\int e^{c(a+bx)}\tan^3(d+ex)\,dx$

3.19.1 Optimal result

Integrand size = 18, antiderivative size = 194

$$\int e^{c(a+bx)}\tan^3(d+ex)\,dx=\frac{ie^{c(a+bx)}}{bc}-\frac{6ie^{c(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}+\frac{12ie^{c(a+bx)}\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}-\frac{8ie^{c(a+bx)}\text{Hypergeometric2F1}\left(3,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}$$

output
```
I*exp(c*(b*x+a))/b/c-6*I*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/2
*I*b*c/e],-exp(2*I*(e*x+d)))/b/c+12*I*exp(c*(b*x+a))*hypergeom([2, -1/2*I*
b*c/e],[1-1/2*I*b*c/e],-exp(2*I*(e*x+d)))/b/c-8*I*exp(c*(b*x+a))*hypergeom
([3, -1/2*I*b*c/e],[1-1/2*I*b*c/e],-exp(2*I*(e*x+d)))/b/c
```

3.19.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.09

$$\int e^{c(a+bx)}\tan^3(d+ex)\,dx$$
$$=\frac{1}{2}e^{c(a+bx)}\left(\frac{2(b^2c^2-2e^2)\,e^{2id}\left(bce^{2iex}\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},-e^{2i(d+ex)}\right)-(bc+2ie)\,\text{Hyper}\right.}{bc(ibc-2e)e^2\left(1+e^{2id}\right)}+\frac{\sec^2(d+ex)}{e}-\frac{bc\sec(d)\sec(d+ex)\sin(ex)}{e^2}-\frac{2\tan(d)}{bc}\right)$$

input
```
Integrate[E^(c*(a + b*x))*Tan[d + e*x]^3,x]
```

output
```
(E^(c*(a + b*x))*((2*(b^2*c^2 - 2*e^2)*E^((2*I)*d)*(b*c*E^((2*I)*e*x)*Hype
rgeometric2F1[1, 1 - ((I/2)*b*c)/e, 2 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x)
)] - (b*c + (2*I)*e)*Hypergeometric2F1[1, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c
)/e, -E^((2*I)*(d + e*x))]))/(b*c*(I*b*c - 2*e)*e^2*(1 + E^((2*I)*d))) + S
ec[d + e*x]^2/e - (b*c*Sec[d]*Sec[d + e*x]*Sin[e*x])/e^2 - (2*Tan[d])/(b*c
)))/2
```

3.19.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\tan^3(d+ex)\,dx$$

$$\Big\downarrow \text{4942}$$

$$-i\int \left(-e^{c(a+bx)}+\frac{6e^{c(a+bx)}}{1+e^{2i(d+ex)}}-\frac{12e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^2}+\frac{8e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{6e^{c(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}-\frac{12e^{c(a+bx)}\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},\right.}{bc}\right.$$

input
```
Int[E^(c*(a + b*x))*Tan[d + e*x]^3,x]
```

output
```
(-I)*(-(E^(c*(a + b*x))/(b*c)) + (6*E^(c*(a + b*x))*Hypergeometric2F1[1, (
(-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c) - (12*E^(
c*(a + b*x))*Hypergeometric2F1[2, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^
((2*I)*(d + e*x))])/(b*c) + (8*E^(c*(a + b*x))*Hypergeometric2F1[3, ((-1/2
*I)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c))
```

3.19.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

3.19.4 Maple [F]

$$\int e^{c(xb+a)} \tan(ex+d)^3 \, dx$$

input
```
int(exp(c*(b*x+a))*tan(e*x+d)^3,x)
```

output
```
int(exp(c*(b*x+a))*tan(e*x+d)^3,x)
```

3.19.5 Fricas [F]

$$\int e^{c(a+bx)} \tan^3(d+ex) \, dx = \int e^{((bx+a)c)} \tan(ex+d)^3 \, dx$$

input
```
integrate(exp(c*(b*x+a))*tan(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(e^(b*c*x + a*c)*tan(e*x + d)^3, x)
```

3.19.6 Sympy [F]

$$\int e^{c(a+bx)} \tan^3(d+ex) \, dx = e^{ac} \int e^{bcx} \tan^3(d+ex) \, dx$$

input
```
integrate(exp(c*(b*x+a))*tan(e*x+d)**3,x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*tan(d + e*x)**3, x)
```

3.19.7 Maxima [F]

$$\int e^{c(a+bx)}\tan^3(d+ex)\,dx = \int e^{((bx+a)c)}\tan(ex+d)^3\,dx$$

input
```
integrate(exp(c*(b*x+a))*tan(e*x+d)^3,x, algorithm="maxima")
```

output
```
(4*e*cos(2*e*x + 2*d)^2*e^(b*c*x + a*c) - b*c*e^(b*c*x + a*c)*sin(2*e*x +
2*d) + 4*e*e^(b*c*x + a*c)*sin(2*e*x + 2*d)^2 + 2*e*cos(2*e*x + 2*d)*e^(b*
c*x + a*c) + (b*c*e^(b*c*x + a*c)*sin(2*e*x + 2*d) + 2*e*cos(2*e*x + 2*d)*
e^(b*c*x + a*c))*cos(4*e*x + 4*d) + (b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c) +
 (b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(4*e*x + 4*d)^2 + 4*(b^2*c^2*e^4
*e^(a*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d)^2 + (b^2*c^2*e^4*e^(a*c) - 2*e^
6*e^(a*c))*sin(4*e*x + 4*d)^2 + 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*si
n(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*
sin(2*e*x + 2*d)^2 + 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c) + 2*(b^2*c^2*e
^4*e^(a*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) + 4*(b^2*c^
2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d))*integrate(e^(b*c*x)*sin(2
*e*x + 2*d)/(e^4*cos(2*e*x + 2*d)^2 + e^4*sin(2*e*x + 2*d)^2 + 2*e^4*cos(2
*e*x + 2*d) + e^4), x) - (b*c*cos(2*e*x + 2*d)*e^(b*c*x + a*c) + b*c*e^(b*
c*x + a*c) - 2*e*e^(b*c*x + a*c)*sin(2*e*x + 2*d))*sin(4*e*x + 4*d))/(e^2*
cos(4*e*x + 4*d)^2 + 4*e^2*cos(2*e*x + 2*d)^2 + e^2*sin(4*e*x + 4*d)^2 + 4
*e^2*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*e^2*sin(2*e*x + 2*d)^2 + 4*e^2*
cos(2*e*x + 2*d) + e^2 + 2*(2*e^2*cos(2*e*x + 2*d) + e^2)*cos(4*e*x + 4*d)
)
```

3.19.8 Giac [F]

$$\int e^{c(a+bx)}\tan^3(d+ex)\,dx = \int e^{((bx+a)c)}\tan(ex+d)^3\,dx$$

input
```
integrate(exp(c*(b*x+a))*tan(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(e^((b*x + a)*c)*tan(e*x + d)^3, x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\tan^3(d+ex)\,dx = \int e^{c\,(a+b\,x)}\tan(d+e\,x)^3\,dx$$

input
```
int(exp(c*(a + b*x))*tan(d + e*x)^3,x)
```

output
```
int(exp(c*(a + b*x))*tan(d + e*x)^3, x)
```

3.20 $\int e^{c(a+bx)} \tan^2(d+ex)\,dx$

3.20.1 Optimal result

Integrand size = 18, antiderivative size = 130

$$\int e^{c(a+bx)} \tan^2(d+ex)\,dx = -\frac{e^{c(a+bx)}}{bc} + \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc} - \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}$$

output
```
-exp(c*(b*x+a))/b/c+4*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/2*I*
b*c/e],-exp(2*I*(e*x+d)))/b/c-4*exp(c*(b*x+a))*hypergeom([2, -1/2*I*b*c/e]
,[1-1/2*I*b*c/e],-exp(2*I*(e*x+d)))/b/c
```

3.20.2 Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.35

$$\int e^{c(a+bx)} \tan^2(d+ex)\,dx = e^{c(a+bx)}\left(-\frac{1}{bc} + \frac{2e^{2id}\left(ibce^{2iex}\,\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},-e^{2i(d+ex)}\right)+(-ibc+2e)\,\text{Hypergeometric2F1}\left(1,\right.\right.}{(bc+2ie)e\left(1+e^{2id}\right)} + \frac{\sec(d)\sec(d+ex)\sin(ex)}{e}\right)$$

input
```
Integrate[E^(c*(a + b*x))*Tan[d + e*x]^2,x]
```

output
```
E^(c*(a + b*x))*(-(1/(b*c)) + (2*E^((2*I)*d)*(I*b*c*E^((2*I)*e*x)*Hypergeo
metric2F1[1, 1 - ((I/2)*b*c)/e, 2 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))] +
 ((-I)*b*c + 2*e)*Hypergeometric2F1[1, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e
, -E^((2*I)*(d + e*x))]))/((b*c + (2*I)*e)*e*(1 + E^((2*I)*d))) + (Sec[d]*
Sec[d + e*x]*Sin[e*x])/e)
```

3.20.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\tan^2(d+ex)\,dx$$

$$\Big\downarrow \text{4942}$$

$$-\int \left(e^{c(a+bx)}-\frac{4e^{c(a+bx)}}{1+e^{2i(d+ex)}}+\frac{4e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}-\frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}-\frac{e^{c(a+bx)}}{bc}$$

input
```
Int[E^(c*(a + b*x))*Tan[d + e*x]^2,x]
```

output
```
-(E^(c*(a + b*x))/(b*c)) + (4*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-1/2*
I)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c) - (4*E^(c*(a +
b*x))*Hypergeometric2F1[2, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*
(d + e*x))])/(b*c)
```

3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

3.20.4 Maple [F]

$$\int \mathrm{e}^{c(xb+a)} \tan(ex+d)^2 \, dx$$

input
```
int(exp(c*(b*x+a))*tan(e*x+d)^2,x)
```

output
```
int(exp(c*(b*x+a))*tan(e*x+d)^2,x)
```

3.20.5 Fricas [F]

$$\int e^{c(a+bx)} \tan^2(d+ex) \, dx = \int e^{((bx+a)c)} \tan(ex+d)^2 \, dx$$

input
```
integrate(exp(c*(b*x+a))*tan(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(e^(b*c*x + a*c)*tan(e*x + d)^2, x)
```

3.20.6 Sympy [F]

$$\int e^{c(a+bx)} \tan^2(d+ex) \, dx = e^{ac} \int e^{bcx} \tan^2(d+ex) \, dx$$

input
```
integrate(exp(c*(b*x+a))*tan(e*x+d)**2,x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*tan(d + e*x)**2, x)
```

3.20.7 Maxima [F]

$$\int e^{c(a+bx)}\tan^2(d+ex)\,dx = \int e^{((bx+a)c)}\tan{(ex+d)}^2\,dx$$

input
```
integrate(exp(c*(b*x+a))*tan(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(e*cos(2*e*x + 2*d)^2*e^(b*c*x + a*c) - 2*b*c*e^(b*c*x + a*c)*sin(2*e*x +
 2*d) + e*e^(b*c*x + a*c)*sin(2*e*x + 2*d)^2 + 2*e*cos(2*e*x + 2*d)*e^(b*c
*x + a*c) + e*e^(b*c*x + a*c) + 2*(b^2*c^2*e^2*cos(2*e*x + 2*d)^2 + b^2*c^
2*e^2*sin(2*e*x + 2*d)^2 + 2*b^2*c^2*e^2*cos(2*e*x + 2*d) + b^2*c^2*e^2)*i
ntegrate(e^(b*c*x + a*c)*sin(2*e*x + 2*d)/(e^2*cos(2*e*x + 2*d)^2 + e^2*si
n(2*e*x + 2*d)^2 + 2*e^2*cos(2*e*x + 2*d) + e^2), x))/(b*c*e*cos(2*e*x + 2
*d)^2 + b*c*e*sin(2*e*x + 2*d)^2 + 2*b*c*e*cos(2*e*x + 2*d) + b*c*e)
```

3.20.8 Giac [F]

$$\int e^{c(a+bx)}\tan^2(d+ex)\,dx = \int e^{((bx+a)c)}\tan{(ex+d)}^2\,dx$$

input
```
integrate(exp(c*(b*x+a))*tan(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(e^((b*x + a)*c)*tan(e*x + d)^2, x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\tan^2(d+ex)\,dx = \int e^{c\,(a+b\,x)}\,\tan(d+e\,x)^2\,dx$$

input
```
int(exp(c*(a + b*x))*tan(d + e*x)^2,x)
```

output
```
int(exp(c*(a + b*x))*tan(d + e*x)^2, x)
```

3.21 $\int e^{c(a+bx)} \tan(d+ex)\, dx$

3.21.1 Optimal result

Integrand size = 16, antiderivative size = 78

$$\int e^{c(a+bx)} \tan(d+ex)\, dx = -\frac{ie^{c(a+bx)}}{bc} + \frac{2ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}$$

output
```
-I*exp(c*(b*x+a))/b/c+2*I*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/
2*I*b*c/e],-exp(2*I*(e*x+d)))/b/c
```

3.21.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 166 vs. 2(78) = 156.

Time = 0.33 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.13

$$\int e^{c(a+bx)} \tan(d+ex)\, dx = \frac{e^{c(a+bx)}\left(2bce^{2i(d+ex)}\,\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},-e^{2i(d+ex)}\right)-(bc+2ie)\left(1-e^{2id}+2e^{2id}\,\text{Hype}\right.\right.}{bc(ibc-2e)\left(1+e^{2id}\right)}$$

input
```
Integrate[E^(c*(a + b*x))*Tan[d + e*x],x]
```

output
```
(E^(c*(a + b*x))*(2*b*c*E^((2*I)*(d + e*x))*Hypergeometric2F1[1, 1 - ((I/2
)*b*c)/e, 2 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))] - (b*c + (2*I)*e)*(1 -
E^((2*I)*d) + 2*E^((2*I)*d)*Hypergeometric2F1[1, ((-1/2*I)*b*c)/e, 1 - ((I
/2)*b*c)/e, -E^((2*I)*(d + e*x))])))/(b*c*(I*b*c - 2*e)*(1 + E^((2*I)*d)))
```

3.21.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\tan(d+ex)\,dx$$

$$\Big\downarrow \text{4942}$$

$$i\int \left(\frac{2e^{c(a+bx)}}{1+e^{2i(d+ex)}}-e^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{e^{c(a+bx)}}{bc}+\frac{2e^{c(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}\right)$$

input
```
Int[E^(c*(a + b*x))*Tan[d + e*x],x]
```

output
```
I*(-(E^(c*(a + b*x))/(b*c)) + (2*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-1
/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c))
```

3.21.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

3.21.4 Maple [F]

$$\int e^{c(xb+a)} \tan(ex+d)\, dx$$

input
```
int(exp(c*(b*x+a))*tan(e*x+d),x)
```

output
```
int(exp(c*(b*x+a))*tan(e*x+d),x)
```

3.21.5 Fricas [F]

$$\int e^{c(a+bx)} \tan(d+ex)\, dx = \int e^{((bx+a)c)} \tan(ex+d)\, dx$$

input
```
integrate(exp(c*(b*x+a))*tan(e*x+d),x, algorithm="fricas")
```

output
```
integral(e^(b*c*x + a*c)*tan(e*x + d), x)
```

3.21.6 Sympy [F]

$$\int e^{c(a+bx)} \tan(d+ex)\, dx = e^{ac} \int e^{bcx} \tan(d+ex)\, dx$$

input
```
integrate(exp(c*(b*x+a))*tan(e*x+d),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*tan(d + e*x), x)
```

3.21.7 Maxima [F]

$$\int e^{c(a+bx)} \tan(d+ex)\, dx = \int e^{((bx+a)c)} \tan\left(ex+d\right)\, dx$$

input
```
integrate(exp(c*(b*x+a))*tan(e*x+d),x, algorithm="maxima")
```

output
```
integrate(e^((b*x + a)*c)*tan(e*x + d), x)
```

3.21.8 Giac [F]

$$\int e^{c(a+bx)} \tan(d+ex)\, dx = \int e^{((bx+a)c)} \tan\left(ex+d\right)\, dx$$

input
```
integrate(exp(c*(b*x+a))*tan(e*x+d),x, algorithm="giac")
```

output
```
integrate(e^((b*x + a)*c)*tan(e*x + d), x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \tan(d+ex)\, dx = \int e^{c\,(a+b\,x)} \tan(d+e\,x)\, dx$$

input
```
int(exp(c*(a + b*x))*tan(d + e*x),x)
```

output
```
int(exp(c*(a + b*x))*tan(d + e*x), x)
```

3.22 $\int e^{c(a+bx)} \cot(d+ex)\,dx$

3.22.1 Optimal result

Integrand size = 16, antiderivative size = 76

$$\int e^{c(a+bx)}\cot(d+ex)\,dx = \frac{ie^{c(a+bx)}}{bc} - \frac{2ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}$$

output
```
I*exp(c*(b*x+a))/b/c-2*I*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/2
*I*b*c/e],exp(2*I*(e*x+d)))/b/c
```

3.22.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 163 vs. 2(76) = 152.

Time = 1.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.14

$$\int e^{c(a+bx)}\cot(d+ex)\,dx$$
$$= \frac{e^{c(a+bx)}\left(2ibce^{2i(d+ex)}\,\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},e^{2i(d+ex)}\right)+i(bc+2ie)\left(1+e^{2id}-2e^{2id}\,\text{Hype}\right.\right.}{bc(bc+2ie)\left(-1+e^{2id}\right)}$$

input
```
Integrate[E^(c*(a + b*x))*Cot[d + e*x],x]
```

output
```
(E^(c*(a + b*x))*((2*I)*b*c*E^((2*I)*(d + e*x))*Hypergeometric2F1[1, 1 - (
(I/2)*b*c)/e, 2 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))] + I*(b*c + (2*I)*e)*
(1 + E^((2*I)*d) - 2*E^((2*I)*d)*Hypergeometric2F1[1, ((-1/2*I)*b*c)/e, 1
- ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])))/(b*c*(b*c + (2*I)*e)*(-1 + E^((2*
I)*d)))
```

3.22.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\cot(d+ex)\,dx$$

$$\Big\downarrow \ 4943$$

$$-i\int \left(\frac{2e^{c(a+bx)}}{1-e^{2i(d+ex)}}-e^{c(a+bx)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-i\left(-\frac{e^{c(a+bx)}}{bc}+\frac{2e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}\right)$$

input
```
Int[E^(c*(a + b*x))*Cot[d + e*x],x]
```

output
```
(-I)*(-(E^(c*(a + b*x))/(b*c)) + (2*E^(c*(a + b*x))*Hypergeometric2F1[1, (
(-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c))
```

3.22.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

3.22.4 Maple [F]

$$\int e^{c(xb+a)} \cot(ex+d)\, dx$$

input
```
int(exp(c*(b*x+a))*cot(e*x+d),x)
```

output
```
int(exp(c*(b*x+a))*cot(e*x+d),x)
```

3.22.5 Fricas [F]

$$\int e^{c(a+bx)} \cot(d+ex)\, dx = \int \cot(ex+d)\, e^{((bx+a)c)}\, dx$$

input
```
integrate(exp(c*(b*x+a))*cot(e*x+d),x, algorithm="fricas")
```

output
```
integral(cot(e*x + d)*e^(b*c*x + a*c), x)
```

3.22.6 Sympy [F]

$$\int e^{c(a+bx)} \cot(d+ex)\, dx = e^{ac} \int e^{bcx} \cot(d+ex)\, dx$$

input
```
integrate(exp(c*(b*x+a))*cot(e*x+d),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*cot(d + e*x), x)
```

3.22.7 Maxima [F]

$$\int e^{c(a+bx)}\cot(d+ex)\,dx = \int \cot\left(ex+d\right)e^{\left((bx+a)c\right)}\,dx$$

input
```
integrate(exp(c*(b*x+a))*cot(e*x+d),x, algorithm="maxima")
```

output
```
integrate(cot(e*x + d)*e^((b*x + a)*c), x)
```

3.22.8 Giac [F]

$$\int e^{c(a+bx)}\cot(d+ex)\,dx = \int \cot\left(ex+d\right)e^{\left((bx+a)c\right)}\,dx$$

input
```
integrate(exp(c*(b*x+a))*cot(e*x+d),x, algorithm="giac")
```

output
```
integrate(cot(e*x + d)*e^((b*x + a)*c), x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\cot(d+ex)\,dx = \int \cot(d+e\,x)\;\mathrm{e}^{c\,(a+b\,x)}\,dx$$

input
```
int(cot(d + e*x)*exp(c*(a + b*x)),x)
```

output
```
int(cot(d + e*x)*exp(c*(a + b*x)), x)
```

3.23 $\int e^{c(a+bx)} \cot^2(d+ex)\,dx$

3.23.1 Optimal result

Integrand size = 18, antiderivative size = 126

$$\int e^{c(a+bx)} \cot^2(d+ex)\,dx = -\frac{e^{c(a+bx)}}{bc} + \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc} - \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}$$

output
```
-exp(c*(b*x+a))/b/c+4*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/2*I*
b*c/e],exp(2*I*(e*x+d)))/b/c-4*exp(c*(b*x+a))*hypergeom([2, -1/2*I*b*c/e],
[1-1/2*I*b*c/e],exp(2*I*(e*x+d)))/b/c
```

3.23.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.36

$$\int e^{c(a+bx)} \cot^2(d+ex)\,dx = e^{c(a+bx)}\left(-\frac{1}{bc} + \frac{2e^{2id}\left(ibce^{2iex}\,\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},e^{2i(d+ex)}\right)+(-ibc+2e)\,\text{Hypergeometric2F1}\left(1,-\right.\right.}{(bc+2ie)e\left(-1+e^{2id}\right)} + \frac{\csc(d)\csc(d+ex)\sin(ex)}{e}\right)$$

input
```
Integrate[E^(c*(a + b*x))*Cot[d + e*x]^2,x]
```

output
```
E^(c*(a + b*x))*(-(1/(b*c)) + (2*E^((2*I)*d)*(I*b*c*E^((2*I)*e*x)*Hypergeo
metric2F1[1, 1 - ((I/2)*b*c)/e, 2 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))] +
((-I)*b*c + 2*e)*Hypergeometric2F1[1, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e,
 E^((2*I)*(d + e*x))]))/((b*c + (2*I)*e)*e*(-1 + E^((2*I)*d))) + (Csc[d]*C
sc[d + e*x]*Sin[e*x])/e)
```

3.23.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \cot^2(d+ex) \, dx$$

$$\Big\downarrow \text{4943}$$

$$-\int \left(e^{c(a+bx)} - \frac{4e^{c(a+bx)}}{1-e^{2i(d+ex)}} + \frac{4e^{c(a+bx)}}{\left(1-e^{2i(d+ex)}\right)^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4e^{c(a+bx)} \text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc} - \frac{4e^{c(a+bx)} \text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc} - \frac{e^{c(a+bx)}}{bc}$$

input
```
Int[E^(c*(a + b*x))*Cot[d + e*x]^2,x]
```

output
```
-(E^(c*(a + b*x))/(b*c)) + (4*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-1/2*
I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c) - (4*E^(c*(a + b
*x))*Hypergeometric2F1[2, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d
 + e*x))])/(b*c)
```

3.23.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

3.23.4 Maple [F]

$$\int e^{c(xb+a)} \cot (ex+d)^2 \, dx$$

input
```
int(exp(c*(b*x+a))*cot(e*x+d)^2,x)
```

output
```
int(exp(c*(b*x+a))*cot(e*x+d)^2,x)
```

3.23.5 Fricas [F]

$$\int e^{c(a+bx)} \cot^2(d+ex) \, dx = \int \cot (ex+d)^2 e^{((bx+a)c)} \, dx$$

input
```
integrate(exp(c*(b*x+a))*cot(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(cot(e*x + d)^2*e^(b*c*x + a*c), x)
```

3.23.6 Sympy [F]

$$\int e^{c(a+bx)} \cot^2(d+ex) \, dx = e^{ac} \int e^{bcx} \cot^2 (d+ex) \, dx$$

input
```
integrate(exp(c*(b*x+a))*cot(e*x+d)**2,x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*cot(d + e*x)**2, x)
```

3.23.7 Maxima [F]

$$\int e^{c(a+bx)}\cot^2(d+ex)\,dx = \int \cot(ex+d)^2 e^{((bx+a)c)}\,dx$$

input
```
integrate(exp(c*(b*x+a))*cot(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(e*cos(2*e*x + 2*d)^2*e^(b*c*x + a*c) + 2*b*c*e^(b*c*x + a*c)*sin(2*e*x +
 2*d) + e*e^(b*c*x + a*c)*sin(2*e*x + 2*d)^2 - 2*e*cos(2*e*x + 2*d)*e^(b*c
*x + a*c) + e*e^(b*c*x + a*c) + (b^2*c^2*e^2*cos(2*e*x + 2*d)^2 + b^2*c^2*
e^2*sin(2*e*x + 2*d)^2 - 2*b^2*c^2*e^2*cos(2*e*x + 2*d) + b^2*c^2*e^2)*int
egrate(e^(b*c*x + a*c)*sin(e*x + d)/(e^2*cos(e*x + d)^2 + e^2*sin(e*x + d)
^2 + 2*e^2*cos(e*x + d) + e^2), x) - (b^2*c^2*e^2*cos(2*e*x + 2*d)^2 + b^2
*c^2*e^2*sin(2*e*x + 2*d)^2 - 2*b^2*c^2*e^2*cos(2*e*x + 2*d) + b^2*c^2*e^2
)*integrate(e^(b*c*x + a*c)*sin(e*x + d)/(e^2*cos(e*x + d)^2 + e^2*sin(e*x
 + d)^2 - 2*e^2*cos(e*x + d) + e^2), x))/(b*c*e*cos(2*e*x + 2*d)^2 + b*c*e
*sin(2*e*x + 2*d)^2 - 2*b*c*e*cos(2*e*x + 2*d) + b*c*e)
```

3.23.8 Giac [F]

$$\int e^{c(a+bx)}\cot^2(d+ex)\,dx = \int \cot(ex+d)^2 e^{((bx+a)c)}\,dx$$

input
```
integrate(exp(c*(b*x+a))*cot(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(cot(e*x + d)^2*e^((b*x + a)*c), x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\cot^2(d+ex)\,dx = \int \cot(d+e\,x)^2\,\mathrm{e}^{c\,(a+b\,x)}\,dx$$

input
```
int(cot(d + e*x)^2*exp(c*(a + b*x)),x)
```

output
```
int(cot(d + e*x)^2*exp(c*(a + b*x)), x)
```

3.24 $\int e^{c(a+bx)} \cot^3(d+ex)\,dx$

3.24.1 Optimal result

Integrand size = 18, antiderivative size = 188

$$\begin{aligned}\int e^{c(a+bx)} \cot^3(d+ex)\,dx = & -\frac{ie^{c(a+bx)}}{bc} \\ & + \frac{6ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc} \\ & - \frac{12ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc} \\ & + \frac{8ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}\end{aligned}$$

output
```
-I*exp(c*(b*x+a))/b/c+6*I*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/
2*I*b*c/e],exp(2*I*(e*x+d)))/b/c-12*I*exp(c*(b*x+a))*hypergeom([2, -1/2*I*
b*c/e],[1-1/2*I*b*c/e],exp(2*I*(e*x+d)))/b/c+8*I*exp(c*(b*x+a))*hypergeom(
[3, -1/2*I*b*c/e],[1-1/2*I*b*c/e],exp(2*I*(e*x+d)))/b/c
```

3.24.2 Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.12

$$\int e^{c(a+bx)}\cot^3(d+ex)\,dx = \frac{1}{2}e^{c(a+bx)}\left(-\frac{2\cot(d)}{bc}-\frac{\csc^2(d+ex)}{e}\right.$$
$$+\frac{2(b^2c^2-2e^2)\,e^{2id}\left(ibce^{2iex}\,\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},e^{2i(d+ex)}\right)+(-ibc+2e)\,\text{Hypergeomet}\right.}{bc(bc+2ie)e^2\left(-1+e^{2id}\right)}$$
$$\left.+\frac{bc\csc(d)\csc(d+ex)\sin(ex)}{e^2}\right)$$

input
```
Integrate[E^(c*(a + b*x))*Cot[d + e*x]^3,x]
```

output
```
(E^(c*(a + b*x))*((-2*Cot[d])/(b*c) - Csc[d + e*x]^2/e + (2*(b^2*c^2 - 2*e
^2)*E^((2*I)*d)*(I*b*c*E^((2*I)*e*x)*Hypergeometric2F1[1, 1 - ((I/2)*b*c)/
e, 2 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))] + ((-I)*b*c + 2*e)*Hypergeometr
ic2F1[1, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))]))/(b*c*
(b*c + (2*I)*e)*e^2*(-1 + E^((2*I)*d))) + (b*c*Csc[d]*Csc[d + e*x]*Sin[e*x
])/e^2))/2
```

3.24.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\cot^3(d+ex)\,dx$$
$$\Big\downarrow \text{4943}$$
$$i\int\left(-e^{c(a+bx)}+\frac{6e^{c(a+bx)}}{1-e^{2i(d+ex)}}-\frac{12e^{c(a+bx)}}{\left(1-e^{2i(d+ex)}\right)^2}+\frac{8e^{c(a+bx)}}{\left(1-e^{2i(d+ex)}\right)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$i\left(\frac{6e^{c(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}-\frac{12e^{c(a+bx)}\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}\right.$$

input
```
Int[E^(c*(a + b*x))*Cot[d + e*x]^3,x]
```

output
```
I*(-(E^(c*(a + b*x))/(b*c)) + (6*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-1
/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c) - (12*E^(c*(a
 + b*x))*Hypergeometric2F1[2, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I
)*(d + e*x))])/(b*c) + (8*E^(c*(a + b*x))*Hypergeometric2F1[3, ((-1/2*I)*b
*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c))
```

3.24.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

3.24.4 Maple [F]

$$\int e^{c(xb+a)}\cot(ex+d)^3\,dx$$

input
```
int(exp(c*(b*x+a))*cot(e*x+d)^3,x)
```

output
```
int(exp(c*(b*x+a))*cot(e*x+d)^3,x)
```

3.24.5 Fricas [F]

$$\int e^{c(a+bx)} \cot^3(d+ex)\,dx = \int \cot(ex+d)^3 e^{((bx+a)c)}\,dx$$

input
```
integrate(exp(c*(b*x+a))*cot(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(cot(e*x + d)^3*e^(b*c*x + a*c), x)
```

3.24.6 Sympy [F]

$$\int e^{c(a+bx)} \cot^3(d+ex)\,dx = e^{ac} \int e^{bcx} \cot^3(d+ex)\,dx$$

input
```
integrate(exp(c*(b*x+a))*cot(e*x+d)**3,x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*cot(d + e*x)**3, x)
```

3.24.7 Maxima [F]

$$\int e^{c(a+bx)} \cot^3(d+ex)\,dx = \int \cot(ex+d)^3 e^{((bx+a)c)}\,dx$$

input
```
integrate(exp(c*(b*x+a))*cot(e*x+d)^3,x, algorithm="maxima")
```

output
```
-(4*e*cos(2*e*x + 2*d)^2*e^(b*c*x + a*c) + b*c*e^(b*c*x + a*c)*sin(2*e*x +
 2*d) + 4*e*e^(b*c*x + a*c)*sin(2*e*x + 2*d)^2 - 2*e*cos(2*e*x + 2*d)*e^(b
*c*x + a*c) - (b*c*e^(b*c*x + a*c)*sin(2*e*x + 2*d) + 2*e*cos(2*e*x + 2*d)
*e^(b*c*x + a*c))*cos(4*e*x + 4*d) + 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c
) + (b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(4*e*x + 4*d)^2 + 4*(b^2*c^2*
e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d)^2 + (b^2*c^2*e^4*e^(a*c) - 2
*e^6*e^(a*c))*sin(4*e*x + 4*d)^2 - 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))
*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c
))*sin(2*e*x + 2*d)^2 + 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c) - 2*(b^2*c^
2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) - 4*(b^2
*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d))*integrate(1/4*e^(b*c*x
)*sin(e*x + d)/(e^4*cos(e*x + d)^2 + e^4*sin(e*x + d)^2 + 2*e^4*cos(e*x +
d) + e^4), x) - 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c) + (b^2*c^2*e^4*e^(a
*c) - 2*e^6*e^(a*c))*cos(4*e*x + 4*d)^2 + 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e
^(a*c))*cos(2*e*x + 2*d)^2 + (b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(4*e
*x + 4*d)^2 - 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(4*e*x + 4*d)*sin
(2*e*x + 2*d) + 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(2*e*x + 2*d)^2
 + 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c) - 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6
*e^(a*c))*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) - 4*(b^2*c^2*e^4*e^(a*c) - 2*
e^6*e^(a*c))*cos(2*e*x + 2*d))*integrate(1/4*e^(b*c*x)*sin(e*x + d)/(e^...
```

3.24.8 Giac [F]

$$\int e^{c(a+bx)} \cot^3(d+ex)\,dx = \int \cot(ex+d)^3 e^{((bx+a)c)}\,dx$$

input
```
integrate(exp(c*(b*x+a))*cot(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(cot(e*x + d)^3*e^((b*x + a)*c), x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\cot^3(d+ex)\,dx = \int \cot(d+e\,x)^3\,\mathrm{e}^{c\,(a+b\,x)}\,dx$$

input
```
int(cot(d + e*x)^3*exp(c*(a + b*x)),x)
```

output
```
int(cot(d + e*x)^3*exp(c*(a + b*x)), x)
```

3.25 $\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)\,dx$

3.25.1 Optimal result

Integrand size = 27, antiderivative size = 76

$$\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)\,dx = \frac{iF^{a+bx}}{b\log(F)}-\frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},ie^{i(c+dx)}\right)}{b\log(F)}$$

output
```
I*F^(b*x+a)/b/ln(F)-2*I*F^(b*x+a)*hypergeom([1, -I*b*ln(F)/d],[1-I*b*ln(F)
/d],I*exp(I*(d*x+c)))/b/ln(F)
```

3.25.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.75

$$\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)\,dx = \frac{F^{a+bx}\left(be^{i(c+dx)}\,\text{Hypergeometric2F1}\left(1,1-\frac{ib\log(F)}{d},2-\frac{ib\log(F)}{d},ie^{i(c+dx)}\right)\log(F)+\text{Hypergeometric2F1}\left(\right.\right.}{b\log(F)(id+b\log(F))}$$

input
```
Integrate[F^(a + b*x)*Tan[Pi/4 + (-c - d*x)/2],x]
```

output
```
(F^(a + b*x)*(b*E^(I*(c + d*x))*Hypergeometric2F1[1, 1 - (I*b*Log[F])/d, 2
 - (I*b*Log[F])/d, I*E^(I*(c + d*x))]*Log[F] + Hypergeometric2F1[1, ((-I)*
b*Log[F])/d, 1 - (I*b*Log[F])/d, I*E^(I*(c + d*x))]*(d - I*b*Log[F])))/(b*
Log[F]*(I*d + b*Log[F]))
```

3.25.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4967, 25, 25, 4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+bx}\tan\left(\frac{1}{2}(-c-dx)+\frac{\pi}{4}\right)dx$$
$$\Big\downarrow \text{4967}$$
$$\int -F^{a+bx}\tan\left(\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{4}\right)dx$$
$$\Big\downarrow \text{25}$$
$$-\int -F^{a+bx}\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx$$
$$\Big\downarrow \text{25}$$
$$\int F^{a+bx}\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx$$
$$\Big\downarrow \text{4943}$$
$$-i\int\left(\frac{2F^{a+bx}}{1-e^{\frac{1}{2}i(2c+2dx+\pi)}}-F^{a+bx}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-i\left(-\frac{F^{a+bx}}{b\log(F)}+\frac{2F^{a+bx}\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},e^{\frac{1}{2}i(2c+2dx+\pi)}\right)}{b\log(F)}\right)$$

input
```
Int[F^(a + b*x)*Tan[Pi/4 + (-c - d*x)/2],x]
```

output
```
(-I)*(-(F^(a + b*x)/(b*Log[F])) + (2*F^(a + b*x)*Hypergeometric2F1[1, ((-I
)*b*Log[F])/d, 1 - (I*b*Log[F])/d, E^((I/2)*(2*c + Pi + 2*d*x))])/(b*Log[F
]))
```

3.25.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

rule 4967
```
Int[(F_)^((c_.)*(u_))*(G_)[v_]^(n_.), x_Symbol] :> Int[F^(c*ExpandToSum[u,
x])*G[ExpandToSum[v, x]]^n, x] /; FreeQ[{F, c, n}, x] && TrigQ[G] && Linear
Q[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

3.25.4 Maple [F]

$$\int F^{xb+a}\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)dx$$

input
```
int(F^(b*x+a)*cot(1/2*c+1/4*Pi+1/2*d*x),x)
```

output
```
int(F^(b*x+a)*cot(1/2*c+1/4*Pi+1/2*d*x),x)
```

3.25.5 Fricas [F]

$$\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)\,dx=\int F^{bx+a}\cot\left(\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\,dx$$

input
```
integrate(F^(b*x+a)*cot(1/2*c+1/4*pi+1/2*d*x),x, algorithm="fricas")
```

output
```
integral(F^(b*x + a)*cot(1/4*pi + 1/2*d*x + 1/2*c), x)
```

3.25.6 Sympy [F]

$$\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)\,dx=\int F^{a+bx}\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)\,dx$$

input
```
integrate(F**(b*x+a)*cot(1/2*c+1/4*pi+1/2*d*x),x)
```

output
```
Integral(F**(a + b*x)*cot(c/2 + d*x/2 + pi/4), x)
```

3.25.7 Maxima [F]

$$\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)\,dx=\int F^{bx+a}\cot\left(\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\,dx$$

input
```
integrate(F^(b*x+a)*cot(1/2*c+1/4*pi+1/2*d*x),x, algorithm="maxima")
```

output
```
integrate(F^(b*x + a)*cot(1/4*pi + 1/2*d*x + 1/2*c), x)
```

3.25.8 Giac [F]

$$\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)\,dx=\int F^{bx+a}\cot\left(\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\,dx$$

input
```
integrate(F^(b*x+a)*cot(1/2*c+1/4*pi+1/2*d*x),x, algorithm="giac")
```

output
```
integrate(F^(b*x + a)*cot(1/4*pi + 1/2*d*x + 1/2*c), x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)\,dx=\int F^{a+b\,x}\cot\left(\frac{\Pi}{4}+\frac{c}{2}+\frac{d\,x}{2}\right)\,dx$$

input
```
int(F^(a + b*x)*cot(Pi/4 + c/2 + (d*x)/2),x)
```

output
```
int(F^(a + b*x)*cot(Pi/4 + c/2 + (d*x)/2), x)
```

3.26 $\int F^{c(a+bx)} \sec^n(d+ex)\,dx$

3.26.1 Optimal result

Integrand size = 18, antiderivative size = 100

$$\int F^{c(a+bx)} \sec^n(d+ex)\,dx$$
$$= \frac{\left(1+e^{2i(d+ex)}\right)^n F^{ac+bcx}\,\text{Hypergeometric2F1}\left(n, \frac{en-ibc\log(F)}{2e}, \frac{1}{2}\left(2+n-\frac{ibc\log(F)}{e}\right), -e^{2i(d+ex)}\right)\sec^n(d+ex}{ien+bc\log(F)}$$

output
```
(1+exp(2*I*(e*x+d)))^n*F^(b*c*x+a*c)*hypergeom([n, 1/2*(e*n-I*b*c*ln(F))/e
],[1+1/2*n-1/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))*sec(e*x+d)^n/(b*c*ln(F)+I
*e*n)
```

3.26.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)} \sec^n(d+ex)\,dx =$$
$$-\frac{i\left(1+e^{2i(d+ex)}\right)^n F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(n, \frac{en-ibc\log(F)}{2e}, \frac{1}{2}\left(2+n-\frac{ibc\log(F)}{e}\right), -e^{2i(d+ex)}\right)\sec^n(d+}{en-ibc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sec[d + e*x]^n,x]
```

output
```
((-I)*(1 + E^((2*I)*(d + e*x)))^n*F^(c*(a + b*x))*Hypergeometric2F1[n, (e*
n - I*b*c*Log[F])/(2*e), (2 + n - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x)
)]*Sec[d + e*x]^n)/(e*n - I*b*c*Log[F])
```

3.26.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4954, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sec^n(d+ex)\,dx$$

$$\Big\downarrow \text{4954}$$

$$e^{-in(d+ex)}\left(1+e^{2i(d+ex)}\right)^n\sec^n(d+ex)\int e^{idn+iexn}\left(1+e^{2i(d+ex)}\right)^{-n}F^{ac+bxc}dx$$

$$\Big\downarrow \text{2689}$$

$$\frac{e^{-in(d+ex)+idn+ienx}\left(1+e^{2i(d+ex)}\right)^nF^{ac+bcx}\sec^n(d+ex)\,\text{Hypergeometric2F1}\left(n,\frac{en-ibc\log(F)}{2e},\frac{1}{2}\left(n-\frac{ibc\log(F)}{e}+2\right.\right.}{bc\log(F)+ien}$$

input
```
Int[F^(c*(a + b*x))*Sec[d + e*x]^n,x]
```

output
```
(E^(I*d*n + I*e*n*x - I*n*(d + e*x))*(1 + E^((2*I)*(d + e*x)))^n*F^(a*c +
b*c*x)*Hypergeometric2F1[n, (e*n - I*b*c*Log[F])/(2*e), (2 + n - (I*b*c*Lo
g[F])/e)/2, -E^((2*I)*(d + e*x))]*Sec[d + e*x]^n)/(I*e*n + b*c*Log[F])
```

3.26.3.1 Defintions of rubi rules used

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 4954
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[(1 + E^(2*I*(d + e*x)))^n*(Sec[d + e*x]^n/E^(I*n*(d + e*x)))
Int[SimplifyIntegrand[F^(c*(a + b*x))*(E^(I*n*(d + e*x))/(1 + E^(2*I*(d + e
*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[n]
```

3.26.4 Maple [F]

$$\int F^{c(xb+a)} \sec(ex+d)^n \, dx$$

input
```
int(F^(c*(b*x+a))*sec(e*x+d)^n,x)
```

output
```
int(F^(c*(b*x+a))*sec(e*x+d)^n,x)
```

3.26.5 Fricas [F]

$$\int F^{c(a+bx)} \sec^n(d+ex) \, dx = \int F^{(bx+a)c} \sec(ex+d)^n \, dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^n,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sec(e*x + d)^n, x)
```

3.26.6 Sympy [F]

$$\int F^{c(a+bx)}\sec^n(d+ex)\,dx=\int F^{c(a+bx)}\sec^n{(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*sec(e*x+d)**n,x)
```

output
```
Integral(F**(c*(a + b*x))*sec(d + e*x)**n, x)
```

3.26.7 Maxima [F]

$$\int F^{c(a+bx)}\sec^n(d+ex)\,dx=\int F^{(bx+a)c}\sec{(ex+d)}^n\;dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^n,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)*c)*sec(e*x + d)^n, x)
```

3.26.8 Giac [F]

$$\int F^{c(a+bx)}\sec^n(d+ex)\,dx=\int F^{(bx+a)c}\sec{(ex+d)}^n\;dx$$

input
```
integrate(F^(c*(b*x+a))*sec(e*x+d)^n,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sec(e*x + d)^n, x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sec^n(d+ex)\,dx=\int F^{c\,(a+b\,x)}\left(\frac{1}{\cos\left(d+e\,x\right)}\right)^n\,dx$$

input
```
int(F^(c*(a + b*x))*(1/cos(d + e*x))^n,x)
```

output
```
int(F^(c*(a + b*x))*(1/cos(d + e*x))^n, x)
```

3.27 $\int F^{c(a+bx)} \csc^n(d+ex)\,dx$

3.27.1 Optimal result

Integrand size = 18, antiderivative size = 102

$$\int F^{c(a+bx)}\csc^n(d+ex)\,dx = -\frac{\left(1-e^{-2i(d+ex)}\right)^n F^{ac+bcx}\csc^n(d+ex)\,\text{Hypergeometric2F1}\left(n,\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(2+n+\frac{ibc\log(F)}{e}\right),e^{-2i(d+}\right.}{ien-bc\log(F)}$$

output
```
-(1-1/exp(2*I*(e*x+d)))^n*F^(b*c*x+a*c)*csc(e*x+d)^n*hypergeom([n, 1/2*(I*
b*c*ln(F)+e*n)/e],[1+1/2*n+1/2*I*b*c*ln(F)/e],exp(-2*I*(e*x+d)))/(I*e*n-b*
c*ln(F))
```

3.27.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}\csc^n(d+ex)\,dx = \frac{i\left(1-e^{-2i(d+ex)}\right)^n F^{c(a+bx)}\csc^n(d+ex)\,\text{Hypergeometric2F1}\left(n,\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(2+n+\frac{ibc\log(F)}{e}\right),e^{-2i(d+e}\right.}{en+ibc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Csc[d + e*x]^n,x]
```

output
```
(I*(1 - E^((-2*I)*(d + e*x)))^n*F^(c*(a + b*x))*Csc[d + e*x]^n*Hypergeomet
ric2F1[n, (e*n + I*b*c*Log[F])/(2*e), (2 + n + (I*b*c*Log[F])/e)/2, E^((-2
*I)*(d + e*x))])/(e*n + I*b*c*Log[F])
```

3.27.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4955, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\csc^n(d+ex)\,dx$$

$$\Big\downarrow \text{ 4955}$$

$$e^{in(d+ex)}\left(1-e^{-2i(d+ex)}\right)^n\csc^n(d+ex)\int e^{-idn-iexn}\left(1-e^{-2i(d+ex)}\right)^{-n}F^{ac+bxc}dx$$

$$\Big\downarrow \text{ 2689}$$

$$-\frac{e^{in(d+ex)-idn-ienx}\left(1-e^{-2i(d+ex)}\right)^n F^{ac+bcx}\csc^n(d+ex)\,\text{Hypergeometric2F1}\left(n,\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(n+\frac{ibc\log(F)}{e}+\right.\right.}{-bc\log(F)+ien}$$

input
```
Int[F^(c*(a + b*x))*Csc[d + e*x]^n,x]
```

output
```
-((E^((-I)*d*n - I*e*n*x + I*n*(d + e*x))*(1 - E^((-2*I)*(d + e*x)))^n*F^(
a*c + b*c*x)*Csc[d + e*x]^n*Hypergeometric2F1[n, (e*n + I*b*c*Log[F])/(2*e
), (2 + n + (I*b*c*Log[F])/e)/2, E^((-2*I)*(d + e*x))])/(I*e*n - b*c*Log[F
]))
```

3.27.3.1 Defintions of rubi rules used

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 4955
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(1 - E^(-2*I*(d + e*x)))^n*(Csc[d + e*x]^n/E^((-I)*n*(d + e*x))
)   Int[SimplifyIntegrand[F^(c*(a + b*x))*(1/(E^(I*n*(d + e*x))*(1 - E^(-2*
I*(d + e*x)))^n)), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ
[n]
```

3.27.4 Maple [F]

$$\int F^{c(xb+a)} \csc(ex+d)^n \, dx$$

input
```
int(F^(c*(b*x+a))*csc(e*x+d)^n,x)
```

output
```
int(F^(c*(b*x+a))*csc(e*x+d)^n,x)
```

3.27.5 Fricas [F]

$$\int F^{c(a+bx)} \csc^n(d+ex) \, dx = \int F^{(bx+a)c} \csc(ex+d)^n \, dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^n,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csc(e*x + d)^n, x)
```

3.27.6 Sympy [F]

$$\int F^{c(a+bx)} \csc^n(d+ex)\,dx = \int F^{c(a+bx)} \csc^n{(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*csc(e*x+d)**n,x)
```

output
```
Integral(F**(c*(a + b*x))*csc(d + e*x)**n, x)
```

3.27.7 Maxima [F]

$$\int F^{c(a+bx)} \csc^n(d+ex)\,dx = \int F^{(bx+a)c} \csc{(ex+d)}^n \,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^n,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)*c)*csc(e*x + d)^n, x)
```

3.27.8 Giac [F]

$$\int F^{c(a+bx)} \csc^n(d+ex)\,dx = \int F^{(bx+a)c} \csc{(ex+d)}^n \,dx$$

input
```
integrate(F^(c*(b*x+a))*csc(e*x+d)^n,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csc(e*x + d)^n, x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)} \csc^n(d+ex)\,dx = \int F^{c\,(a+b\,x)} \left(\frac{1}{\sin\left(d+e\,x\right)}\right)^n dx$$

input
```
int(F^(c*(a + b*x))*(1/sin(d + e*x))^n,x)
```

output
```
int(F^(c*(a + b*x))*(1/sin(d + e*x))^n, x)
```

3.28 $\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx$

3.28.1 Optimal result

Integrand size = 21, antiderivative size = 139

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx = -\frac{e^{-id}F^{ac}(fx)^m\Gamma(1+m,x(ie-bc\log(F)))(x(ie-bc\log(F)))^{-m}}{2(e+ibc\log(F))} - \frac{e^{id}F^{ac}(fx)^m\Gamma(1+m,-x(ie+bc\log(F)))(-x(ie+bc\log(F)))^{-m}}{2(e-ibc\log(F))}$$

output
```
-1/2*F^(a*c)*(f*x)^m*GAMMA(1+m,x*(I*e-b*c*ln(F)))/exp(I*d)/((x*(I*e-b*c*ln
(F)))^m)/(e+I*b*c*ln(F))-1/2*exp(I*d)*F^(a*c)*(f*x)^m*GAMMA(1+m,-x*(b*c*ln
(F)+I*e))/(e-I*b*c*ln(F))/((-x*(b*c*ln(F)+I*e))^m)
```

3.28.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx = \frac{1}{2}F^{ac}(fx)^m(x(-ie-bc\log(F)))^{-m}\left(-ix\Gamma(1+m,iex-bcx\log(F))(ix(e+ibc\log(F)))^{-1-m}(-iex-bcx\log(F))^m(\cos(d)-i\sin(d)) - \frac{\Gamma(1+m,-iex-bcx\log(F))(\cos(d)+i\sin(d))}{e-ibc\log(F)}\right)$$

input
```
Integrate[F^(c*(a + b*x))*(f*x)^m*Sin[d + e*x],x]
```

output
```
(F^(a*c)*(f*x)^m*((-I)*x*Gamma[1 + m, I*e*x - b*c*x*Log[F]]*(I*x*(e + I*b*
c*Log[F]))^(-1 - m)*((-I)*e*x - b*c*x*Log[F])^m*(Cos[d] - I*Sin[d]) - (Gam
ma[1 + m, (-I)*e*x - b*c*x*Log[F]]*(Cos[d] + I*Sin[d]))/(e - I*b*c*Log[F])
))/(2*(x*((-I)*e - b*c*Log[F]))^m)
```

3.28.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \sin(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \; 7292$$

$$\int (fx)^m \sin(d+ex)F^{ac+bcx}dx$$

$$\Big\downarrow \; 7299$$

$$\int (fx)^m \sin(d+ex)F^{ac+bcx}dx$$

input
```
Int[F^(c*(a + b*x))*(f*x)^m*Sin[d + e*x],x]
```

output
```
$Aborted
```

3.28.3.1 Defintions of rubi rules used

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.28.4 Maple [F]

$$\int F^{c(xb+a)}(fx)^m \sin(ex+d)\,dx$$

input
```
int(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx = \frac{(i\,bc\log(F)-e)e^{\left(ac\log(F)-m\log\left(-\frac{bc\log(F)-i\,e}{f}\right)-i\,d\right)}\Gamma(m+1,-bcx\log(F)+i\,ex)+(-i\,bc\log(F)-e)e^{\left(ac\log\right.}}{2\left(b^2c^2\log(F)^2+e^2\right)}$$

input
```
integrate(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x, algorithm="fricas")
```

output
```
1/2*((I*b*c*log(F) - e)*e^(a*c*log(F) - m*log(-(b*c*log(F) - I*e)/f) - I*d
)*gamma(m + 1, -b*c*x*log(F) + I*e*x) + (-I*b*c*log(F) - e)*e^(a*c*log(F)
- m*log(-(b*c*log(F) + I*e)/f) + I*d)*gamma(m + 1, -b*c*x*log(F) - I*e*x))
/(b^2*c^2*log(F)^2 + e^2)
```

3.28.6 Sympy [F]

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx = \int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*x)**m*sin(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*(f*x)**m*sin(d + e*x), x)
```

3.28.7 Maxima [F]

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx = \int (fx)^m F^{(bx+a)c} \sin(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x, algorithm="maxima")
```

output
```
integrate((f*x)^m*F^((b*x + a)*c)*sin(e*x + d), x)
```

3.28.8 Giac [F]

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx = \int (fx)^m F^{(bx+a)c} \sin(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x, algorithm="giac")
```

output
```
integrate((f*x)^m*F^((b*x + a)*c)*sin(e*x + d), x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx = \int F^{c\,(a+b\,x)} \sin(d+e\,x)\,(f\,x)^m\,dx$$

input
```
int(F^(c*(a + b*x))*sin(d + e*x)*(f*x)^m,x)
```

output
```
int(F^(c*(a + b*x))*sin(d + e*x)*(f*x)^m, x)
```

3.29 $\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx$

3.29.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \mathrm{Int}\left(F^{ac+bcx}(fx)^m \csc(d+ex),x\right)$$

output
```
CannotIntegrate(F^(b*c*x+a*c)*(f*x)^m*csc(e*x+d),x)
```

3.29.2 Mathematica [N/A]

Not integrable

Time = 16.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x],x]
```

output
```
Integrate[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x], x]
```

3.29.3 Rubi [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7292, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \csc(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \text{ 7292}$$
$$\int (fx)^m \csc(d+ex)F^{ac+bcx}dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int (fx)^m \csc(d+ex)F^{ac+bcx}dx$$

input
```
Int[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x],x]
```

output
```
$Aborted
```

3.29.3.1 Defintions of rubi rules used

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.29.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{F^{c(xb+a)}(fx)^m}{\sin(ex+d)}dx$$

input
```
int(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x)
```

3.29.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \int \frac{(fx)^m F^{(bx+a)c}}{\sin(ex+d)}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x, algorithm="fricas")
```

output
```
integral((f*x)^m*F^(b*c*x + a*c)/sin(e*x + d), x)
```

3.29.6 Sympy [N/A]

Not integrable

Time = 2.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \int \frac{F^{c(a+bx)}(fx)^m}{\sin(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*x)**m/sin(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*(f*x)**m/sin(d + e*x), x)
```

3.29.7 Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \int \frac{(fx)^m F^{(bx+a)c}}{\sin(ex+d)}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x, algorithm="maxima")
```

output
```
integrate((f*x)^m*F^((b*x + a)*c)/sin(e*x + d), x)
```

3.29.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \int \frac{(fx)^m F^{(bx+a)c}}{\sin(ex+d)}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x, algorithm="giac")
```

output
```
integrate((f*x)^m*F^((b*x + a)*c)/sin(e*x + d), x)
```

3.29.9 Mupad [N/A]

Not integrable

Time = 27.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}\,(f\,x)^m}{\sin(d+e\,x)}\,dx$$

input
```
int((F^(c*(a + b*x))*(f*x)^m)/sin(d + e*x),x)
```

output
```
int((F^(c*(a + b*x))*(f*x)^m)/sin(d + e*x), x)
```

3.30 $\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx$

3.30.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \operatorname{Int}\left(F^{ac+bcx}(fx)^m \csc^2(d+ex),x\right)$$

output
```
CannotIntegrate(F^(b*c*x+a*c)*(f*x)^m*csc(e*x+d)^2,x)
```

3.30.2 Mathematica [N/A]

Not integrable

Time = 14.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x]^2,x]
```

output
```
Integrate[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x]^2, x]
```

3.30.3 Rubi [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7292, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \csc^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{7292}$$

$$\int (fx)^m \csc^2(d+ex)F^{ac+bcx}dx$$

$$\Big\downarrow \text{7299}$$

$$\int (fx)^m \csc^2(d+ex)F^{ac+bcx}dx$$

input
```
Int[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x]^2,x]
```

output
```
$Aborted
```

3.30.3.1 Defintions of rubi rules used

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.30.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(xb+a)}(fx)^m}{\sin(ex+d)^2}dx$$

input
```
int(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x)
```

3.30.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \int \frac{(fx)^m F^{(bx+a)c}}{\sin(ex+d)^2}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(-(f*x)^m*F^(b*c*x + a*c)/(cos(e*x + d)^2 - 1), x)
```

3.30.6 Sympy [N/A]

Not integrable

Time = 2.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \int \frac{F^{c(a+bx)}(fx)^m}{\sin^2(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*x)**m/sin(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*x)**m/sin(d + e*x)**2, x)
```

3.30.7 Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \int \frac{(fx)^m F^{(bx+a)c}}{\sin(ex+d)^2}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((f*x)^m*F^((b*x + a)*c)/sin(e*x + d)^2, x)
```

3.30.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \int \frac{(fx)^m F^{(bx+a)c}}{\sin(ex+d)^2}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((f*x)^m*F^((b*x + a)*c)/sin(e*x + d)^2, x)
```

3.30.9 Mupad [N/A]

Not integrable

Time = 28.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}\,(f\,x)^m}{\sin(d+e\,x)^2}\,dx$$

input
```
int((F^(c*(a + b*x))*(f*x)^m)/sin(d + e*x)^2,x)
```

output
```
int((F^(c*(a + b*x))*(f*x)^m)/sin(d + e*x)^2, x)
```

3.31 $\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx$

3.31.1 Optimal result

Integrand size = 44, antiderivative size = 24

$$\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx = F^{ac+bcx}(fx)^{-1+m}\sin(d+ex)$$

output
```
F^(b*c*x+a*c)*(f*x)^(-1+m)*sin(e*x+d)
```

3.31.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx = fF^{ac+bcx}x(fx)^{-2+m}\sin(d+ex)$$

input
```
Integrate[f*F^(c*(a + b*x))*(f*x)^(-2 + m)*(e*x*Cos[d + e*x] + (-1 + m + b
*c*x*Log[F])*Sin[d + e*x]),x]
```

output
```
f*F^(a*c + b*c*x)*x*(f*x)^(-2 + m)*Sin[d + e*x]
```

3.31.3 Rubi [A] (verified)

Time = 2.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {27, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f(fx)^{m-2}F^{c(a+bx)}(\sin(d+ex)(bcx\log(F)+m-1)+ex\cos(d+ex))\,dx$$

$$\Big\downarrow \text{ 27}$$

$$f\int F^{c(a+bx)}(fx)^{m-2}(ex\cos(d+ex)-(-m-bcx\log(F)+1)\sin(d+ex))dx$$

$$\Big\downarrow \text{ 7292}$$

$$f\int F^{ac+bxc}(fx)^{m-2}(ex\cos(d+ex)-(-m-bcx\log(F)+1)\sin(d+ex))dx$$

$$\Big\downarrow \text{ 7293}$$

$$f\int \left(F^{ac+bxc}(m+bcx\log(F)-1)\sin(d+ex)(fx)^{m-2}+\frac{eF^{ac+bxc}\cos(d+ex)(fx)^{m-1}}{f}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$(fx)^{m-1}\sin(d+ex)F^{ac+bcx}$$

input
```
Int[f*F^(c*(a + b*x))*(f*x)^(-2 + m)*(e*x*Cos[d + e*x] + (-1 + m + b*c*x*L
og[F])*Sin[d + e*x]),x]
```

output
```
F^(a*c + b*c*x)*(f*x)^(-1 + m)*Sin[d + e*x]
```

3.31.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.31.4 Maple [A] (verified)

Time = 5.93 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

method	result	size
parallelrisch	$\frac{F^{c(xb+a)}(fx)^m \sin(ex+d)}{fx}$	28
risch	$\frac{F^{c(xb+a)} \sin(ex+d) x^m f^m \mathrm{e}^{\frac{i\pi \operatorname{csgn}(ifx) m(\operatorname{csgn}(ifx)-\operatorname{csgn}(ix))(-\operatorname{csgn}(ifx)+\operatorname{csgn}(if))}{2}}}{xf}$	69

input
```
int(f*F^(c*(b*x+a))*(f*x)^(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*ln(F))*sin(e*
x+d)),x,method=_RETURNVERBOSE)
```

output
```
1/f*F^(c*(b*x+a))/x*(f*x)^m*sin(e*x+d)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx$$
$$=(fx)^{m-2}\,F^{bcx+ac}fx\sin\left(ex+d\right)$$

input
```
integrate(f*F^(c*(b*x+a))*(f*x)^(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*log(F))
*sin(e*x+d)),x, algorithm="fricas")
```

output
```
(f*x)^(m - 2)*F^(b*c*x + a*c)*f*x*sin(e*x + d)
```

3.31.6 Sympy [F]

$$\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx$$
$$=f\left(\int\left(-F^{ac+bcx}(fx)^{m-2}\sin\left(d+ex\right)\right)\,dx+\int F^{ac+bcx}m(fx)^{m-2}\sin\left(d+ex\right)dx+\int F^{ac+bcx}ex(fx)^{m-2}\cos\left(d+ex\right)dx+\int F^{ac+bcx}bcx(fx)^{m-2}\log\left(F\right)\sin\left(d+ex\right)dx\right)$$

input
```
integrate(f*F**(c*(b*x+a))*(f*x)**(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*ln(F)
)*sin(e*x+d)),x)
```

output
```
f*(Integral(-F**(a*c + b*c*x)*(f*x)**(m - 2)*sin(d + e*x), x) + Integral(F
**(a*c + b*c*x)*m*(f*x)**(m - 2)*sin(d + e*x), x) + Integral(F**(a*c + b*c
*x)*e*x*(f*x)**(m - 2)*cos(d + e*x), x) + Integral(F**(a*c + b*c*x)*b*c*x*
(f*x)**(m - 2)*log(F)*sin(d + e*x), x))
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx=\frac{F^{ac}f^{m-1}e^{(bcx\log(F)+m\log(x))}\sin\left(ex+d\right)}{x}$$

input
```
integrate(f*F^(c*(b*x+a))*(f*x)^(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*log(F))
*sin(e*x+d)),x, algorithm="maxima")
```

output
```
F^(a*c)*f^(m - 1)*e^(b*c*x*log(F) + m*log(x))*sin(e*x + d)/x
```

3.31.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6346 vs. $2(24) = 48$.

Time = 0.61 (sec) , antiderivative size = 6346, normalized size of antiderivative = 264.42

$$\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx$$
$$= \text{Too large to display}$$

input
```
integrate(f*F^(c*(b*x+a))*(f*x)^(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*log(F))
*sin(e*x+d)),x, algorithm="giac")
```

output
```
(x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)
*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f)
- 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x
 - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x
))^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4
*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*
pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*
tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*
pi*a*c - 1/2*d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x
)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*f
loor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1
/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(
f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(
-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi
*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) -
1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*tan(1/4*pi*a*
c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m
*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*
pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/
4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x...
```

3.31.9 Mupad [B] (verification not implemented)

Time = 29.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx = \frac{F^{c\,(a+b\,x)}\,\sin\left(d+e\,x\right)\,\left(f\,x\right)^m}{f\,x}$$

input
```
int(F^(c*(a + b*x))*f*(f*x)^(m - 2)*(sin(d + e*x)*(m + b*c*x*log(F) - 1) +
 e*x*cos(d + e*x)),x)
```

output
```
(F^(c*(a + b*x))*sin(d + e*x)*(f*x)^m)/(f*x)
```

3.32 $\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx$

3.32.1 Optimal result

Integrand size = 42, antiderivative size = 23

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx = fF^{c(a+bx)}x(fx)^m\sin(d+ex)$$

output
```
f*F^(c*(b*x+a))*x*(f*x)^m*sin(e*x+d)
```

3.32.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx = fF^{ac+bcx}x(fx)^m\sin(d+ex)$$

input
```
Integrate[f*F^(c*(a + b*x))*(f*x)^m*(e*x*Cos[d + e*x] + (1 + m + b*c*x*Log
[F])*Sin[d + e*x]),x]
```

output
```
f*F^(a*c + b*c*x)*x*(f*x)^m*Sin[d + e*x]
```

3.32.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f(fx)^m F^{c(a+bx)}(\sin(d+ex)(bcx\log(F)+m+1)+ex\cos(d+ex))\,dx$$

$$\Big\downarrow \text{ 27}$$

$$f\int F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F)+1)\sin(d+ex))dx$$

$$\Big\downarrow \text{ 7292}$$

$$f\int F^{ac+bxc}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F)+1)\sin(d+ex))dx$$

$$\Big\downarrow \text{ 7293}$$

$$f\int \left(F^{ac+bxc}(m+bcx\log(F)+1)\sin(d+ex)(fx)^m+\frac{eF^{ac+bxc}\cos(d+ex)(fx)^{m+1}}{f}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$f\left((m+1)\int F^{ac+bxc}(fx)^m\sin(d+ex)dx+\frac{e\int F^{ac+bxc}(fx)^{m+1}\cos(d+ex)dx}{f}+\frac{bc\log(F)\int F^{ac+bxc}(fx)^{m+1}\sin}{f}\right.$$

input
```
Int[f*F^(c*(a + b*x))*(f*x)^m*(e*x*Cos[d + e*x] + (1 + m + b*c*x*Log[F])*S
in[d + e*x]),x]
```

output
```
$Aborted
```

3.32.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.32.4 Maple [A] (verified)

Time = 5.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result
parallelrisch	$f\,F^{c(xb+a)}x(fx)^{m}\sin\left(ex+d\right)$
risch	$-\frac{i x^{m} f^{m} F^{c(xb+a)} x f \left(\mathrm{e}^{iex}\mathrm{e}^{id}\mathrm{e}^{-\frac{i\pi\,\mathrm{csgn}(ifx)^{3} m}{2}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ifx)^{2}\,\mathrm{csgn}(if) m}{2}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ifx)^{2}\,\mathrm{csgn}(ix) m}{2}}\mathrm{e}^{-\frac{i\pi\,\mathrm{csgn}(ifx)\,\mathrm{csgn}(if)\,\mathrm{csgn}(ix) m}{2}}\right.}{2}$

input
```
int(f*F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(1+m+b*c*x*ln(F))*sin(e*x+d)),
x,method=_RETURNVERBOSE)
```

output
```
f*F^(c*(b*x+a))*x*(f*x)^m*sin(e*x+d)
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx$$
$$=(fx)^{m}\,F^{bcx+ac}fx\sin\left(ex+d\right)$$

input
```
integrate(f*F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(1+m+b*c*x*log(F))*sin(e
*x+d)),x, algorithm="fricas")
```

output
```
(f*x)^m*F^(b*c*x + a*c)*f*x*sin(e*x + d)
```

3.32.6 Sympy [F]

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx$$
$$=f\left(\int F^{ac+bcx}(fx)^m\sin(d+ex)\,dx+\int F^{ac+bcx}m(fx)^m\sin(d+ex)\,dx\right.$$
$$\left.+\int F^{ac+bcx}ex(fx)^m\cos(d+ex)\,dx+\int F^{ac+bcx}bcx(fx)^m\log(F)\sin(d+ex)\,dx\right)$$

input
```
integrate(f*F**(c*(b*x+a))*(f*x)**m*(e*x*cos(e*x+d)+(1+m+b*c*x*ln(F))*sin(
e*x+d)),x)
```

output
```
f*(Integral(F**(a*c + b*c*x)*(f*x)**m*sin(d + e*x), x) + Integral(F**(a*c
+ b*c*x)*m*(f*x)**m*sin(d + e*x), x) + Integral(F**(a*c + b*c*x)*e*x*(f*x)
**m*cos(d + e*x), x) + Integral(F**(a*c + b*c*x)*b*c*x*(f*x)**m*log(F)*sin
(d + e*x), x))
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx$$
$$=F^{ac}f^{m+1}xe^{(bcx\log(F)+m\log(x))}\sin(ex+d)$$

input
```
integrate(f*F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(1+m+b*c*x*log(F))*sin(e
*x+d)),x, algorithm="maxima")
```

output
```
F^(a*c)*f^(m + 1)*x*e^(b*c*x*log(F) + m*log(x))*sin(e*x + d)
```

3.32.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4746 vs. $2(23) = 46$.

Time = 0.53 (sec) , antiderivative size = 4746, normalized size of antiderivative = 206.35

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx = \text{Too large to display}$$

input
```
integrate(f*F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(1+m+b*c*x*log(F))*sin(e
*x+d)),x, algorithm="giac")
```

output
```
(x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*
c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4
*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*x*sg
n(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*
sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1
/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) - x*abs(F
)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(
F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sg
n(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*x*sgn(F) - 1
/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) +
 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*
c + 1/2*d)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*
e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4
*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1
/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*
c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*
m*sgn(x) - 1/2*pi*m - 1/2*e*x)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)
^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x
*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*
x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi...
```

3.32.9 Mupad [B] (verification not implemented)

Time = 28.99 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx$$
$$= F^{c\,(a+b\,x)}\,f\,x\,\sin\left(d+e\,x\right)\,\left(f\,x\right)^m$$

input
```
int(F^(c*(a + b*x))*f*(f*x)^m*(sin(d + e*x)*(m + b*c*x*log(F) + 1) + e*x*c
os(d + e*x)),x)
```

output
```
F^(c*(a + b*x))*f*x*sin(d + e*x)*(f*x)^m
```

3.32. $\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx$

3.33 $\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx$

3.33.1 Optimal result

Integrand size = 43, antiderivative size = 22

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx = F^{ac+bcx}(fx)^m\sin(d+ex)$$

output
```
F^(b*c*x+a*c)*(f*x)^m*sin(e*x+d)
```

3.33.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx = F^{ac+bcx}(fx)^m\sin(d+ex)$$

input
```
Integrate[(F^(c*(a + b*x))*(f*x)^m*(e*x*Cos[d + e*x] + (m + b*c*x*Log[F])*
Sin[d + e*x]))/x,x]
```

output
```
F^(a*c + b*c*x)*(f*x)^m*Sin[d + e*x]
```

3.33.3 Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {8, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m F^{c(a+bx)}(\sin(d+ex)(bcx\log(F)+m)+ex\cos(d+ex))}{x} \, dx$$

$$\Big\downarrow \text{8}$$

$$f\int F^{c(a+bx)}(fx)^{m-1}(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))dx$$

$$\Big\downarrow \text{7292}$$

$$f\int F^{ac+bxc}(fx)^{m-1}(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))dx$$

$$\Big\downarrow \text{7293}$$

$$f\int \left(F^{ac+bxc}(m+bcx\log(F))\sin(d+ex)(fx)^{m-1}+\frac{eF^{ac+bxc}\cos(d+ex)(fx)^m}{f}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$(fx)^m\sin(d+ex)F^{ac+bcx}$$

input
```
Int[(F^(c*(a + b*x))*(f*x)^m*(e*x*Cos[d + e*x] + (m + b*c*x*Log[F])*Sin[d
+ e*x]))/x,x]
```

output
```
F^(a*c + b*c*x)*(f*x)^m*Sin[d + e*x]
```

3.33.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.33.4 Maple [A] (verified)

Time = 5.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

method	result
parallelrisch	$F^{c(xb+a)}(fx)^m \sin(ex+d)$
risch	$-\frac{i x^m f^m F^{c(xb+a)}\left(\mathrm{e}^{iex}\mathrm{e}^{id}\mathrm{e}^{-\frac{i\pi\,\mathrm{csgn}(ifx)^3 m}{2}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ifx)^2\,\mathrm{csgn}(if)m}{2}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ifx)^2\,\mathrm{csgn}(ix)m}{2}}\mathrm{e}^{-\frac{i\pi\,\mathrm{csgn}(ifx)\,\mathrm{csgn}(if)\,\mathrm{csgn}(ix)m}{2}} - \right.}{2}$

input
```
int(F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(m+b*c*x*ln(F))*sin(e*x+d))/x,x,
method=_RETURNVERBOSE)
```

output
```
F^(c*(b*x+a))*(f*x)^m*sin(e*x+d)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx = (fx)^m F^{bcx+ac}\sin(ex+d)$$

input
```
integrate(F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(m+b*c*x*log(F))*sin(e*x+d
))/x,x, algorithm="fricas")
```

output
```
(f*x)^m*F^(b*c*x + a*c)*sin(e*x + d)
```

3.33.6 Sympy [F]

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx = \int \frac{F^{c(a+bx)}(fx)^m\left(bcx\log\left(F\right)\sin\left(d+ex\right)+ex\cos\left(d+ex\right)+m\sin\left(d+ex\right)\right)}{x}\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*x)**m*(e*x*cos(e*x+d)+(m+b*c*x*ln(F))*sin(e*x+
d))/x,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*x)**m*(b*c*x*log(F)*sin(d + e*x) + e*x*cos(d
+ e*x) + m*sin(d + e*x))/x, x)
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx = F^{ac}f^m e^{(bcx\log(F)+m\log(x))}\sin\left(ex+d\right)$$

input
```
integrate(F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(m+b*c*x*log(F))*sin(e*x+d
))/x,x, algorithm="maxima")
```

output
```
F^(a*c)*f^m*e^(b*c*x*log(F) + m*log(x))*sin(e*x + d)
```

3.33.8 Giac [F]

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx = \int \frac{(ex\cos\left(ex+d\right)+(bcx\log\left(F\right)+m)\sin\left(ex+d\right))(fx)^m F^{(bx+a)c}}{x}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(m+b*c*x*log(F))*sin(e*x+d
))/x,x, algorithm="giac")
```

output
```
integrate((e*x*cos(e*x + d) + (b*c*x*log(F) + m)*sin(e*x + d))*(f*x)^m*F^(
(b*x + a)*c)/x, x)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 27.98 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx = F^{c\,(a+b\,x)}\,\sin\left(d+e\,x\right)\,\left(f\,x\right)^m$$

input
```
int((F^(c*(a + b*x))*(f*x)^m*(sin(d + e*x)*(m + b*c*x*log(F)) + e*x*cos(d
+ e*x)))/x,x)
```

output
```
F^(c*(a + b*x))*sin(d + e*x)*(f*x)^m
```

3.34 $\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx$

3.34.1 Optimal result

Integrand size = 35, antiderivative size = 17

$$\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx=F^{c(a+bx)}x\sin(d+ex)$$

output
```
F^(c*(b*x+a))*x*sin(e*x+d)
```

3.34.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx=F^{ac+bcx}x\sin(d+ex)$$

input
```
Integrate[F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (1 + b*c*x*Log[F])*Sin[d + e
*x]),x]
```

output
```
F^(a*c + b*c*x)*x*Sin[d + e*x]
```

3.34.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 327 vs. $2(17) = 34$.

Time = 0.95 (sec) , antiderivative size = 327, normalized size of antiderivative = 19.24, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(\sin(d+ex)(bcx\log(F)+1)+ex\cos(d+ex))\,dx$$

$$\Big\downarrow \text{7292}$$

$$\int F^{ac+bcx}(\sin(d+ex)(bcx\log(F)+1)+ex\cos(d+ex))dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(ex\cos(d+ex)F^{ac+bcx}+\sin(d+ex)F^{ac+bcx}(bcx\log(F)+1)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2c^2x\log^2(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{e^2x\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{bc\log(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{bce^2\log(F)\sin(d+ex)F^{ac+bcx}}{\left(b^2c^2\log^2(F)+e^2\right)^2}-\frac{e\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{b^2c^2e\log^2(F)\cos(d+ex)F^{ac+bcx}}{\left(b^2c^2\log^2(F)+e^2\right)^2}+\frac{e^3\cos(d+ex)F^{ac+bcx}}{\left(b^2c^2\log^2(F)+e^2\right)^2}-\frac{b^3c^3\log^3(F)\sin(d+ex)F^{ac+bcx}}{\left(b^2c^2\log^2(F)+e^2\right)^2}$$

input
```
Int[F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (1 + b*c*x*Log[F])*Sin[d + e*x]),x
]
```

output
```
(e^3*F^(a*c + b*c*x)*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)^2 + (b^2*c^2*e
*F^(a*c + b*c*x)*Cos[d + e*x]*Log[F]^2)/(e^2 + b^2*c^2*Log[F]^2)^2 - (e*F^
(a*c + b*c*x)*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2) - (b*c*e^2*F^(a*c + b
*c*x)*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)^2 - (b^3*c^3*F^(a*c +
b*c*x)*Log[F]^3*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)^2 + (e^2*F^(a*c + b
*c*x)*x*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2) + (b*c*F^(a*c + b*c*x)*Log[
F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2) + (b^2*c^2*F^(a*c + b*c*x)*x*Log
[F]^2*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)
```

3.34.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.34.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result
risch	$F^{c(xb+a)}x\sin\left(ex+d\right)$
parallelrisch	$F^{c(xb+a)}x\sin\left(ex+d\right)$
norman	$\frac{2x\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}$
parts	$\frac{\frac{e\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{e^{2}+b^{2}c^{2}\ln(F)^{2}}-\frac{e\,\mathrm{e}^{c(xb+a)\ln(F)}}{e^{2}+b^{2}c^{2}\ln(F)^{2}}+\frac{2bc\ln(F)\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^{2}+b^{2}c^{2}\ln(F)^{2}}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}+\frac{bc\ln(F)\left(\frac{ex\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{e^{2}+b^{2}c^{2}\ln(F)^{2}}\right.}{}$

input
```
int(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*ln(F))*sin(e*x+d)),x,method=_RE
TURNVERBOSE)
```

output
```
F^(c*(b*x+a))*x*sin(e*x+d)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx = F^{bcx+ac}x\sin{(ex+d)}$$

input
```
integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*log(F))*sin(e*x+d)),x, al
gorithm="fricas")
```

output
```
F^(b*c*x + a*c)*x*sin(e*x + d)
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx = F^{ac+bcx}x\sin{(d+ex)}$$

input
```
integrate(F**(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*ln(F))*sin(e*x+d)),x)
```

output
```
F**(a*c + b*c*x)*x*sin(d + e*x)
```

3.34.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1382 vs. $2(17) = 34$.

Time = 0.33 (sec) , antiderivative size = 1382, normalized size of antiderivative = 81.29

$$\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*log(F))*sin(e*x+d)),x, al
gorithm="maxima")
```

output
```
1/2*((F^(a*c)*b^2*c^2*log(F)^2*sin(d) + 2*F^(a*c)*b*c*e*cos(d)*log(F) - F^
(a*c)*e^2*sin(d) - (F^(a*c)*b^3*c^3*log(F)^3*sin(d) + F^(a*c)*b^2*c^2*e*co
s(d)*log(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(d) + F^(a*c)*e^3*cos(d))*x)*F^(
b*c*x)*cos(e*x + 2*d) - (F^(a*c)*b^2*c^2*log(F)^2*sin(d) - 2*F^(a*c)*b*c*e
*cos(d)*log(F) - F^(a*c)*e^2*sin(d) - (F^(a*c)*b^3*c^3*log(F)^3*sin(d) - F
^(a*c)*b^2*c^2*e*cos(d)*log(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(d) - F^(a*c)
*e^3*cos(d))*x)*F^(b*c*x)*cos(e*x) - (F^(a*c)*b^2*c^2*cos(d)*log(F)^2 - 2*
F^(a*c)*b*c*e*log(F)*sin(d) - F^(a*c)*e^2*cos(d) - (F^(a*c)*b^3*c^3*cos(d)
*log(F)^3 - F^(a*c)*b^2*c^2*e*log(F)^2*sin(d) + F^(a*c)*b*c*e^2*cos(d)*log
(F) - F^(a*c)*e^3*sin(d))*x)*F^(b*c*x)*sin(e*x + 2*d) - (F^(a*c)*b^2*c^2*c
os(d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(d) - F^(a*c)*e^2*cos(d) - (F^(
a*c)*b^3*c^3*cos(d)*log(F)^3 + F^(a*c)*b^2*c^2*e*log(F)^2*sin(d) + F^(a*c)
*b*c*e^2*cos(d)*log(F) + F^(a*c)*e^3*sin(d))*x)*F^(b*c*x)*sin(e*x))*b*c*lo
g(F)/(b^4*c^4*cos(d)^2*log(F)^4 + b^4*c^4*log(F)^4*sin(d)^2 + (cos(d)^2 +
sin(d)^2)*e^4 + 2*(b^2*c^2*cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2)*
e^2) - 1/2*((F^(a*c)*b^2*c^2*cos(d)*log(F)^2 - 2*F^(a*c)*b*c*e*log(F)*sin(
d) - F^(a*c)*e^2*cos(d) - (F^(a*c)*b^3*c^3*cos(d)*log(F)^3 - F^(a*c)*b^2*c
^2*e*log(F)^2*sin(d) + F^(a*c)*b*c*e^2*cos(d)*log(F) - F^(a*c)*e^3*sin(d))
*x)*F^(b*c*x)*cos(e*x + 2*d) + (F^(a*c)*b^2*c^2*cos(d)*log(F)^2 + 2*F^(a*c
)*b*c*e*log(F)*sin(d) - F^(a*c)*e^2*cos(d) - (F^(a*c)*b^3*c^3*cos(d)*lo...
```

3.34.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 1941, normalized size of antiderivative = 114.18

$$\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx=\text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*log(F))*sin(e*x+d)),x, al
gorithm="giac")
```

output
```
-1/4*((pi*b^2*c^2*x*log(F)*sgn(F) - pi*b^2*c^2*x*log(F) - 2*I*b^2*c^2*x*lo
g(F)*log(abs(F)) - I*pi*b*c*e*x*sgn(F) + I*pi*b*c*e*x + 2*b*c*e*x*log(F) -
 2*b*c*e*x*log(abs(F)) + pi*b*c*sgn(F) - pi*b*c - 2*I*e^2*x + 2*I*b*c*log(
F) - 2*I*b*c*log(abs(F)) + 4*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x
+ 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(pi^2*b^2*c^2*sgn(F) +
 2*I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs
(F)) + 2*b^2*c^2*log(abs(F))^2 - 2*pi*b*c*e*sgn(F) + 2*pi*b*c*e + 4*I*b*c*
e*log(abs(F)) - 2*e^2) - (pi*b^2*c^2*x*log(F)*sgn(F) - pi*b^2*c^2*x*log(F)
 + 2*I*b^2*c^2*x*log(F)*log(abs(F)) - I*pi*b*c*e*x*sgn(F) + I*pi*b*c*e*x +
 2*b*c*e*x*log(F) + 2*b*c*e*x*log(abs(F)) + pi*b*c*sgn(F) - pi*b*c - 2*I*e
^2*x - 2*I*b*c*log(F) + 2*I*b*c*log(abs(F)))*e^(-1/2*I*pi*b*c*x*sgn(F) + 1
/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(pi^2*b^
2*c^2*sgn(F) - 2*I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 + 2*I*pi*b
^2*c^2*log(abs(F)) + 2*b^2*c^2*log(abs(F))^2 - 2*pi*b*c*e*sgn(F) + 2*pi*b*
c*e - 4*I*b*c*e*log(abs(F)) - 2*e^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F
))) + 1/4*I*((-I*pi*b^2*c^2*x*log(F)*sgn(F) + I*pi*b^2*c^2*x*log(F) - 2*b^
2*c^2*x*log(F)*log(abs(F)) - pi*b*c*e*x*sgn(F) + pi*b*c*e*x - 2*I*b*c*e*x*
log(F) + 2*I*b*c*e*x*log(abs(F)) - I*pi*b*c*sgn(F) + I*pi*b*c - 2*e^2*x +
2*b*c*log(F) - 2*b*c*log(abs(F)) - 4*I*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I
*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(pi^2*b^2...
```

3.34.9 Mupad [B] (verification not implemented)

Time = 27.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx=F^{c\,(a+b\,x)}\,x\,\sin\left(d+e\,x\right)$$

input
```
int(F^(c*(a + b*x))*(sin(d + e*x)*(b*c*x*log(F) + 1) + e*x*cos(d + e*x)),x
)
```

output
```
F^(c*(a + b*x))*x*sin(d + e*x)
```

3.35 $\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx$

3.35.1 Optimal result

Integrand size = 30, antiderivative size = 16

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx=F^{c(a+bx)}\sin(d+ex)$$

output
```
F^(c*(b*x+a))*sin(e*x+d)
```

3.35.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx=F^{c(a+bx)}\sin(d+ex)$$

input
```
Integrate[F^(c*(a + b*x))*(e*Cos[d + e*x] + b*c*Log[F]*Sin[d + e*x]),x]
```

output
```
F^(c*(a + b*x))*Sin[d + e*x]
```

3.35.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(bc\log(F)\sin(d+ex)+e\cos(d+ex))\,dx$$

$$\Big\downarrow \text{2726}$$

$$\sin(d+ex)F^{c(a+bx)}$$

input
```
Int[F^(c*(a + b*x))*(e*Cos[d + e*x] + b*c*Log[F]*Sin[d + e*x]),x]
```

output
```
F^(c*(a + b*x))*Sin[d + e*x]
```

3.35.3.1 Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

3.35.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result
risch	$F^{c(xb+a)}\sin(ex+d)$
parallelrisch	$F^{c(xb+a)}\sin(ex+d)$
norman	$\frac{2\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$
parts	$\frac{\frac{ebc\ln(F)\mathrm{e}^{c(xb+a)\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{2e^2\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}-\frac{ebc\ln(F)\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}+\frac{\frac{ebc\ln(F)\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\right.}{e^2+b^2c^2\ln(F)^2}}{}$

input
```
int(F^(c*(b*x+a))*(e*cos(e*x+d)+b*c*ln(F)*sin(e*x+d)),x,method=_RETURNVERB
OSE)
```

output
```
F^(c*(b*x+a))*sin(e*x+d)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx=F^{bcx+ac}\sin\left(ex+d\right)$$

input
```
integrate(F^(c*(b*x+a))*(e*cos(e*x+d)+b*c*log(F)*sin(e*x+d)),x, algorithm=
"fricas")
```

output
```
F^(b*c*x + a*c)*sin(e*x + d)
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx=F^{ac+bcx}\sin\left(d+ex\right)$$

input
```
integrate(F**(c*(b*x+a))*(e*cos(e*x+d)+b*c*ln(F)*sin(e*x+d)),x)
```

output
```
F**(a*c + b*c*x)*sin(d + e*x)
```

3.35.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(16) = 32.

Time = 0.27 (sec) , antiderivative size = 392, normalized size of antiderivative = 24.50

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx=$$
$$-\frac{((F^{ac}bc\log\left(F\right)\sin\left(d\right)+F^{ac}e\cos\left(d\right))F^{bcx}\cos\left(ex+2\,d\right)-(F^{ac}bc\log\left(F\right)\sin\left(d\right)-F^{ac}e\cos\left(d\right))F^{bcx}\cos}{2\left(b^2c^2\cos\left(d\right)^2\log\left(F\right)^2+\right.}$$
$$+\frac{((F^{ac}bc\cos\left(d\right)\log\left(F\right)-F^{ac}e\sin\left(d\right))F^{bcx}\cos\left(ex+2\,d\right)+(F^{ac}bc\cos\left(d\right)\log\left(F\right)+F^{ac}e\sin\left(d\right))F^{bcx}\cos}{2\left(b^2c^2\cos\left(d\right)^2\log\left(F\right)^2+b^2c\right.}$$

input
```
integrate(F^(c*(b*x+a))*(e*cos(e*x+d)+b*c*log(F)*sin(e*x+d)),x, algorithm=
"maxima")
```

output
```
-1/2*((F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x + 2
*d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x) -
(F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x + 2*d) -
(F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x))*b*c*log
(F)/(b^2*c^2*cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + s
in(d)^2)*e^2) + 1/2*((F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c
*x)*cos(e*x + 2*d) + (F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c
*x)*cos(e*x) + (F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*si
n(e*x + 2*d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*si
n(e*x))*e/(b^2*c^2*cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)
^2 + sin(d)^2)*e^2)
```

3.35.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 639, normalized size of antiderivative = 39.94

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(e*cos(e*x+d)+b*c*log(F)*sin(e*x+d)),x, algorithm=
"giac")
```

output
```
-I*((b*c*log(F) - I*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*p
i*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c
 + 4*b*c*log(abs(F)) + 4*I*e) - (b*c*log(F) - I*e)*e^(-1/2*I*pi*b*c*x*sgn(
F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-
2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(
abs(F)) + a*c*log(abs(F))) - ((-I*b*c*log(F) - e)*e^(1/2*I*pi*b*c*x*sgn(F)
 - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(2*I
*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e) + (-I*b*c*log(F)
- e)*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/
2*I*pi*a*c - I*e*x - I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs
(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*((b*c*log(F) +
I*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2
*I*pi*a*c - I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F
)) - 4*I*e) - (b*c*log(F) + I*e)*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*
x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*sgn(F)
+ 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(
abs(F))) - ((I*b*c*log(F) - e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x +
 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*
I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e) + (I*b*c*log(F) - e)*e^(-1/2*I*pi*b*
c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*e*...
```

3.35.9 Mupad [B] (verification not implemented)

Time = 26.96 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx = F^{c\,(a+b\,x)}\,\sin\left(d+e\,x\right)$$

input
```
int(F^(c*(a + b*x))*(e*cos(d + e*x) + b*c*sin(d + e*x)*log(F)),x)
```

output
```
F^(c*(a + b*x))*sin(d + e*x)
```

3.36 $\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx$

3.36.1 Optimal result

Integrand size = 38, antiderivative size = 20

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx=\frac{F^{ac+bcx}\sin(d+ex)}{x}$$

output
```
F^(b*c*x+a*c)*sin(e*x+d)/x
```

3.36.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx=\frac{F^{ac+bcx}\sin(d+ex)}{x}$$

input
```
Integrate[(F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (-1 + b*c*x*Log[F])*Sin[d +
 e*x]))/x^2,x]
```

output
```
(F^(a*c + b*c*x)*Sin[d + e*x])/x
```

3.36.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}(\sin(d+ex)(bcx\log(F)-1)+ex\cos(d+ex))}{x^2}\,dx$$

$$\Big\downarrow \text{ 7292}$$

$$\int \frac{F^{ac+bcx}(\sin(d+ex)(bcx\log(F)-1)+ex\cos(d+ex))}{x^2}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{\sin(d+ex)F^{ac+bcx}(bcx\log(F)-1)}{x^2}+\frac{e\cos(d+ex)F^{ac+bcx}}{x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sin(d+ex)F^{ac+bcx}}{x}$$

input
```
Int[(F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (-1 + b*c*x*Log[F])*Sin[d + e*x])
)/x^2,x]
```

output
```
(F^(a*c + b*c*x)*Sin[d + e*x])/x
```

3.36.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.36.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{F^{c(xb+a)}\sin(ex+d)}{x}$	20
parallelrisch	$\frac{F^{c(xb+a)}\sin(ex+d)}{x}$	20
norman	$\frac{2\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)x}$	40

input
```
int(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*ln(F))*sin(e*x+d))/x^2,x,metho
d=_RETURNVERBOSE)
```

output
```
1/x*F^(c*(b*x+a))*sin(e*x+d)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx=\frac{F^{bcx+ac}\sin{(ex+d)}}{x}$$

input
```
integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*log(F))*sin(e*x+d))/x^2,
x, algorithm="fricas")
```

output
```
F^(b*c*x + a*c)*sin(e*x + d)/x
```

3.36.6 Sympy [F]

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx$$
$$=\int \frac{F^{c(a+bx)}(bcx\log{(F)}\sin{(d+ex)}+ex\cos{(d+ex)}-\sin{(d+ex)})}{x^2}\,dx$$

input
```
integrate(F**(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*ln(F))*sin(e*x+d))/x**2
,x)
```

output
```
Integral(F**(c*(a + b*x))*(b*c*x*log(F)*sin(d + e*x) + e*x*cos(d + e*x) -
sin(d + e*x))/x**2, x)
```

3.36.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.76 (sec) , antiderivative size = 564, normalized size of antiderivative = 28.20

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*log(F))*sin(e*x+d))/x^2,
x, algorithm="maxima")
```

output
```
-1/4*F^(a*c)*b*c*(I*conjugate(gamma(-1, -(b*c*log(F) + I*e)*x)) - I*conjug
ate(gamma(-1, -(b*c*log(F) - I*e)*x)) - I*gamma(-1, -(b*c*log(F) + I*e)*x)
 + I*gamma(-1, -(b*c*log(F) - I*e)*x))*cos(d)*log(F) - 1/4*F^(a*c)*b*c*(co
njugate(gamma(-1, -(b*c*log(F) + I*e)*x)) + conjugate(gamma(-1, -(b*c*log(
F) - I*e)*x)) + gamma(-1, -(b*c*log(F) + I*e)*x) + gamma(-1, -(b*c*log(F)
- I*e)*x))*log(F)*sin(d) - 1/4*(F^(a*c)*(I*Ei((b*c*log(F) + I*e)*x) - I*Ei
((b*c*log(F) - I*e)*x) - I*conjugate(Ei((b*c*log(F) + I*e)*x)) + I*conjuga
te(Ei((b*c*log(F) - I*e)*x)))*cos(d) - F^(a*c)*(Ei((b*c*log(F) + I*e)*x) +
 Ei((b*c*log(F) - I*e)*x) + conjugate(Ei((b*c*log(F) + I*e)*x)) + conjugat
e(Ei((b*c*log(F) - I*e)*x)))*sin(d))*b*c*log(F) + 1/4*(F^(a*c)*(Ei((b*c*lo
g(F) + I*e)*x) + Ei((b*c*log(F) - I*e)*x) + conjugate(Ei((b*c*log(F) + I*e
)*x)) + conjugate(Ei((b*c*log(F) - I*e)*x)))*cos(d) - F^(a*c)*(-I*Ei((b*c*
log(F) + I*e)*x) + I*Ei((b*c*log(F) - I*e)*x) + I*conjugate(Ei((b*c*log(F)
 + I*e)*x)) - I*conjugate(Ei((b*c*log(F) - I*e)*x)))*sin(d))*e - 1/4*(F^(a
*c)*(conjugate(gamma(-1, -(b*c*log(F) + I*e)*x)) + conjugate(gamma(-1, -(b
*c*log(F) - I*e)*x)) + gamma(-1, -(b*c*log(F) + I*e)*x) + gamma(-1, -(b*c*
log(F) - I*e)*x))*cos(d) + F^(a*c)*(-I*conjugate(gamma(-1, -(b*c*log(F) +
I*e)*x)) + I*conjugate(gamma(-1, -(b*c*log(F) - I*e)*x)) + I*gamma(-1, -(b
*c*log(F) + I*e)*x) - I*gamma(-1, -(b*c*log(F) - I*e)*x))*sin(d))*e
```

3.36.8 Giac [F]

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx$$
$$=\int \frac{(ex\cos(ex+d)+(bcx\log(F)-1)\sin(ex+d))F^{(bx+a)c}}{x^2}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*log(F))*sin(e*x+d))/x^2,
x, algorithm="giac")
```

output
```
integrate((e*x*cos(e*x + d) + (b*c*x*log(F) - 1)*sin(e*x + d))*F^((b*x + a
)*c)/x^2, x)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 27.72 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx=\frac{F^{c\,(a+b\,x)}\,\sin\left(d+e\,x\right)}{x}$$

input
```
int((F^(c*(a + b*x))*(sin(d + e*x)*(b*c*x*log(F) - 1) + e*x*cos(d + e*x)))
/x^2,x)
```

output
```
(F^(c*(a + b*x))*sin(d + e*x))/x
```

3.37 $\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx$

3.37.1 Optimal result

Integrand size = 38, antiderivative size = 20

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx = \frac{F^{ac+bcx}\sin(d+ex)}{x^2}$$

output
```
F^(b*c*x+a*c)*sin(e*x+d)/x^2
```

3.37.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx = \frac{F^{ac+bcx}\sin(d+ex)}{x^2}$$

input
```
Integrate[(F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (-2 + b*c*x*Log[F])*Sin[d +
 e*x]))/x^3,x]
```

output
```
(F^(a*c + b*c*x)*Sin[d + e*x])/x^2
```

3.37.3 Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}(\sin(d+ex)(bcx\log(F)-2)+ex\cos(d+ex))}{x^3}\,dx$$

$$\Big\downarrow \text{7292}$$

$$\int \frac{F^{ac+bcx}(\sin(d+ex)(bcx\log(F)-2)+ex\cos(d+ex))}{x^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{\sin(d+ex)F^{ac+bcx}(bcx\log(F)-2)}{x^3}+\frac{e\cos(d+ex)F^{ac+bcx}}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(d+ex)F^{ac+bcx}}{x^2}$$

input
```
Int[(F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (-2 + b*c*x*Log[F])*Sin[d + e*x])
)/x^3,x]
```

output
```
(F^(a*c + b*c*x)*Sin[d + e*x])/x^2
```

3.37.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.37.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{\sin(ex+d)F^{c(xb+a)}}{x^2}$	20
parallelrisch	$\frac{\sin(ex+d)F^{c(xb+a)}}{x^2}$	20
norman	$\frac{2\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)x^2}$	40

input
```
int(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*ln(F))*sin(e*x+d))/x^3,x,metho
d=_RETURNVERBOSE)
```

output
```
sin(e*x+d)*F^(c*(b*x+a))/x^2
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx = \frac{F^{bcx+ac}\sin(ex+d)}{x^2}$$

input
```
integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*log(F))*sin(e*x+d))/x^3,
x, algorithm="fricas")
```

output
```
F^(b*c*x + a*c)*sin(e*x + d)/x^2
```

3.37.6 Sympy [F]

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx = \int \frac{F^{c(a+bx)}(bcx\log(F)\sin(d+ex)+ex\cos(d+ex)-2\sin(d+ex))}{x^3}\,dx$$

input
```
integrate(F**(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*ln(F))*sin(e*x+d))/x**3
,x)
```

output
```
Integral(F**(c*(a + b*x))*(b*c*x*log(F)*sin(d + e*x) + e*x*cos(d + e*x) -
2*sin(d + e*x))/x**3, x)
```

3.37.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.08 (sec) , antiderivative size = 1069, normalized size of antiderivative = 53.45

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*log(F))*sin(e*x+d))/x^3,
x, algorithm="maxima")
```

output
```
-1/2*F^(a*c)*b^2*c^2*(-I*conjugate(gamma(-2, -(b*c*log(F) + I*e)*x)) + I*c
onjugate(gamma(-2, -(b*c*log(F) - I*e)*x)) + I*gamma(-2, -(b*c*log(F) + I*
e)*x) - I*gamma(-2, -(b*c*log(F) - I*e)*x))*cos(d)*log(F)^2 + 1/2*F^(a*c)*
b^2*c^2*(conjugate(gamma(-2, -(b*c*log(F) + I*e)*x)) + conjugate(gamma(-2,
 -(b*c*log(F) - I*e)*x)) + gamma(-2, -(b*c*log(F) + I*e)*x) + gamma(-2, -(
b*c*log(F) - I*e)*x))*log(F)^2*sin(d) + 1/4*(F^(a*c)*b*c*(I*conjugate(gamm
a(-1, -(b*c*log(F) + I*e)*x)) - I*conjugate(gamma(-1, -(b*c*log(F) - I*e)*
x)) - I*gamma(-1, -(b*c*log(F) + I*e)*x) + I*gamma(-1, -(b*c*log(F) - I*e)
*x))*cos(d)*log(F) + F^(a*c)*b*c*(conjugate(gamma(-1, -(b*c*log(F) + I*e)*
x)) + conjugate(gamma(-1, -(b*c*log(F) - I*e)*x)) + gamma(-1, -(b*c*log(F)
 + I*e)*x) + gamma(-1, -(b*c*log(F) - I*e)*x))*log(F)*sin(d) + (F^(a*c)*(c
onjugate(gamma(-1, -(b*c*log(F) + I*e)*x)) + conjugate(gamma(-1, -(b*c*log
(F) - I*e)*x)) + gamma(-1, -(b*c*log(F) + I*e)*x) + gamma(-1, -(b*c*log(F)
 - I*e)*x))*cos(d) + F^(a*c)*(-I*conjugate(gamma(-1, -(b*c*log(F) + I*e)*x
)) + I*conjugate(gamma(-1, -(b*c*log(F) - I*e)*x)) + I*gamma(-1, -(b*c*log
(F) + I*e)*x) - I*gamma(-1, -(b*c*log(F) - I*e)*x))*sin(d))*e)*b*c*log(F)
- 1/2*(F^(a*c)*(I*conjugate(gamma(-2, -(b*c*log(F) + I*e)*x)) - I*conjugat
e(gamma(-2, -(b*c*log(F) - I*e)*x)) - I*gamma(-2, -(b*c*log(F) + I*e)*x) +
 I*gamma(-2, -(b*c*log(F) - I*e)*x))*cos(d) + F^(a*c)*(conjugate(gamma(-2,
 -(b*c*log(F) + I*e)*x)) + conjugate(gamma(-2, -(b*c*log(F) - I*e)*x)) ...
```

3.37.8 Giac [F]

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx$$
$$=\int \frac{(ex\cos(ex+d)+(bcx\log(F)-2)\sin(ex+d))F^{(bx+a)c}}{x^3}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*log(F))*sin(e*x+d))/x^3,
x, algorithm="giac")
```

output
```
integrate((e*x*cos(e*x + d) + (b*c*x*log(F) - 2)*sin(e*x + d))*F^((b*x + a
)*c)/x^3, x)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 27.84 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx=\frac{F^{c\,(a+b\,x)}\,\sin\left(d+e\,x\right)}{x^2}$$

input
```
int((F^(c*(a + b*x))*(sin(d + e*x)*(b*c*x*log(F) - 2) + e*x*cos(d + e*x)))
/x^3,x)
```

output
```
(F^(c*(a + b*x))*sin(d + e*x))/x^2
```

3.38 $\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx$

3.38.1 Optimal result

Integrand size = 20, antiderivative size = 63

$$\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx=-\frac{de^{a+bx}\cos(2c+2dx)}{b^2+4d^2}+\frac{be^{a+bx}\sin(2c+2dx)}{2\,(b^2+4d^2)}$$

output
```
-d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)+1/2*b*exp(b*x+a)*sin(2*d*x+2*c)/(
b^2+4*d^2)
```

3.38.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx=\frac{e^{a+bx}(-2d\cos(2(c+dx))+b\sin(2(c+dx)))}{2\,(b^2+4d^2)}$$

input
```
Integrate[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x],x]
```

output
```
(E^(a + b*x)*(-2*d*Cos[2*(c + d*x)] + b*Sin[2*(c + d*x)]))/(2*(b^2 + 4*d^2
))
```

3.38.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4972, 27, 4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sin(c+dx)\cos(c+dx)\,dx$$

$$\Big\downarrow \text{4972}$$

$$\int \frac{1}{2}e^{a+bx}\sin(2c+2dx)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int e^{a+bx}\sin(2c+2dx)dx$$

$$\Big\downarrow \text{4932}$$

$$\frac{1}{2}\left(\frac{be^{a+bx}\sin(2c+2dx)}{b^2+4d^2}-\frac{2de^{a+bx}\cos(2c+2dx)}{b^2+4d^2}\right)$$

input
```
Int[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x],x]
```

output
```
((-2*d*E^(a + b*x)*Cos[2*c + 2*d*x])/(b^2 + 4*d^2) + (b*E^(a + b*x)*Sin[2*
c + 2*d*x])/(b^2 + 4*d^2))/2
```

3.38.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

3.38.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

method	result	size
parallelrisch	$\frac{e^{xb+a}(b\sin(2dx+2c)-2d\cos(2dx+2c))}{2b^2+8d^2}$	45
risch	$-\frac{i e^{xb+a}(4id\cos(2dx+2c)-2ib\sin(2dx+2c))}{4(2id+b)(2id-b)}$	55
default	$-\frac{d\,e^{xb+a}\cos(2dx+2c)}{b^2+4d^2}+\frac{b\,e^{xb+a}\sin(2dx+2c)}{2b^2+8d^2}$	60
norman	$\frac{-\frac{d\,e^{xb+a}}{b^2+4d^2}+\frac{2b\,e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2+4d^2}-\frac{2b\,e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{b^2+4d^2}+\frac{6d\,e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{b^2+4d^2}-\frac{d\,e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{b^2+4d^2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$	160

input
```
int(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)*(b*sin(2*d*x+2*c)-2*d*cos(2*d*x+2*c))/(2*b^2+8*d^2)
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx = \frac{b\cos(dx+c)\,e^{(bx+a)}\sin(dx+c)-\left(2\,d\cos(dx+c)^2-d\right)e^{(bx+a)}}{b^2+4\,d^2}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x, algorithm="fricas")
```

output
```
(b*cos(d*x + c)*e^(b*x + a)*sin(d*x + c) - (2*d*cos(d*x + c)^2 - d)*e^(b*x
 + a))/(b^2 + 4*d^2)
```

3.38.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 325, normalized size of antiderivative = 5.16

$$\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx$$

$$= \begin{cases} xe^{a}\sin(c)\cos(c) & \text{for } b = 0 \\ \frac{ixe^{a}e^{-2idx}\sin^{2}(c+dx)}{4} + \frac{xe^{a}e^{-2idx}\sin(c+dx)\cos(c+dx)}{2} - \frac{ixe^{a}e^{-2idx}\cos^{2}(c+dx)}{4} + \frac{ie^{a}e^{-2idx}\sin(c+dx)\cos(c+dx)}{4d} & \text{for } b = - \\ -\frac{ixe^{a}e^{2idx}\sin^{2}(c+dx)}{4} + \frac{xe^{a}e^{2idx}\sin(c+dx)\cos(c+dx)}{2} + \frac{ixe^{a}e^{2idx}\cos^{2}(c+dx)}{4} - \frac{ie^{a}e^{2idx}\sin(c+dx)\cos(c+dx)}{4d} & \text{for } b = 2i \\ \frac{be^{a}e^{bx}\sin(c+dx)\cos(c+dx)}{b^{2}+4d^{2}} + \frac{de^{a}e^{bx}\sin^{2}(c+dx)}{b^{2}+4d^{2}} - \frac{de^{a}e^{bx}\cos^{2}(c+dx)}{b^{2}+4d^{2}} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x)
```

output
```
Piecewise((x*exp(a)*sin(c)*cos(c), Eq(b, 0) & Eq(d, 0)), (I*x*exp(a)*exp(-
2*I*d*x)*sin(c + d*x)**2/4 + x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)*cos(c + d
*x)/2 - I*x*exp(a)*exp(-2*I*d*x)*cos(c + d*x)**2/4 + I*exp(a)*exp(-2*I*d*x
)*sin(c + d*x)*cos(c + d*x)/(4*d), Eq(b, -2*I*d)), (-I*x*exp(a)*exp(2*I*d*
x)*sin(c + d*x)**2/4 + x*exp(a)*exp(2*I*d*x)*sin(c + d*x)*cos(c + d*x)/2 +
 I*x*exp(a)*exp(2*I*d*x)*cos(c + d*x)**2/4 - I*exp(a)*exp(2*I*d*x)*sin(c +
 d*x)*cos(c + d*x)/(4*d), Eq(b, 2*I*d)), (b*exp(a)*exp(b*x)*sin(c + d*x)*c
os(c + d*x)/(b**2 + 4*d**2) + d*exp(a)*exp(b*x)*sin(c + d*x)**2/(b**2 + 4*
d**2) - d*exp(a)*exp(b*x)*cos(c + d*x)**2/(b**2 + 4*d**2), True))
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx = -\frac{(2\,d\cos(2\,dx+2\,c) - b\sin(2\,dx+2\,c))e^{(bx+a)}}{2\,(b^{2}+4\,d^{2})}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*(2*d*cos(2*d*x + 2*c) - b*sin(2*d*x + 2*c))*e^(b*x + a)/(b^2 + 4*d^2)
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx = -\frac{1}{2}\left(\frac{2\,d\cos\left(2\,dx+2\,c\right)}{b^2+4\,d^2} - \frac{b\sin\left(2\,dx+2\,c\right)}{b^2+4\,d^2}\right)e^{(bx+a)}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x, algorithm="giac")
```

output
```
-1/2*(2*d*cos(2*d*x + 2*c)/(b^2 + 4*d^2) - b*sin(2*d*x + 2*c)/(b^2 + 4*d^2
))*e^(b*x + a)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 27.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx = -\frac{e^{a+b\,x}\,\left(2\,d\,\cos\left(2\,c+2\,d\,x\right)-b\,\sin\left(2\,c+2\,d\,x\right)\right)}{2\,\left(b^2+4\,d^2\right)}$$

input
```
int(cos(c + d*x)*exp(a + b*x)*sin(c + d*x),x)
```

output
```
-(exp(a + b*x)*(2*d*cos(2*c + 2*d*x) - b*sin(2*c + 2*d*x)))/(2*(b^2 + 4*d^
2))
```

3.39 $\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx$

3.39.1 Optimal result

Integrand size = 22, antiderivative size = 119

$$\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx=\frac{be^{a+bx}\cos(c+dx)}{4(b^2+d^2)}-\frac{be^{a+bx}\cos(3c+3dx)}{4(b^2+9d^2)}+\frac{de^{a+bx}\sin(c+dx)}{4(b^2+d^2)}-\frac{3de^{a+bx}\sin(3c+3dx)}{4(b^2+9d^2)}$$

output
```
1/4*b*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-1/4*b*exp(b*x+a)*cos(3*d*x+3*c)/(b^2
+9*d^2)+1/4*d*exp(b*x+a)*sin(d*x+c)/(b^2+d^2)-3/4*d*exp(b*x+a)*sin(3*d*x+3
*c)/(b^2+9*d^2)
```

3.39.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.62

$$\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx=\frac{1}{4}e^{a+bx}\left(\frac{b\cos(c+dx)+d\sin(c+dx)}{b^2+d^2}-\frac{b\cos(3(c+dx))+3d\sin(3(c+dx))}{b^2+9d^2}\right)$$

input
```
Integrate[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x]^2,x]
```

output
```
(E^(a + b*x)*((b*Cos[c + d*x] + d*Sin[c + d*x])/(b^2 + d^2) - (b*Cos[3*(c
+ d*x)] + 3*d*Sin[3*(c + d*x)])/(b^2 + 9*d^2)))/4
```

3.39.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4972, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sin^2(c+dx)\cos(c+dx)\,dx$$

$$\Big\downarrow \text{4972}$$

$$\int \left(\frac{1}{4}e^{a+bx}\cos(c+dx)-\frac{1}{4}e^{a+bx}\cos(3c+3dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{de^{a+bx}\sin(c+dx)}{4(b^2+d^2)}-\frac{3de^{a+bx}\sin(3c+3dx)}{4(b^2+9d^2)}+\frac{be^{a+bx}\cos(c+dx)}{4(b^2+d^2)}-\frac{be^{a+bx}\cos(3c+3dx)}{4(b^2+9d^2)}$$

input
```
Int[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x]^2,x]
```

output
```
(b*E^(a + b*x)*Cos[c + d*x])/(4*(b^2 + d^2)) - (b*E^(a + b*x)*Cos[3*c + 3*
d*x])/(4*(b^2 + 9*d^2)) + (d*E^(a + b*x)*Sin[c + d*x])/(4*(b^2 + d^2)) - (
3*d*E^(a + b*x)*Sin[3*c + 3*d*x])/(4*(b^2 + 9*d^2))
```

3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

3.39.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

method	result
parallelrisch	$-\frac{3\,\mathrm{e}^{x b+a}\left(\frac{\left(b^{3}+b\,d^{2}\right)\cos\left(3dx+3c\right)}{3}+\left(b^{2}d+d^{3}\right)\sin\left(3dx+3c\right)-\frac{\left(b^{2}+9d^{2}\right)\left(\cos\left(dx+c\right)b+d\sin\left(dx+c\right)\right)}{3}\right)}{4\left(b^{4}+10b^{2}d^{2}+9d^{4}\right)}$
default	$\frac{b\,\mathrm{e}^{x b+a}\cos\left(dx+c\right)}{4b^{2}+4d^{2}}-\frac{b\,\mathrm{e}^{x b+a}\cos\left(3dx+3c\right)}{4\left(b^{2}+9d^{2}\right)}+\frac{d\,\mathrm{e}^{x b+a}\sin\left(dx+c\right)}{4b^{2}+4d^{2}}-\frac{3d\,\mathrm{e}^{x b+a}\sin\left(3dx+3c\right)}{4\left(b^{2}+9d^{2}\right)}$
risch	$-\frac{\mathrm{e}^{x b+a}\left(\left(-2b^{3}-18b\,d^{2}\right)\cos\left(dx+c\right)-2d\left(b^{2}+9d^{2}\right)\sin\left(dx+c\right)+\left(2b^{3}+2b\,d^{2}\right)\cos\left(3dx+3c\right)+6d\left(b^{2}+d^{2}\right)\sin\left(3dx+3c\right)\right)}{8\left(3id+b\right)\left(id+b\right)\left(id-b\right)\left(3id-b\right)}$
norman	$\frac{\frac{2b\,d^{2}\mathrm{e}^{x b+a}}{b^{4}+10b^{2}d^{2}+9d^{4}}-\frac{2b\,d^{2}\mathrm{e}^{x b+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{b^{4}+10b^{2}d^{2}+9d^{4}}+\frac{2b\left(2b^{2}+3d^{2}\right)\mathrm{e}^{x b+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{b^{4}+10b^{2}d^{2}+9d^{4}}-\frac{2b\left(2b^{2}+3d^{2}\right)\mathrm{e}^{x b+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{b^{4}+10b^{2}d^{2}+9d^{4}}-\frac{4b^{2}d\,\mathrm{e}^{x b+a}\tan\left(\frac{}{}\right.}{b^{4}+10b^{2}d^{2}+9}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}$

input
```
int(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/4*exp(b*x+a)*(1/3*(b^3+b*d^2)*cos(3*d*x+3*c)+(b^2*d+d^3)*sin(3*d*x+3*c)
-1/3*(b^2+9*d^2)*(cos(d*x+c)*b+d*sin(d*x+c)))/(b^4+10*b^2*d^2+9*d^4)
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx$$
$$=\frac{\left(b^2d+3\,d^3-3\left(b^2d+d^3\right)\cos\left(dx+c\right)^2\right)e^{(bx+a)}\sin\left(dx+c\right)-\left(\left(b^3+bd^2\right)\cos\left(dx+c\right)^3-\left(b^3+3\,bd^2\right)\cos\left(\right.\right.}{b^4+10\,b^2d^2+9\,d^4}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
((b^2*d + 3*d^3 - 3*(b^2*d + d^3)*cos(d*x + c)^2)*e^(b*x + a)*sin(d*x + c)
 - ((b^3 + b*d^2)*cos(d*x + c)^3 - (b^3 + 3*b*d^2)*cos(d*x + c))*e^(b*x +
a))/(b^4 + 10*b^2*d^2 + 9*d^4)
```

3.39.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.51 (sec) , antiderivative size = 1030, normalized size of antiderivative = 8.66

$$\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)**2,x)
```

output
```
Piecewise((x*exp(a)*sin(c)**2*cos(c), Eq(b, 0) & Eq(d, 0)), (I*x*exp(a)*ex
p(-3*I*d*x)*sin(c + d*x)**3/8 + 3*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**2*c
os(c + d*x)/8 - 3*I*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8
- x*exp(a)*exp(-3*I*d*x)*cos(c + d*x)**3/8 + exp(a)*exp(-3*I*d*x)*sin(c +
d*x)**3/(24*d) - exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d) +
 I*exp(a)*exp(-3*I*d*x)*cos(c + d*x)**3/(8*d), Eq(b, -3*I*d)), (I*x*exp(a)
*exp(-I*d*x)*sin(c + d*x)**3/8 + x*exp(a)*exp(-I*d*x)*sin(c + d*x)**2*cos(
c + d*x)/8 + I*x*exp(a)*exp(-I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + x*exp
(a)*exp(-I*d*x)*cos(c + d*x)**3/8 + exp(a)*exp(-I*d*x)*sin(c + d*x)**3/(8*
d) - exp(a)*exp(-I*d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d) + I*exp(a)*exp(
-I*d*x)*cos(c + d*x)**3/(8*d), Eq(b, -I*d)), (-I*x*exp(a)*exp(I*d*x)*sin(c
 + d*x)**3/8 + x*exp(a)*exp(I*d*x)*sin(c + d*x)**2*cos(c + d*x)/8 - I*x*ex
p(a)*exp(I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + x*exp(a)*exp(I*d*x)*cos(c
 + d*x)**3/8 + exp(a)*exp(I*d*x)*sin(c + d*x)**3/(8*d) - exp(a)*exp(I*d*x)
*sin(c + d*x)*cos(c + d*x)**2/(4*d) - I*exp(a)*exp(I*d*x)*cos(c + d*x)**3/
(8*d), Eq(b, I*d)), (-I*x*exp(a)*exp(3*I*d*x)*sin(c + d*x)**3/8 + 3*x*exp(
a)*exp(3*I*d*x)*sin(c + d*x)**2*cos(c + d*x)/8 + 3*I*x*exp(a)*exp(3*I*d*x)
*sin(c + d*x)*cos(c + d*x)**2/8 - x*exp(a)*exp(3*I*d*x)*cos(c + d*x)**3/8
+ exp(a)*exp(3*I*d*x)*sin(c + d*x)**3/(24*d) - exp(a)*exp(3*I*d*x)*sin(c +
 d*x)*cos(c + d*x)**2/(4*d) - I*exp(a)*exp(3*I*d*x)*cos(c + d*x)**3/(8*...
```

3.39.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 538 vs. 2(107) = 214.

Time = 0.24 (sec) , antiderivative size = 538, normalized size of antiderivative = 4.52

$$\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx =$$

$$-\frac{(b^3\cos(3c)e^a + bd^2\cos(3c)e^a + 3b^2de^a\sin(3c) + 3d^3e^a\sin(3c))\cos(3dx)e^{(bx)} + (b^3\cos(3c)e^a + ba}{}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/8*((b^3*cos(3*c)*e^a + b*d^2*cos(3*c)*e^a + 3*b^2*d*e^a*sin(3*c) + 3*d^
3*e^a*sin(3*c))*cos(3*d*x)*e^(b*x) + (b^3*cos(3*c)*e^a + b*d^2*cos(3*c)*e^
a - 3*b^2*d*e^a*sin(3*c) - 3*d^3*e^a*sin(3*c))*cos(3*d*x + 6*c)*e^(b*x) -
(b^3*cos(3*c)*e^a + 9*b*d^2*cos(3*c)*e^a - b^2*d*e^a*sin(3*c) - 9*d^3*e^a*
sin(3*c))*cos(d*x + 4*c)*e^(b*x) - (b^3*cos(3*c)*e^a + 9*b*d^2*cos(3*c)*e^
a + b^2*d*e^a*sin(3*c) + 9*d^3*e^a*sin(3*c))*cos(d*x - 2*c)*e^(b*x) + (3*b
^2*d*cos(3*c)*e^a + 3*d^3*cos(3*c)*e^a - b^3*e^a*sin(3*c) - b*d^2*e^a*sin(
3*c))*e^(b*x)*sin(3*d*x) + (3*b^2*d*cos(3*c)*e^a + 3*d^3*cos(3*c)*e^a + b^
3*e^a*sin(3*c) + b*d^2*e^a*sin(3*c))*e^(b*x)*sin(3*d*x + 6*c) - (b^2*d*cos
(3*c)*e^a + 9*d^3*cos(3*c)*e^a + b^3*e^a*sin(3*c) + 9*b*d^2*e^a*sin(3*c))*
e^(b*x)*sin(d*x + 4*c) - (b^2*d*cos(3*c)*e^a + 9*d^3*cos(3*c)*e^a - b^3*e^
a*sin(3*c) - 9*b*d^2*e^a*sin(3*c))*e^(b*x)*sin(d*x - 2*c))/(b^4*cos(3*c)^2
 + b^4*sin(3*c)^2 + 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 + 10*(b^2*cos(3*c)^2 +
 b^2*sin(3*c)^2)*d^2)
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx = -\frac{1}{4}\left(\frac{b\cos\left(3\,dx+3\,c\right)}{b^2+9\,d^2}+\frac{3\,d\sin\left(3\,dx+3\,c\right)}{b^2+9\,d^2}\right)e^{(bx+a)} + \frac{1}{4}\left(\frac{b\cos\left(dx+c\right)}{b^2+d^2}+\frac{d\sin\left(dx+c\right)}{b^2+d^2}\right)e^{(bx+a)}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/4*(b*cos(3*d*x + 3*c)/(b^2 + 9*d^2) + 3*d*sin(3*d*x + 3*c)/(b^2 + 9*d^2
))*e^(b*x + a) + 1/4*(b*cos(d*x + c)/(b^2 + d^2) + d*sin(d*x + c)/(b^2 + d
^2))*e^(b*x + a)
```

3.39.9 Mupad [B] (verification not implemented)

Time = 27.34 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.39

$$\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx = \frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(d\,x\right)-\sin\left(d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(c\right)-\sin\left(c\right)\,1\mathrm{i}\right)}{8\,\left(b-d\,1\mathrm{i}\right)} - \frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(3\,d\,x\right)+\sin\left(3\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(3\,c\right)+\sin\left(3\,c\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{8\,\left(-3\,d+b\,1\mathrm{i}\right)} + \frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(d\,x\right)+\sin\left(d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(c\right)+\sin\left(c\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{8\,\left(-d+b\,1\mathrm{i}\right)} - \frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(3\,d\,x\right)-\sin\left(3\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(3\,c\right)-\sin\left(3\,c\right)\,1\mathrm{i}\right)}{8\,\left(b-d\,3\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)*exp(a + b*x)*sin(c + d*x)^2,x)
```

output
```
(exp(a + b*x)*(cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i))/(8*(b - d*1i)
) - (exp(a + b*x)*(cos(3*d*x) + sin(3*d*x)*1i)*(cos(3*c) + sin(3*c)*1i)*1i
)/(8*(b*1i - 3*d)) + (exp(a + b*x)*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(
c)*1i)*1i)/(8*(b*1i - d)) - (exp(a + b*x)*(cos(3*d*x) - sin(3*d*x)*1i)*(co
s(3*c) - sin(3*c)*1i))/(8*(b - d*3i))
```

3.40 $\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx$

3.40.1 Optimal result

Integrand size = 22, antiderivative size = 129

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx=-\frac{de^{a+bx}\cos(2c+2dx)}{2(b^2+4d^2)}+\frac{de^{a+bx}\cos(4c+4dx)}{2(b^2+16d^2)}+\frac{be^{a+bx}\sin(2c+2dx)}{4(b^2+4d^2)}-\frac{be^{a+bx}\sin(4c+4dx)}{8(b^2+16d^2)}$$

output
```
-1/2*d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)+1/2*d*exp(b*x+a)*cos(4*d*x+4*
c)/(b^2+16*d^2)+1/4*b*exp(b*x+a)*sin(2*d*x+2*c)/(b^2+4*d^2)-1/8*b*exp(b*x+
a)*sin(4*d*x+4*c)/(b^2+16*d^2)
```

3.40.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.64

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx=\frac{1}{8}e^{a+bx}\left(\frac{2(-2d\cos(2(c+dx))+b\sin(2(c+dx)))}{b^2+4d^2}+\frac{4d\cos(4(c+dx))-b\sin(4(c+dx))}{b^2+16d^2}\right)$$

input
```
Integrate[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x]^3,x]
```

output
```
(E^(a + b*x)*((2*(-2*d*Cos[2*(c + d*x)] + b*Sin[2*(c + d*x)]))/(b^2 + 4*d^
2) + (4*d*Cos[4*(c + d*x)] - b*Sin[4*(c + d*x)])/(b^2 + 16*d^2)))/8
```

3.40.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4972, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sin^3(c+dx)\cos(c+dx)\,dx$$

$$\Big\downarrow \text{4972}$$

$$\int \left(\frac{1}{4}e^{a+bx}\sin(2c+2dx)-\frac{1}{8}e^{a+bx}\sin(4c+4dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{be^{a+bx}\sin(2c+2dx)}{4\left(b^2+4d^2\right)}-\frac{be^{a+bx}\sin(4c+4dx)}{8\left(b^2+16d^2\right)}-\frac{de^{a+bx}\cos(2c+2dx)}{2\left(b^2+4d^2\right)}+\frac{de^{a+bx}\cos(4c+4dx)}{2\left(b^2+16d^2\right)}$$

input
```
Int[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x]^3,x]
```

output
```
-1/2*(d*E^(a + b*x)*Cos[2*c + 2*d*x])/(b^2 + 4*d^2) + (d*E^(a + b*x)*Cos[4
*c + 4*d*x])/(2*(b^2 + 16*d^2)) + (b*E^(a + b*x)*Sin[2*c + 2*d*x])/(4*(b^2
 + 4*d^2)) - (b*E^(a + b*x)*Sin[4*c + 4*d*x])/(8*(b^2 + 16*d^2))
```

3.40.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

3.40.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

method	result
parallelrisch	$-\frac{\left(\left(b^{3}+4 b\,d^{2}\right) \sin \left(4 d x+4 c\right)+\left(-4 b^{2} d-16 d^{3}\right) \cos \left(4 d x+4 c\right)-2 \left(b^{2}+16 d^{2}\right) \left(b \sin \left(2 d x+2 c\right)-2 d \cos \left(2 d x+2 c\right)\right)\right) {\mathrm e}^{x b+a}}{8 b^{4}+160 b^{2} d^{2}+512 d^{4}}$
default	$-\frac{d\,{\mathrm e}^{x b+a} \cos \left(2 d x+2 c\right)}{2 \left(b^{2}+4 d^{2}\right)}+\frac{d\,{\mathrm e}^{x b+a} \cos \left(4 d x+4 c\right)}{2 b^{2}+32 d^{2}}+\frac{b\,{\mathrm e}^{x b+a} \sin \left(2 d x+2 c\right)}{4 b^{2}+16 d^{2}}-\frac{b\,{\mathrm e}^{x b+a} \sin \left(4 d x+4 c\right)}{8 \left(b^{2}+16 d^{2}\right)}$
risch	$\frac{i {\mathrm e}^{x b+a} \left(-8 i d \left(b^{2}+4 d^{2}\right) \cos \left(4 d x+4 c\right)-i \left(-2 b^{3}-8 b\,d^{2}\right) \sin \left(4 d x+4 c\right)+8 i d \left(b^{2}+16 d^{2}\right) \cos \left(2 d x+2 c\right)-i \left(4 b^{3}+64 b\,d^{2}\right) \sin \left(2 d x+2 c\right)\right)}{16 \left(4 i d+b\right) \left(2 i d+b\right) \left(2 i d-b\right) \left(4 i d-b\right)}$
norman	$\frac{-\frac{6 d^{3} {\mathrm e}^{x b+a}}{b^{4}+20 b^{2} d^{2}+64 d^{4}}-\frac{6 d^{3} {\mathrm e}^{x b+a} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{8}}{b^{4}+20 b^{2} d^{2}+64 d^{4}}+\frac{12 b\,d^{2} {\mathrm e}^{x b+a} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{b^{4}+20 b^{2} d^{2}+64 d^{4}}-\frac{12 b\,d^{2} {\mathrm e}^{x b+a} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{b^{4}+20 b^{2} d^{2}+64 d^{4}}+\frac{4 b \left(2 b^{2}+11 d^{2}\right) {\mathrm e}^{x b+a} \tan \left(\frac{d x}{2}+\frac{}{}\right.}{b^{4}+20 b^{2} d^{2}+64 d^{4}}}{\left(1\right.}$

input
```
int(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-((b^3+4*b*d^2)*sin(4*d*x+4*c)+(-4*b^2*d-16*d^3)*cos(4*d*x+4*c)-2*(b^2+16*
d^2)*(b*sin(2*d*x+2*c)-2*d*cos(2*d*x+2*c)))*exp(b*x+a)/(8*b^4+160*b^2*d^2+
512*d^4)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx =$$
$$-\frac{\left((b^3+4\,bd^2)\cos\left(dx+c\right)^3-(b^3+10\,bd^2)\cos\left(dx+c\right)\right)e^{(bx+a)}\sin\left(dx+c\right)-\left(4\left(b^2d+4\,d^3\right)\cos\left(dx+c\right.\right.}{b^4+20\,b^2d^2+64\,d^4}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
-(((b^3 + 4*b*d^2)*cos(d*x + c)^3 - (b^3 + 10*b*d^2)*cos(d*x + c))*e^(b*x
+ a)*sin(d*x + c) - (4*(b^2*d + 4*d^3)*cos(d*x + c)^4 + b^2*d + 10*d^3 - (
5*b^2*d + 32*d^3)*cos(d*x + c)^2)*e^(b*x + a))/(b^4 + 20*b^2*d^2 + 64*d^4)
```

3.40.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.29 (sec) , antiderivative size = 1353, normalized size of antiderivative = 10.49

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)**3,x)
```

output
```
Piecewise((x*exp(a)*sin(c)**3*cos(c), Eq(b, 0) & Eq(d, 0)), (I*x*exp(a)*ex
p(-4*I*d*x)*sin(c + d*x)**4/16 + x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**3*co
s(c + d*x)/4 - 3*I*x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**2/
8 - x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 + I*x*exp(a)*exp
(-4*I*d*x)*cos(c + d*x)**4/16 - exp(a)*exp(-4*I*d*x)*sin(c + d*x)**4/(24*d
) + 11*I*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/(48*d) + 5*I*ex
p(a)*exp(-4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/(48*d) + exp(a)*exp(-4*I*d
*x)*cos(c + d*x)**4/(24*d), Eq(b, -4*I*d)), (I*x*exp(a)*exp(-2*I*d*x)*sin(
c + d*x)**4/8 + x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/4 + x*
exp(a)*exp(-2*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 - I*x*exp(a)*exp(-2*I*
d*x)*cos(c + d*x)**4/8 + exp(a)*exp(-2*I*d*x)*sin(c + d*x)**4/(16*d) - exp
(a)*exp(-2*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**2/(4*d) + I*exp(a)*exp(-2*
I*d*x)*sin(c + d*x)*cos(c + d*x)**3/(6*d) + exp(a)*exp(-2*I*d*x)*cos(c + d
*x)**4/(48*d), Eq(b, -2*I*d)), (-I*x*exp(a)*exp(2*I*d*x)*sin(c + d*x)**4/8
 + x*exp(a)*exp(2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/4 + x*exp(a)*exp(2*I
*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 + I*x*exp(a)*exp(2*I*d*x)*cos(c + d*x
)**4/8 + exp(a)*exp(2*I*d*x)*sin(c + d*x)**4/(16*d) - exp(a)*exp(2*I*d*x)*
sin(c + d*x)**2*cos(c + d*x)**2/(4*d) - I*exp(a)*exp(2*I*d*x)*sin(c + d*x)
*cos(c + d*x)**3/(6*d) + exp(a)*exp(2*I*d*x)*cos(c + d*x)**4/(48*d), Eq(b,
 2*I*d)), (-I*x*exp(a)*exp(4*I*d*x)*sin(c + d*x)**4/16 + x*exp(a)*exp(4...
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(117) = 234.

Time = 0.22 (sec) , antiderivative size = 550, normalized size of antiderivative = 4.26

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx$$
$$= \frac{\left(4\,b^2d\cos\left(4\,c\right)e^a + 16\,d^3\cos\left(4\,c\right)e^a - b^3e^a\sin\left(4\,c\right) - 4\,bd^2e^a\sin\left(4\,c\right)\right)\cos\left(4\,dx\right)e^{(bx)} + \left(4\,b^2d\cos\left(4\,c\right)e^a + \right.}{}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/16*((4*b^2*d*cos(4*c)*e^a + 16*d^3*cos(4*c)*e^a - b^3*e^a*sin(4*c) - 4*b
*d^2*e^a*sin(4*c))*cos(4*d*x)*e^(b*x) + (4*b^2*d*cos(4*c)*e^a + 16*d^3*cos
(4*c)*e^a + b^3*e^a*sin(4*c) + 4*b*d^2*e^a*sin(4*c))*cos(4*d*x + 8*c)*e^(b
*x) - 2*(2*b^2*d*cos(4*c)*e^a + 32*d^3*cos(4*c)*e^a + b^3*e^a*sin(4*c) + 1
6*b*d^2*e^a*sin(4*c))*cos(2*d*x + 6*c)*e^(b*x) - 2*(2*b^2*d*cos(4*c)*e^a +
 32*d^3*cos(4*c)*e^a - b^3*e^a*sin(4*c) - 16*b*d^2*e^a*sin(4*c))*cos(2*d*x
 - 2*c)*e^(b*x) - (b^3*cos(4*c)*e^a + 4*b*d^2*cos(4*c)*e^a + 4*b^2*d*e^a*s
in(4*c) + 16*d^3*e^a*sin(4*c))*e^(b*x)*sin(4*d*x) - (b^3*cos(4*c)*e^a + 4*
b*d^2*cos(4*c)*e^a - 4*b^2*d*e^a*sin(4*c) - 16*d^3*e^a*sin(4*c))*e^(b*x)*s
in(4*d*x + 8*c) + 2*(b^3*cos(4*c)*e^a + 16*b*d^2*cos(4*c)*e^a - 2*b^2*d*e^
a*sin(4*c) - 32*d^3*e^a*sin(4*c))*e^(b*x)*sin(2*d*x + 6*c) + 2*(b^3*cos(4*
c)*e^a + 16*b*d^2*cos(4*c)*e^a + 2*b^2*d*e^a*sin(4*c) + 32*d^3*e^a*sin(4*c
))*e^(b*x)*sin(2*d*x - 2*c))/(b^4*cos(4*c)^2 + b^4*sin(4*c)^2 + 64*(cos(4*
c)^2 + sin(4*c)^2)*d^4 + 20*(b^2*cos(4*c)^2 + b^2*sin(4*c)^2)*d^2)
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx = \frac{1}{8}\left(\frac{4\,d\cos\left(4\,dx+4\,c\right)}{b^2+16\,d^2}-\frac{b\sin\left(4\,dx+4\,c\right)}{b^2+16\,d^2}\right)e^{(bx+a)} - \frac{1}{4}\left(\frac{2\,d\cos\left(2\,dx+2\,c\right)}{b^2+4\,d^2}-\frac{b\sin\left(2\,dx+2\,c\right)}{b^2+4\,d^2}\right)e^{(bx+a)}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*(4*d*cos(4*d*x + 4*c)/(b^2 + 16*d^2) - b*sin(4*d*x + 4*c)/(b^2 + 16*d^
2))*e^(b*x + a) - 1/4*(2*d*cos(2*d*x + 2*c)/(b^2 + 4*d^2) - b*sin(2*d*x +
2*c)/(b^2 + 4*d^2))*e^(b*x + a)
```

3.40.9 Mupad [B] (verification not implemented)

Time = 28.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.38

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx$$
$$= -\frac{e^{a+b\,x}\,(\cos(2\,d\,x)-\sin(2\,d\,x)\;1\mathrm{i})\;(\cos(2\,c)-\sin(2\,c)\;1\mathrm{i})}{8\;(2\,d+b\,1\mathrm{i})}$$
$$+\frac{e^{a+b\,x}\,(\cos(4\,d\,x)-\sin(4\,d\,x)\;1\mathrm{i})\;(\cos(4\,c)-\sin(4\,c)\;1\mathrm{i})}{16\;(4\,d+b\,1\mathrm{i})}$$
$$-\frac{e^{a+b\,x}\,(\cos(2\,d\,x)+\sin(2\,d\,x)\;1\mathrm{i})\;(\cos(2\,c)+\sin(2\,c)\;1\mathrm{i})\;1\mathrm{i}}{8\;(b+d\,2\mathrm{i})}$$
$$+\frac{e^{a+b\,x}\,(\cos(4\,d\,x)+\sin(4\,d\,x)\;1\mathrm{i})\;(\cos(4\,c)+\sin(4\,c)\;1\mathrm{i})\;1\mathrm{i}}{16\;(b+d\,4\mathrm{i})}$$

input
```
int(cos(c + d*x)*exp(a + b*x)*sin(c + d*x)^3,x)
```

output
```
(exp(a + b*x)*(cos(4*d*x) - sin(4*d*x)*1i)*(cos(4*c) - sin(4*c)*1i))/(16*(
b*1i + 4*d)) - (exp(a + b*x)*(cos(2*d*x) - sin(2*d*x)*1i)*(cos(2*c) - sin(
2*c)*1i))/(8*(b*1i + 2*d)) - (exp(a + b*x)*(cos(2*d*x) + sin(2*d*x)*1i)*(c
os(2*c) + sin(2*c)*1i)*1i)/(8*(b + d*2i)) + (exp(a + b*x)*(cos(4*d*x) + si
n(4*d*x)*1i)*(cos(4*c) + sin(4*c)*1i)*1i)/(16*(b + d*4i))
```

3.41 $\int e^{a+bx}\cos^2(c+dx)\sin(c+dx)\,dx$

3.41.1 Optimal result

Integrand size = 22, antiderivative size = 119

$$\int e^{a+bx}\cos^2(c+dx)\sin(c+dx)\,dx=-\frac{de^{a+bx}\cos(c+dx)}{4(b^2+d^2)}-\frac{3de^{a+bx}\cos(3c+3dx)}{4(b^2+9d^2)}+\frac{be^{a+bx}\sin(c+dx)}{4(b^2+d^2)}+\frac{be^{a+bx}\sin(3c+3dx)}{4(b^2+9d^2)}$$

output
```
-1/4*d*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-3/4*d*exp(b*x+a)*cos(3*d*x+3*c)/(b^
2+9*d^2)+1/4*b*exp(b*x+a)*sin(d*x+c)/(b^2+d^2)+1/4*b*exp(b*x+a)*sin(3*d*x+
3*c)/(b^2+9*d^2)
```

3.41.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.62

$$\int e^{a+bx}\cos^2(c+dx)\sin(c+dx)\,dx=\frac{1}{4}e^{a+bx}\left(\frac{-d\cos(c+dx)+b\sin(c+dx)}{b^2+d^2}+\frac{-3d\cos(3(c+dx))+b\sin(3(c+dx))}{b^2+9d^2}\right)$$

input
```
Integrate[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x],x]
```

output
```
(E^(a + b*x)*((-(d*Cos[c + d*x]) + b*Sin[c + d*x])/(b^2 + d^2) + (-3*d*Cos
[3*(c + d*x)] + b*Sin[3*(c + d*x)])/(b^2 + 9*d^2)))/4
```

3.41.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4972, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sin(c+dx)\cos^2(c+dx)\,dx$$

$$\Big\downarrow \ 4972$$

$$\int \left(\frac{1}{4}e^{a+bx}\sin(c+dx)+\frac{1}{4}e^{a+bx}\sin(3c+3dx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{be^{a+bx}\sin(c+dx)}{4(b^2+d^2)}+\frac{be^{a+bx}\sin(3c+3dx)}{4(b^2+9d^2)}-\frac{de^{a+bx}\cos(c+dx)}{4(b^2+d^2)}-\frac{3de^{a+bx}\cos(3c+3dx)}{4(b^2+9d^2)}$$

input
```
Int[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x],x]
```

output
```
-1/4*(d*E^(a + b*x)*Cos[c + d*x])/(b^2 + d^2) - (3*d*E^(a + b*x)*Cos[3*c +
 3*d*x])/(4*(b^2 + 9*d^2)) + (b*E^(a + b*x)*Sin[c + d*x])/(4*(b^2 + d^2))
+ (b*E^(a + b*x)*Sin[3*c + 3*d*x])/(4*(b^2 + 9*d^2))
```

3.41.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

3.41.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{\left(\left(-3 b^{2} d -3 d^{3}\right) \cos \left(3 d x +3 c \right)+\left(b^{3}+b \,d^{2}\right) \sin \left(3 d x +3 c \right)+\left(b^{2}+9 d^{2}\right) \left(b \sin \left(d x +c \right)-d \cos \left(d x +c \right)\right)\right) {\mathrm e}^{x b +a}}{4 b^{4}+40 b^{2} d^{2}+36 d^{4}}$
default	$-\frac{d \,{\mathrm e}^{x b +a} \cos \left(d x +c \right)}{4 \left(b^{2}+d^{2}\right)}-\frac{3 d \,{\mathrm e}^{x b +a} \cos \left(3 d x +3 c \right)}{4 \left(b^{2}+9 d^{2}\right)}+\frac{b \,{\mathrm e}^{x b +a} \sin \left(d x +c \right)}{4 b^{2}+4 d^{2}}+\frac{b \,{\mathrm e}^{x b +a} \sin \left(3 d x +3 c \right)}{4 b^{2}+36 d^{2}}$
risch	$-\frac{i {\mathrm e}^{x b +a} \left(-2 i d \left(b^{2}+9 d^{2}\right) \cos \left(d x +c \right)+i \left(2 b^{3}+18 b \,d^{2}\right) \sin \left(d x +c \right)-6 i d \left(b^{2}+d^{2}\right) \cos \left(3 d x +3 c \right)-i \left(-2 b^{3}-2 b \,d^{2}\right) \sin \left(3 d x +3 c \right)\right)}{8 \left(3 i d +b \right) \left(i d +b \right) \left(i d -b \right) \left(3 i d -b \right)}$
norman	$\frac{\frac{d \left(b^{2}+3 d^{2}\right) {\mathrm e}^{x b +a} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{6}}{b^{4}+10 b^{2} d^{2}+9 d^{4}}+\frac{d \left(11 b^{2}+9 d^{2}\right) {\mathrm e}^{x b +a} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{b^{4}+10 b^{2} d^{2}+9 d^{4}}-\frac{d \left(b^{2}+3 d^{2}\right) {\mathrm e}^{x b +a}}{b^{4}+10 b^{2} d^{2}+9 d^{4}}-\frac{4 b \left(b^{2}-d^{2}\right) {\mathrm e}^{x b +a} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{b^{4}+10 b^{2} d^{2}+9 d^{4}}+\frac{2 b \left(b^{2}+3 d^{2}\right) \cdots}{b^{4}+\cdots}}{\left(1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)^{3}}$

input
```
int(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
((-3*b^2*d-3*d^3)*cos(3*d*x+3*c)+(b^3+b*d^2)*sin(3*d*x+3*c)+(b^2+9*d^2)*(b
*sin(d*x+c)-d*cos(d*x+c)))*exp(b*x+a)/(4*b^4+40*b^2*d^2+36*d^4)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\cos^2(c+dx)\sin(c+dx)\,dx = \frac{\left(2\,bd^{2}+\left(b^{3}+bd^{2}\right)\cos\left(dx+c\right)^{2}\right)e^{\left(bx+a\right)}\sin\left(dx+c\right)+\left(2\,b^{2}d\cos\left(dx+c\right)-3\left(b^{2}d+d^{3}\right)\cos\left(dx+c\right)^{3}\right)e^{(bx\cdots}}{b^{4}+10\,b^{2}d^{2}+9\,d^{4}}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c),x, algorithm="fricas")
```

output
```
((2*b*d^2 + (b^3 + b*d^2)*cos(d*x + c)^2)*e^(b*x + a)*sin(d*x + c) + (2*b^
2*d*cos(d*x + c) - 3*(b^2*d + d^3)*cos(d*x + c)^3)*e^(b*x + a))/(b^4 + 10*
b^2*d^2 + 9*d^4)
```

3.41.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.49 (sec) , antiderivative size = 1040, normalized size of antiderivative = 8.74

$$\int e^{a+bx}\cos^2(c+dx)\sin(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)**2*sin(d*x+c),x)
```

output
```
Piecewise((x*exp(a)*sin(c)*cos(c)**2, Eq(b, 0) & Eq(d, 0)), (-x*exp(a)*exp
(-3*I*d*x)*sin(c + d*x)**3/8 + 3*I*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**2*
cos(c + d*x)/8 + 3*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 -
 I*x*exp(a)*exp(-3*I*d*x)*cos(c + d*x)**3/8 + I*exp(a)*exp(-3*I*d*x)*sin(c
 + d*x)**3/(24*d) + I*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)**2/(4
*d) + exp(a)*exp(-3*I*d*x)*cos(c + d*x)**3/(24*d), Eq(b, -3*I*d)), (x*exp(
a)*exp(-I*d*x)*sin(c + d*x)**3/8 - I*x*exp(a)*exp(-I*d*x)*sin(c + d*x)**2*
cos(c + d*x)/8 + x*exp(a)*exp(-I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - I*x
*exp(a)*exp(-I*d*x)*cos(c + d*x)**3/8 - I*exp(a)*exp(-I*d*x)*sin(c + d*x)*
*3/(8*d) - I*exp(a)*exp(-I*d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d) - 3*exp
(a)*exp(-I*d*x)*cos(c + d*x)**3/(8*d), Eq(b, -I*d)), (x*exp(a)*exp(I*d*x)*
sin(c + d*x)**3/8 + I*x*exp(a)*exp(I*d*x)*sin(c + d*x)**2*cos(c + d*x)/8 +
 x*exp(a)*exp(I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + I*x*exp(a)*exp(I*d*x
)*cos(c + d*x)**3/8 + I*exp(a)*exp(I*d*x)*sin(c + d*x)**3/(8*d) + I*exp(a)
*exp(I*d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d) - 3*exp(a)*exp(I*d*x)*cos(c
 + d*x)**3/(8*d), Eq(b, I*d)), (-x*exp(a)*exp(3*I*d*x)*sin(c + d*x)**3/8 -
 3*I*x*exp(a)*exp(3*I*d*x)*sin(c + d*x)**2*cos(c + d*x)/8 + 3*x*exp(a)*exp
(3*I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + I*x*exp(a)*exp(3*I*d*x)*cos(c +
 d*x)**3/8 - I*exp(a)*exp(3*I*d*x)*sin(c + d*x)**3/(24*d) - I*exp(a)*exp(3
*I*d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d) + exp(a)*exp(3*I*d*x)*cos(c ...
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 538 vs. 2(107) = 214.

Time = 0.23 (sec) , antiderivative size = 538, normalized size of antiderivative = 4.52

$$\int e^{a+bx}\cos^2(c+dx)\sin(c+dx)\,dx = -\frac{(3\,b^2d\cos(3\,c)\,e^a + 3\,d^3\cos(3\,c)\,e^a - b^3e^a\sin(3\,c) - bd^2e^a\sin(3\,c))\cos(3\,dx)\,e^{(bx)} + (3\,b^2d\cos(3\,c)\,e^a +}{}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c),x, algorithm="maxima")
```

output
```
-1/8*((3*b^2*d*cos(3*c)*e^a + 3*d^3*cos(3*c)*e^a - b^3*e^a*sin(3*c) - b*d^
2*e^a*sin(3*c))*cos(3*d*x)*e^(b*x) + (3*b^2*d*cos(3*c)*e^a + 3*d^3*cos(3*c
)*e^a + b^3*e^a*sin(3*c) + b*d^2*e^a*sin(3*c))*cos(3*d*x + 6*c)*e^(b*x) +
(b^2*d*cos(3*c)*e^a + 9*d^3*cos(3*c)*e^a + b^3*e^a*sin(3*c) + 9*b*d^2*e^a*
sin(3*c))*cos(d*x + 4*c)*e^(b*x) + (b^2*d*cos(3*c)*e^a + 9*d^3*cos(3*c)*e^
a - b^3*e^a*sin(3*c) - 9*b*d^2*e^a*sin(3*c))*cos(d*x - 2*c)*e^(b*x) - (b^3
*cos(3*c)*e^a + b*d^2*cos(3*c)*e^a + 3*b^2*d*e^a*sin(3*c) + 3*d^3*e^a*sin(
3*c))*e^(b*x)*sin(3*d*x) - (b^3*cos(3*c)*e^a + b*d^2*cos(3*c)*e^a - 3*b^2*
d*e^a*sin(3*c) - 3*d^3*e^a*sin(3*c))*e^(b*x)*sin(3*d*x + 6*c) - (b^3*cos(3
*c)*e^a + 9*b*d^2*cos(3*c)*e^a - b^2*d*e^a*sin(3*c) - 9*d^3*e^a*sin(3*c))*
e^(b*x)*sin(d*x + 4*c) - (b^3*cos(3*c)*e^a + 9*b*d^2*cos(3*c)*e^a + b^2*d*
e^a*sin(3*c) + 9*d^3*e^a*sin(3*c))*e^(b*x)*sin(d*x - 2*c))/(b^4*cos(3*c)^2
 + b^4*sin(3*c)^2 + 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 + 10*(b^2*cos(3*c)^2 +
 b^2*sin(3*c)^2)*d^2)
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.84

$$\int e^{a+bx}\cos^2(c+dx)\sin(c+dx)\,dx = -\frac{1}{4}\left(\frac{3\,d\cos\left(3\,dx+3\,c\right)}{b^2+9\,d^2}-\frac{b\sin\left(3\,dx+3\,c\right)}{b^2+9\,d^2}\right)e^{(bx+a)} - \frac{1}{4}\left(\frac{d\cos\left(dx+c\right)}{b^2+d^2}-\frac{b\sin\left(dx+c\right)}{b^2+d^2}\right)e^{(bx+a)}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c),x, algorithm="giac")
```

output
```
-1/4*(3*d*cos(3*d*x + 3*c)/(b^2 + 9*d^2) - b*sin(3*d*x + 3*c)/(b^2 + 9*d^2
))*e^(b*x + a) - 1/4*(d*cos(d*x + c)/(b^2 + d^2) - b*sin(d*x + c)/(b^2 + d
^2))*e^(b*x + a)
```

3.41.9 Mupad [B] (verification not implemented)

Time = 28.84 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.40

$$\int e^{a+bx}\cos^2(c+dx)\sin(c+dx)\,dx$$
$$= -\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(d\,x\right)-\sin\left(d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(c\right)-\sin\left(c\right)\,1\mathrm{i}\right)}{8\,\left(d+b\,1\mathrm{i}\right)}$$
$$-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(d\,x\right)+\sin\left(d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(c\right)+\sin\left(c\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{8\,\left(b+d\,1\mathrm{i}\right)}$$
$$-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(3\,d\,x\right)-\sin\left(3\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(3\,c\right)-\sin\left(3\,c\right)\,1\mathrm{i}\right)}{8\,\left(3\,d+b\,1\mathrm{i}\right)}$$
$$-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(3\,d\,x\right)+\sin\left(3\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(3\,c\right)+\sin\left(3\,c\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{8\,\left(b+d\,3\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^2*exp(a + b*x)*sin(c + d*x),x)
```

output
```
- (exp(a + b*x)*(cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i))/(8*(b*1i +
d)) - (exp(a + b*x)*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*1i)/(8*(
b + d*1i)) - (exp(a + b*x)*(cos(3*d*x) - sin(3*d*x)*1i)*(cos(3*c) - sin(3*
c)*1i))/(8*(b*1i + 3*d)) - (exp(a + b*x)*(cos(3*d*x) + sin(3*d*x)*1i)*(cos
(3*c) + sin(3*c)*1i)*1i)/(8*(b + d*3i))
```

3.42 $\int e^{a+bx} \cos^2(c+dx) \sin^2(c+dx)\,dx$

3.42.1 Optimal result

Integrand size = 24, antiderivative size = 79

$$\int e^{a+bx} \cos^2(c+dx) \sin^2(c+dx)\,dx = \frac{e^{a+bx}}{8b} - \frac{be^{a+bx}\cos(4c+4dx)}{8(b^2+16d^2)} - \frac{de^{a+bx}\sin(4c+4dx)}{2(b^2+16d^2)}$$

output
```
1/8*exp(b*x+a)/b-1/8*b*exp(b*x+a)*cos(4*d*x+4*c)/(b^2+16*d^2)-1/2*d*exp(b*
x+a)*sin(4*d*x+4*c)/(b^2+16*d^2)
```

3.42.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$\int e^{a+bx} \cos^2(c+dx) \sin^2(c+dx)\,dx = \frac{e^{a+bx}(b^2+16d^2-b^2\cos(4(c+dx))-4bd\sin(4(c+dx)))}{8(b^3+16bd^2)}$$

input
```
Integrate[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x]^2,x]
```

output
```
(E^(a + b*x)*(b^2 + 16*d^2 - b^2*Cos[4*(c + d*x)] - 4*b*d*Sin[4*(c + d*x)]
))/(8*(b^3 + 16*b*d^2))
```

3.42.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4972, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \sin^2(c+dx) \cos^2(c+dx) \, dx$$

$$\Big\downarrow \text{4972}$$

$$\int \left(\frac{1}{8}e^{a+bx} - \frac{1}{8}e^{a+bx} \cos(4c+4dx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{de^{a+bx} \sin(4c+4dx)}{2(b^2+16d^2)} - \frac{be^{a+bx} \cos(4c+4dx)}{8(b^2+16d^2)} + \frac{e^{a+bx}}{8b}$$

input
```
Int[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x]^2,x]
```

output
```
E^(a + b*x)/(8*b) - (b*E^(a + b*x)*Cos[4*c + 4*d*x])/(8*(b^2 + 16*d^2)) -
(d*E^(a + b*x)*Sin[4*c + 4*d*x])/(2*(b^2 + 16*d^2))
```

3.42.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

3.42.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

method	result
parallelrisch	$-\frac{e^{xb+a}\left(4bd\sin(4dx+4c)+b^{2}\cos(4dx+4c)-b^{2}-16d^{2}\right)}{8(b^{2}+16d^{2})b}$
risch	$\frac{e^{xb+a}\left(-2b^{2}-32d^{2}+2b^{2}\cos(4dx+4c)+8bd\sin(4dx+4c)\right)}{16b(4id+b)(4id-b)}$
default	$\frac{e^{xb+a}}{8b}-\frac{b\,e^{xb+a}\cos(4dx+4c)}{8(b^{2}+16d^{2})}-\frac{d\,e^{xb+a}\sin(4dx+4c)}{2(b^{2}+16d^{2})}$
norman	$\frac{-\frac{4d\,e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^{2}+16d^{2}}+\frac{28d\,e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{b^{2}+16d^{2}}-\frac{28d\,e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{b^{2}+16d^{2}}+\frac{4d\,e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{b^{2}+16d^{2}}+\frac{2d^{2}e^{xb+a}}{\left(b^{2}+16d^{2}\right)b}+\frac{2d^{2}e^{xb+a}\tan\left(\frac{}{}\right.}{\left(b^{2}+16d^{2}\right.}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{4}}$

input
```
int(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*exp(b*x+a)*(4*b*d*sin(4*d*x+4*c)+b^2*cos(4*d*x+4*c)-b^2-16*d^2)/(b^2+
16*d^2)/b
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int e^{a+bx}\cos^{2}(c+dx)\sin^{2}(c+dx)\,dx =$$
$$-\frac{2\left(2\,bd\cos\left(dx+c\right)^{3}-bd\cos\left(dx+c\right)\right)e^{(bx+a)}\sin\left(dx+c\right)+\left(b^{2}\cos\left(dx+c\right)^{4}-b^{2}\cos\left(dx+c\right)^{2}-2\,d^{2}\right)e}{b^{3}+16\,bd^{2}}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(2*(2*b*d*cos(d*x + c)^3 - b*d*cos(d*x + c))*e^(b*x + a)*sin(d*x + c) + (
b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*d^2)*e^(b*x + a))/(b^3 + 16*b*
d^2)
```

3.42.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.58 (sec) , antiderivative size = 850, normalized size of antiderivative = 10.76

$$\int e^{a+bx}\cos^2(c+dx)\sin^2(c+dx)\,dx$$
$$= \begin{cases} xe^a\sin^2(c)\cos^2(c) \\ \left(\frac{x\sin^4(c+dx)}{8}+\frac{x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{x\cos^4(c+dx)}{8}+\frac{\sin^3(c+dx)\cos(c+dx)}{8d}-\frac{\sin(c+dx)\cos^3(c+dx)}{8d}\right)e^a \\ -\frac{xe^ae^{-4idx}\sin^4(c+dx)}{16}+\frac{ixe^ae^{-4idx}\sin^3(c+dx)\cos(c+dx)}{4}+\frac{3xe^ae^{-4idx}\sin^2(c+dx)\cos^2(c+dx)}{8}-\frac{ixe^ae^{-4idx}\sin(c+dx)\cos^3(c+d}{4} \\ -\frac{xe^ae^{4idx}\sin^4(c+dx)}{16}-\frac{ixe^ae^{4idx}\sin^3(c+dx)\cos(c+dx)}{4}+\frac{3xe^ae^{4idx}\sin^2(c+dx)\cos^2(c+dx)}{8}+\frac{ixe^ae^{4idx}\sin(c+dx)\cos^3(c+dx)}{4}- \\ \frac{b^2e^ae^{bx}\sin^2(c+dx)\cos^2(c+dx)}{b^3+16bd^2}+\frac{2bde^ae^{bx}\sin^3(c+dx)\cos(c+dx)}{b^3+16bd^2}-\frac{2bde^ae^{bx}\sin(c+dx)\cos^3(c+dx)}{b^3+16bd^2}+\frac{2d^2e^ae^{bx}\sin^4(c+dx)}{b^3+16bd^2}+\frac{4d^2e^{}}{} \end{cases}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)**2*sin(d*x+c)**2,x)
```

output
```
Piecewise((x*exp(a)*sin(c)**2*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c +
 d*x)**4/8 + x*sin(c + d*x)**2*cos(c + d*x)**2/4 + x*cos(c + d*x)**4/8 + s
in(c + d*x)**3*cos(c + d*x)/(8*d) - sin(c + d*x)*cos(c + d*x)**3/(8*d))*ex
p(a), Eq(b, 0)), (-x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**4/16 + I*x*exp(a)*
exp(-4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/4 + 3*x*exp(a)*exp(-4*I*d*x)*si
n(c + d*x)**2*cos(c + d*x)**2/8 - I*x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)*co
s(c + d*x)**3/4 - x*exp(a)*exp(-4*I*d*x)*cos(c + d*x)**4/16 + I*exp(a)*exp
(-4*I*d*x)*sin(c + d*x)**4/(24*d) + 5*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**3
*cos(c + d*x)/(48*d) - 5*exp(a)*exp(-4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3
/(48*d) + I*exp(a)*exp(-4*I*d*x)*cos(c + d*x)**4/(24*d), Eq(b, -4*I*d)), (
-x*exp(a)*exp(4*I*d*x)*sin(c + d*x)**4/16 - I*x*exp(a)*exp(4*I*d*x)*sin(c
+ d*x)**3*cos(c + d*x)/4 + 3*x*exp(a)*exp(4*I*d*x)*sin(c + d*x)**2*cos(c +
 d*x)**2/8 + I*x*exp(a)*exp(4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 - x*ex
p(a)*exp(4*I*d*x)*cos(c + d*x)**4/16 - I*exp(a)*exp(4*I*d*x)*sin(c + d*x)*
*4/(24*d) + 5*exp(a)*exp(4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/(48*d) - 5*
exp(a)*exp(4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/(48*d) - I*exp(a)*exp(4*I
*d*x)*cos(c + d*x)**4/(24*d), Eq(b, 4*I*d)), (b**2*exp(a)*exp(b*x)*sin(c +
 d*x)**2*cos(c + d*x)**2/(b**3 + 16*b*d**2) + 2*b*d*exp(a)*exp(b*x)*sin(c
+ d*x)**3*cos(c + d*x)/(b**3 + 16*b*d**2) - 2*b*d*exp(a)*exp(b*x)*sin(c +
d*x)*cos(c + d*x)**3/(b**3 + 16*b*d**2) + 2*d**2*exp(a)*exp(b*x)*sin(c ...
```

3.42.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(70) = 140.

Time = 0.21 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.99

$$\int e^{a+bx}\cos^2(c+dx)\sin^2(c+dx)\,dx =$$
$$-\frac{(b^2\cos(4c)e^a + 4bde^a\sin(4c))\cos(4dx)e^{(bx)} + (b^2\cos(4c)e^a - 4bde^a\sin(4c))\cos(4dx+8c)e^{(bx)} +$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/16*((b^2*cos(4*c)*e^a + 4*b*d*e^a*sin(4*c))*cos(4*d*x)*e^(b*x) + (b^2*c
os(4*c)*e^a - 4*b*d*e^a*sin(4*c))*cos(4*d*x + 8*c)*e^(b*x) + (4*b*d*cos(4*
c)*e^a - b^2*e^a*sin(4*c))*e^(b*x)*sin(4*d*x) + (4*b*d*cos(4*c)*e^a + b^2*
e^a*sin(4*c))*e^(b*x)*sin(4*d*x + 8*c) - 2*(b^2*cos(4*c)^2*e^a + b^2*e^a*s
in(4*c)^2 + 16*(cos(4*c)^2*e^a + e^a*sin(4*c)^2)*d^2)*e^(b*x))/(b^3*cos(4*
c)^2 + b^3*sin(4*c)^2 + 16*(b*cos(4*c)^2 + b*sin(4*c)^2)*d^2)
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int e^{a+bx}\cos^2(c+dx)\sin^2(c+dx)\,dx = -\frac{1}{8}\left(\frac{b\cos(4dx+4c)}{b^2+16d^2} + \frac{4d\sin(4dx+4c)}{b^2+16d^2}\right)e^{(bx+a)} + \frac{e^{(bx+a)}}{8b}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/8*(b*cos(4*d*x + 4*c)/(b^2 + 16*d^2) + 4*d*sin(4*d*x + 4*c)/(b^2 + 16*d
^2))*e^(b*x + a) + 1/8*e^(b*x + a)/b
```

3.42.9 Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\cos^2(c+dx)\sin^2(c+dx)\,dx = \frac{e^{a+b\,x}\,\left(b^2+16\,d^2-b^2\,\cos\left(4\,c+4\,d\,x\right)-4\,b\,d\,\sin\left(4\,c+4\,d\,x\right)\right)}{8\,b\,\left(b^2+16\,d^2\right)}$$

input
```
int(cos(c + d*x)^2*exp(a + b*x)*sin(c + d*x)^2,x)
```

output
```
(exp(a + b*x)*(b^2 + 16*d^2 - b^2*cos(4*c + 4*d*x) - 4*b*d*sin(4*c + 4*d*x
)))/(8*b*(b^2 + 16*d^2))
```

3.43 $\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx$

3.43.1 Optimal result

Integrand size = 24, antiderivative size = 183

$$\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx = -\frac{de^{a+bx}\cos(c+dx)}{8(b^2+d^2)} - \frac{3de^{a+bx}\cos(3c+3dx)}{16(b^2+9d^2)} + \frac{5de^{a+bx}\cos(5c+5dx)}{16(b^2+25d^2)} + \frac{be^{a+bx}\sin(c+dx)}{8(b^2+d^2)} + \frac{be^{a+bx}\sin(3c+3dx)}{16(b^2+9d^2)} - \frac{be^{a+bx}\sin(5c+5dx)}{16(b^2+25d^2)}$$

output
```
-1/8*d*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-3/16*d*exp(b*x+a)*cos(3*d*x+3*c)/(b
^2+9*d^2)+5/16*d*exp(b*x+a)*cos(5*d*x+5*c)/(b^2+25*d^2)+1/8*b*exp(b*x+a)*s
in(d*x+c)/(b^2+d^2)+1/16*b*exp(b*x+a)*sin(3*d*x+3*c)/(b^2+9*d^2)-1/16*b*ex
p(b*x+a)*sin(5*d*x+5*c)/(b^2+25*d^2)
```

3.43.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.60

$$\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx = \frac{1}{16}e^{a+bx}\left(\frac{2(-d\cos(c+dx)+b\sin(c+dx))}{b^2+d^2} + \frac{-3d\cos(3(c+dx))+b\sin(3(c+dx))}{b^2+9d^2} + \frac{5d\cos(5(c+dx))-b\sin(5(c+dx))}{b^2+25d^2}\right)$$

input
```
Integrate[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x]^3,x]
```

output
```
(E^(a + b*x)*((2*(-(d*Cos[c + d*x]) + b*Sin[c + d*x]))/(b^2 + d^2) + (-3*d
*Cos[3*(c + d*x)] + b*Sin[3*(c + d*x)])/(b^2 + 9*d^2) + (5*d*Cos[5*(c + d*
x)] - b*Sin[5*(c + d*x)])/(b^2 + 25*d^2)))/16
```

3.43.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4972, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sin^3(c+dx)\cos^2(c+dx)\,dx$$

$$\Big\downarrow \text{4972}$$

$$\int \left(\frac{1}{8}e^{a+bx}\sin(c+dx)+\frac{1}{16}e^{a+bx}\sin(3c+3dx)-\frac{1}{16}e^{a+bx}\sin(5c+5dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{be^{a+bx}\sin(c+dx)}{8(b^2+d^2)}+\frac{be^{a+bx}\sin(3c+3dx)}{16(b^2+9d^2)}-\frac{be^{a+bx}\sin(5c+5dx)}{16(b^2+25d^2)}-\frac{de^{a+bx}\cos(c+dx)}{8(b^2+d^2)}-\frac{3de^{a+bx}\cos(3c+3dx)}{16(b^2+9d^2)}+\frac{5de^{a+bx}\cos(5c+5dx)}{16(b^2+25d^2)}$$

input
```
Int[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x]^3,x]
```

output
```
-1/8*(d*E^(a + b*x)*Cos[c + d*x])/(b^2 + d^2) - (3*d*E^(a + b*x)*Cos[3*c +
 3*d*x])/(16*(b^2 + 9*d^2)) + (5*d*E^(a + b*x)*Cos[5*c + 5*d*x])/(16*(b^2
+ 25*d^2)) + (b*E^(a + b*x)*Sin[c + d*x])/(8*(b^2 + d^2)) + (b*E^(a + b*x)
*Sin[3*c + 3*d*x])/(16*(b^2 + 9*d^2)) - (b*E^(a + b*x)*Sin[5*c + 5*d*x])/(
16*(b^2 + 25*d^2))
```

3.43.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

3.43.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.91

method	result
default	$-\frac{d\,\mathrm{e}^{xb+a}\cos(dx+c)}{8(b^2+d^2)}-\frac{3d\,\mathrm{e}^{xb+a}\cos(3dx+3c)}{16(b^2+9d^2)}+\frac{5d\,\mathrm{e}^{xb+a}\cos(5dx+5c)}{16(b^2+25d^2)}+\frac{b\,\mathrm{e}^{xb+a}\sin(dx+c)}{8b^2+8d^2}+\frac{b\,\mathrm{e}^{xb+a}\sin(3dx+3c)}{16b^2+144d^2}-$
parallelrisch	$\frac{\mathrm{e}^{xb+a}\left(3\left(-\frac{1}{2}b^4d-13d^3b^2-\frac{25}{2}d^5\right)\cos(3dx+3c)+5\left(\frac{1}{2}b^4d+5d^3b^2+\frac{9}{2}d^5\right)\cos(5dx+5c)+\left(\frac{1}{2}b^5+13d^2b^3+\frac{25}{2}d^4b\right)\sin(3dx+3c)+(b^2\right.}{8b^6+280b^4d^2+2072b^2d^4+1800d^6}$
risch	$\frac{i\mathrm{e}^{xb+a}\left(i\left(4b^5+136d^2b^3+900d^4b\right)\sin(dx+c)-4id\left(b^4+34b^2d^2+225d^4\right)\cos(dx+c)+10id\left(b^4+10b^2d^2+9d^4\right)\cos(5dx+5c)-i\left(2b^5+\right.\right.}{32(5id+b)(3id+b)(id+b)(id-b)(3}$

input
```
int(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*d*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-3/16*d*exp(b*x+a)*cos(3*d*x+3*c)/(b
^2+9*d^2)+5/16*d*exp(b*x+a)*cos(5*d*x+5*c)/(b^2+25*d^2)+1/8*b*exp(b*x+a)*s
in(d*x+c)/(b^2+d^2)+1/16*b*exp(b*x+a)*sin(3*d*x+3*c)/(b^2+9*d^2)-1/16*b*ex
p(b*x+a)*sin(5*d*x+5*c)/(b^2+25*d^2)
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.10

$$\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx$$
$$=\frac{\left(2\,b^3d^2+26\,bd^4-\left(b^5+10\,b^3d^2+9\,bd^4\right)\cos\left(dx+c\right)^4+\left(b^5+14\,b^3d^2+13\,bd^4\right)\cos\left(dx+c\right)^2\right)e^{(bx+a)}\sin\left(d\right.}{b^6+3}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
((2*b^3*d^2 + 26*b*d^4 - (b^5 + 10*b^3*d^2 + 9*b*d^4)*cos(d*x + c)^4 + (b^
5 + 14*b^3*d^2 + 13*b*d^4)*cos(d*x + c)^2)*e^(b*x + a)*sin(d*x + c) + (5*(
b^4*d + 10*b^2*d^3 + 9*d^5)*cos(d*x + c)^5 - (7*b^4*d + 82*b^2*d^3 + 75*d^
5)*cos(d*x + c)^3 + 2*(b^4*d + 13*b^2*d^3)*cos(d*x + c))*e^(b*x + a))/(b^6
 + 35*b^4*d^2 + 259*b^2*d^4 + 225*d^6)
```

3.43.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.82 (sec) , antiderivative size = 2958, normalized size of antiderivative = 16.16

$$\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)**2*sin(d*x+c)**3,x)
```

output
```
Piecewise((x*exp(a)*sin(c)**3*cos(c)**2, Eq(b, 0) & Eq(d, 0)), (-x*exp(a)*
exp(-5*I*d*x)*sin(c + d*x)**5/32 + 5*I*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)
**4*cos(c + d*x)/32 + 5*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**3*cos(c + d*x
)**2/16 - 5*I*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/16 -
5*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/32 + I*x*exp(a)*exp(
-5*I*d*x)*cos(c + d*x)**5/32 + 31*I*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**5/(
960*d) + 25*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/(192*d) - I*
exp(a)*exp(-5*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(6*d) - exp(a)*exp(-5
*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) + 41*I*exp(a)*exp(-5*I*d*x)*
sin(c + d*x)*cos(c + d*x)**4/(192*d) + 47*exp(a)*exp(-5*I*d*x)*cos(c + d*x
)**5/(960*d), Eq(b, -5*I*d)), (-x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**5/32
+ 3*I*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/32 + x*exp(a)*ex
p(-3*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/16 + I*x*exp(a)*exp(-3*I*d*x)*
sin(c + d*x)**2*cos(c + d*x)**3/16 + 3*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)
*cos(c + d*x)**4/32 - I*x*exp(a)*exp(-3*I*d*x)*cos(c + d*x)**5/32 - 7*I*ex
p(a)*exp(-3*I*d*x)*sin(c + d*x)**5/(192*d) - 9*exp(a)*exp(-3*I*d*x)*sin(c
+ d*x)**4*cos(c + d*x)/(64*d) + I*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**3*cos
(c + d*x)**2/(6*d) - exp(a)*exp(-3*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/
(3*d) + 25*I*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/(64*d) + 23
*exp(a)*exp(-3*I*d*x)*cos(c + d*x)**5/(192*d), Eq(b, -3*I*d)), (x*exp(a...
```

3.43.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. 2(165) = 330.

Time = 0.27 (sec) , antiderivative size = 1148, normalized size of antiderivative = 6.27

$$\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/32*((5*b^4*d*cos(5*c)*e^a + 50*b^2*d^3*cos(5*c)*e^a + 45*d^5*cos(5*c)*e^
a - b^5*e^a*sin(5*c) - 10*b^3*d^2*e^a*sin(5*c) - 9*b*d^4*e^a*sin(5*c))*cos
(5*d*x)*e^(b*x) + (5*b^4*d*cos(5*c)*e^a + 50*b^2*d^3*cos(5*c)*e^a + 45*d^5
*cos(5*c)*e^a + b^5*e^a*sin(5*c) + 10*b^3*d^2*e^a*sin(5*c) + 9*b*d^4*e^a*s
in(5*c))*cos(5*d*x + 10*c)*e^(b*x) - (3*b^4*d*cos(5*c)*e^a + 78*b^2*d^3*co
s(5*c)*e^a + 75*d^5*cos(5*c)*e^a + b^5*e^a*sin(5*c) + 26*b^3*d^2*e^a*sin(5
*c) + 25*b*d^4*e^a*sin(5*c))*cos(3*d*x + 8*c)*e^(b*x) - (3*b^4*d*cos(5*c)*
e^a + 78*b^2*d^3*cos(5*c)*e^a + 75*d^5*cos(5*c)*e^a - b^5*e^a*sin(5*c) - 2
6*b^3*d^2*e^a*sin(5*c) - 25*b*d^4*e^a*sin(5*c))*cos(3*d*x - 2*c)*e^(b*x) -
 2*(b^4*d*cos(5*c)*e^a + 34*b^2*d^3*cos(5*c)*e^a + 225*d^5*cos(5*c)*e^a +
b^5*e^a*sin(5*c) + 34*b^3*d^2*e^a*sin(5*c) + 225*b*d^4*e^a*sin(5*c))*cos(d
*x + 6*c)*e^(b*x) - 2*(b^4*d*cos(5*c)*e^a + 34*b^2*d^3*cos(5*c)*e^a + 225*
d^5*cos(5*c)*e^a - b^5*e^a*sin(5*c) - 34*b^3*d^2*e^a*sin(5*c) - 225*b*d^4*
e^a*sin(5*c))*cos(d*x - 4*c)*e^(b*x) - (b^5*cos(5*c)*e^a + 10*b^3*d^2*cos(
5*c)*e^a + 9*b*d^4*cos(5*c)*e^a + 5*b^4*d*e^a*sin(5*c) + 50*b^2*d^3*e^a*si
n(5*c) + 45*d^5*e^a*sin(5*c))*e^(b*x)*sin(5*d*x) - (b^5*cos(5*c)*e^a + 10*
b^3*d^2*cos(5*c)*e^a + 9*b*d^4*cos(5*c)*e^a - 5*b^4*d*e^a*sin(5*c) - 50*b^
2*d^3*e^a*sin(5*c) - 45*d^5*e^a*sin(5*c))*e^(b*x)*sin(5*d*x + 10*c) + (b^5
*cos(5*c)*e^a + 26*b^3*d^2*cos(5*c)*e^a + 25*b*d^4*cos(5*c)*e^a - 3*b^4*d*
e^a*sin(5*c) - 78*b^2*d^3*e^a*sin(5*c) - 75*d^5*e^a*sin(5*c))*e^(b*x)*s...
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.85

$$\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx = \frac{1}{16}\left(\frac{5\,d\cos(5\,dx+5\,c)}{b^2+25\,d^2}-\frac{b\sin(5\,dx+5\,c)}{b^2+25\,d^2}\right)e^{(bx+a)} - \frac{1}{16}\left(\frac{3\,d\cos(3\,dx+3\,c)}{b^2+9\,d^2}-\frac{b\sin(3\,dx+3\,c)}{b^2+9\,d^2}\right)e^{(bx+a)} - \frac{1}{8}\left(\frac{d\cos(dx+c)}{b^2+d^2}-\frac{b\sin(dx+c)}{b^2+d^2}\right)e^{(bx+a)}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
1/16*(5*d*cos(5*d*x + 5*c)/(b^2 + 25*d^2) - b*sin(5*d*x + 5*c)/(b^2 + 25*d
^2))*e^(b*x + a) - 1/16*(3*d*cos(3*d*x + 3*c)/(b^2 + 9*d^2) - b*sin(3*d*x
+ 3*c)/(b^2 + 9*d^2))*e^(b*x + a) - 1/8*(d*cos(d*x + c)/(b^2 + d^2) - b*si
n(d*x + c)/(b^2 + d^2))*e^(b*x + a)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 28.58 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.39

$$\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx$$
$$= -\frac{e^{a+b\,x}\,(\cos(d\,x)-\sin(d\,x)\,1i)\,(\cos(c)-\sin(c)\,1i)}{16\,(d+b\,1i)} - \frac{e^{a+b\,x}\,(\cos(d\,x)+\sin(d\,x)\,1i)\,(\cos(c)+\sin(c)\,1i)\,1i}{16\,(b+d\,1i)} - \frac{e^{a+b\,x}\,(\cos(3\,d\,x)-\sin(3\,d\,x)\,1i)\,(\cos(3\,c)-\sin(3\,c)\,1i)}{32\,(3\,d+b\,1i)} + \frac{e^{a+b\,x}\,(\cos(5\,d\,x)-\sin(5\,d\,x)\,1i)\,(\cos(5\,c)-\sin(5\,c)\,1i)}{32\,(5\,d+b\,1i)} - \frac{e^{a+b\,x}\,(\cos(3\,d\,x)+\sin(3\,d\,x)\,1i)\,(\cos(3\,c)+\sin(3\,c)\,1i)\,1i}{32\,(b+d\,3i)} + \frac{e^{a+b\,x}\,(\cos(5\,d\,x)+\sin(5\,d\,x)\,1i)\,(\cos(5\,c)+\sin(5\,c)\,1i)\,1i}{32\,(b+d\,5i)}$$

input
```
int(cos(c + d*x)^2*exp(a + b*x)*sin(c + d*x)^3,x)
```

output
```
(exp(a + b*x)*(cos(5*d*x) - sin(5*d*x)*1i)*(cos(5*c) - sin(5*c)*1i))/(32*(
b*1i + 5*d)) - (exp(a + b*x)*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)
*1i)/(16*(b + d*1i)) - (exp(a + b*x)*(cos(3*d*x) - sin(3*d*x)*1i)*(cos(3*c
) - sin(3*c)*1i))/(32*(b*1i + 3*d)) - (exp(a + b*x)*(cos(d*x) - sin(d*x)*1
i)*(cos(c) - sin(c)*1i))/(16*(b*1i + d)) - (exp(a + b*x)*(cos(3*d*x) + sin
(3*d*x)*1i)*(cos(3*c) + sin(3*c)*1i)*1i)/(32*(b + d*3i)) + (exp(a + b*x)*(
cos(5*d*x) + sin(5*d*x)*1i)*(cos(5*c) + sin(5*c)*1i)*1i)/(32*(b + d*5i))
```

3.44 $\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx$

3.44.1 Optimal result

Integrand size = 22, antiderivative size = 129

$$\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx=-\frac{de^{a+bx}\cos(2c+2dx)}{2(b^2+4d^2)}-\frac{de^{a+bx}\cos(4c+4dx)}{2(b^2+16d^2)}+\frac{be^{a+bx}\sin(2c+2dx)}{4(b^2+4d^2)}+\frac{be^{a+bx}\sin(4c+4dx)}{8(b^2+16d^2)}$$

output
```
-1/2*d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)-1/2*d*exp(b*x+a)*cos(4*d*x+4*
c)/(b^2+16*d^2)+1/4*b*exp(b*x+a)*sin(2*d*x+2*c)/(b^2+4*d^2)+1/8*b*exp(b*x+
a)*sin(4*d*x+4*c)/(b^2+16*d^2)
```

3.44.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.63

$$\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx=\frac{1}{8}e^{a+bx}\left(\frac{2(-2d\cos(2(c+dx))+b\sin(2(c+dx)))}{b^2+4d^2}+\frac{-4d\cos(4(c+dx))+b\sin(4(c+dx))}{b^2+16d^2}\right)$$

input
```
Integrate[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x],x]
```

output
```
(E^(a + b*x)*((2*(-2*d*Cos[2*(c + d*x)] + b*Sin[2*(c + d*x)]))/(b^2 + 4*d^
2) + (-4*d*Cos[4*(c + d*x)] + b*Sin[4*(c + d*x)])/(b^2 + 16*d^2)))/8
```

3.44.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4972, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sin(c+dx)\cos^3(c+dx)\,dx$$

$$\Big\downarrow \text{4972}$$

$$\int \left(\frac{1}{4}e^{a+bx}\sin(2c+2dx)+\frac{1}{8}e^{a+bx}\sin(4c+4dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{be^{a+bx}\sin(2c+2dx)}{4\left(b^2+4d^2\right)}+\frac{be^{a+bx}\sin(4c+4dx)}{8\left(b^2+16d^2\right)}-\frac{de^{a+bx}\cos(2c+2dx)}{2\left(b^2+4d^2\right)}-\frac{de^{a+bx}\cos(4c+4dx)}{2\left(b^2+16d^2\right)}$$

input
```
Int[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x],x]
```

output
```
-1/2*(d*E^(a + b*x)*Cos[2*c + 2*d*x])/(b^2 + 4*d^2) - (d*E^(a + b*x)*Cos[4
*c + 4*d*x])/(2*(b^2 + 16*d^2)) + (b*E^(a + b*x)*Sin[2*c + 2*d*x])/(4*(b^2
 + 4*d^2)) + (b*E^(a + b*x)*Sin[4*c + 4*d*x])/(8*(b^2 + 16*d^2))
```

3.44.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

3.44.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{\left(\left(b^{3}+4 b \,d^{2}\right) \sin \left(4 d x+4 c\right)+\left(-4 b^{2} d-16 d^{3}\right) \cos \left(4 d x+4 c\right)+2 \left(b^{2}+16 d^{2}\right) \left(b \sin \left(2 d x+2 c\right)-2 d \cos \left(2 d x+2 c\right)\right)\right) {\mathrm e}^{x b+a}}{8 b^{4}+160 b^{2} d^{2}+512 d^{4}}$
default	$-\frac{d \,{\mathrm e}^{x b+a} \cos \left(2 d x+2 c\right)}{2 \left(b^{2}+4 d^{2}\right)}-\frac{d \,{\mathrm e}^{x b+a} \cos \left(4 d x+4 c\right)}{2 \left(b^{2}+16 d^{2}\right)}+\frac{b \,{\mathrm e}^{x b+a} \sin \left(2 d x+2 c\right)}{4 b^{2}+16 d^{2}}+\frac{b \,{\mathrm e}^{x b+a} \sin \left(4 d x+4 c\right)}{8 b^{2}+128 d^{2}}$
risch	$-\frac{i {\mathrm e}^{x b+a} \left(-8 i d \left(b^{2}+4 d^{2}\right) \cos \left(4 d x+4 c\right)-i \left(-2 b^{3}-8 b \,d^{2}\right) \sin \left(4 d x+4 c\right)-8 i d \left(b^{2}+16 d^{2}\right) \cos \left(2 d x+2 c\right)-i \left(-4 b^{3}-64 b \,d^{2}\right) \sin \left(2 d x+2\right.}{16 \left(4 i d+b\right) \left(2 i d+b\right) \left(2 i d-b\right) \left(4 i d-b\right)}$
norman	$\frac{-\frac{d \left(b^{2}+10 d^{2}\right) {\mathrm e}^{x b+a}}{b^{4}+20 b^{2} d^{2}+64 d^{4}}-\frac{6 b \left(b^{2}+2 d^{2}\right) {\mathrm e}^{x b+a} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{b^{4}+20 b^{2} d^{2}+64 d^{4}}+\frac{6 b \left(b^{2}+2 d^{2}\right) {\mathrm e}^{x b+a} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{b^{4}+20 b^{2} d^{2}+64 d^{4}}+\frac{2 b \left(b^{2}+10 d^{2}\right) {\mathrm e}^{x b+a} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{b^{4}+20 b^{2} d^{2}+64 d^{4}}-\frac{2 b \left(b^{2}+1\right.}{}}{}$

input
```
int(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
((b^3+4*b*d^2)*sin(4*d*x+4*c)+(-4*b^2*d-16*d^3)*cos(4*d*x+4*c)+2*(b^2+16*d
^2)*(b*sin(2*d*x+2*c)-2*d*cos(2*d*x+2*c)))*exp(b*x+a)/(8*b^4+160*b^2*d^2+5
12*d^4)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx$$
$$=\frac{\left(6\,bd^{2}\cos\left(dx+c\right)+\left(b^{3}+4\,bd^{2}\right)\cos\left(dx+c\right)^{3}\right)e^{(bx+a)}\sin\left(dx+c\right)+\left(3\,b^{2}d\cos\left(dx+c\right)^{2}-4\left(b^{2}d+4\,d^{3}\right)\mathrm{c}\right.}{b^{4}+20\,b^{2}d^{2}+64\,d^{4}}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c),x, algorithm="fricas")
```

output
```
((6*b*d^2*cos(d*x + c) + (b^3 + 4*b*d^2)*cos(d*x + c)^3)*e^(b*x + a)*sin(d
*x + c) + (3*b^2*d*cos(d*x + c)^2 - 4*(b^2*d + 4*d^3)*cos(d*x + c)^4 + 6*d
^3)*e^(b*x + a))/(b^4 + 20*b^2*d^2 + 64*d^4)
```

3.44.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.38 (sec) , antiderivative size = 1357, normalized size of antiderivative = 10.52

$$\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)**3*sin(d*x+c),x)
```

output
```
Piecewise((x*exp(a)*sin(c)*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-I*x*exp(a)*e
xp(-4*I*d*x)*sin(c + d*x)**4/16 - x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**3*c
os(c + d*x)/4 + 3*I*x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**2
/8 + x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 - I*x*exp(a)*ex
p(-4*I*d*x)*cos(c + d*x)**4/16 - exp(a)*exp(-4*I*d*x)*sin(c + d*x)**4/(24*
d) + 5*I*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/(48*d) + 11*I*e
xp(a)*exp(-4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/(48*d) + exp(a)*exp(-4*I*
d*x)*cos(c + d*x)**4/(24*d), Eq(b, -4*I*d)), (I*x*exp(a)*exp(-2*I*d*x)*sin
(c + d*x)**4/8 + x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/4 + x
*exp(a)*exp(-2*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 - I*x*exp(a)*exp(-2*I
*d*x)*cos(c + d*x)**4/8 + exp(a)*exp(-2*I*d*x)*sin(c + d*x)**4/(16*d) + ex
p(a)*exp(-2*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**2/(4*d) - I*exp(a)*exp(-2
*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/(6*d) - 7*exp(a)*exp(-2*I*d*x)*cos(c
+ d*x)**4/(48*d), Eq(b, -2*I*d)), (-I*x*exp(a)*exp(2*I*d*x)*sin(c + d*x)**
4/8 + x*exp(a)*exp(2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/4 + x*exp(a)*exp(
2*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 + I*x*exp(a)*exp(2*I*d*x)*cos(c +
d*x)**4/8 + exp(a)*exp(2*I*d*x)*sin(c + d*x)**4/(16*d) + exp(a)*exp(2*I*d*
x)*sin(c + d*x)**2*cos(c + d*x)**2/(4*d) + I*exp(a)*exp(2*I*d*x)*sin(c + d
*x)*cos(c + d*x)**3/(6*d) - 7*exp(a)*exp(2*I*d*x)*cos(c + d*x)**4/(48*d),
Eq(b, 2*I*d)), (I*x*exp(a)*exp(4*I*d*x)*sin(c + d*x)**4/16 - x*exp(a)*e...
```

3.44.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(117) = 234.

Time = 0.23 (sec) , antiderivative size = 550, normalized size of antiderivative = 4.26

$$\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx =$$

$$-\frac{(4\,b^2d\cos(4\,c)\,e^a+16\,d^3\cos(4\,c)\,e^a-b^3e^a\sin(4\,c)-4\,bd^2e^a\sin(4\,c))\cos(4\,dx)\,e^{(bx)}+(4\,b^2d\cos(4\,c)\,e^{\cdots}}{}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c),x, algorithm="maxima")
```

output
```
-1/16*((4*b^2*d*cos(4*c)*e^a + 16*d^3*cos(4*c)*e^a - b^3*e^a*sin(4*c) - 4*
b*d^2*e^a*sin(4*c))*cos(4*d*x)*e^(b*x) + (4*b^2*d*cos(4*c)*e^a + 16*d^3*co
s(4*c)*e^a + b^3*e^a*sin(4*c) + 4*b*d^2*e^a*sin(4*c))*cos(4*d*x + 8*c)*e^(
b*x) + 2*(2*b^2*d*cos(4*c)*e^a + 32*d^3*cos(4*c)*e^a + b^3*e^a*sin(4*c) +
16*b*d^2*e^a*sin(4*c))*cos(2*d*x + 6*c)*e^(b*x) + 2*(2*b^2*d*cos(4*c)*e^a
+ 32*d^3*cos(4*c)*e^a - b^3*e^a*sin(4*c) - 16*b*d^2*e^a*sin(4*c))*cos(2*d*
x - 2*c)*e^(b*x) - (b^3*cos(4*c)*e^a + 4*b*d^2*cos(4*c)*e^a + 4*b^2*d*e^a*
sin(4*c) + 16*d^3*e^a*sin(4*c))*e^(b*x)*sin(4*d*x) - (b^3*cos(4*c)*e^a + 4
*b*d^2*cos(4*c)*e^a - 4*b^2*d*e^a*sin(4*c) - 16*d^3*e^a*sin(4*c))*e^(b*x)*
sin(4*d*x + 8*c) - 2*(b^3*cos(4*c)*e^a + 16*b*d^2*cos(4*c)*e^a - 2*b^2*d*e
^a*sin(4*c) - 32*d^3*e^a*sin(4*c))*e^(b*x)*sin(2*d*x + 6*c) - 2*(b^3*cos(4
*c)*e^a + 16*b*d^2*cos(4*c)*e^a + 2*b^2*d*e^a*sin(4*c) + 32*d^3*e^a*sin(4*
c))*e^(b*x)*sin(2*d*x - 2*c))/(b^4*cos(4*c)^2 + b^4*sin(4*c)^2 + 64*(cos(4
*c)^2 + sin(4*c)^2)*d^4 + 20*(b^2*cos(4*c)^2 + b^2*sin(4*c)^2)*d^2)
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

$$\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx = -\frac{1}{8}\left(\frac{4\,d\cos\left(4\,dx+4\,c\right)}{b^2+16\,d^2}-\frac{b\sin\left(4\,dx+4\,c\right)}{b^2+16\,d^2}\right)e^{(bx+a)} - \frac{1}{4}\left(\frac{2\,d\cos\left(2\,dx+2\,c\right)}{b^2+4\,d^2}-\frac{b\sin\left(2\,dx+2\,c\right)}{b^2+4\,d^2}\right)e^{(bx+a)}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c),x, algorithm="giac")
```

output
```
-1/8*(4*d*cos(4*d*x + 4*c)/(b^2 + 16*d^2) - b*sin(4*d*x + 4*c)/(b^2 + 16*d
^2))*e^(b*x + a) - 1/4*(2*d*cos(2*d*x + 2*c)/(b^2 + 4*d^2) - b*sin(2*d*x +
 2*c)/(b^2 + 4*d^2))*e^(b*x + a)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.39

$$\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx$$
$$= -\frac{e^{a+b\,x}\,(\cos(2\,d\,x)-\sin(2\,d\,x)\,1\mathrm{i})\,(\cos(2\,c)-\sin(2\,c)\,1\mathrm{i})}{8\,(2\,d+b\,1\mathrm{i})}$$
$$-\frac{e^{a+b\,x}\,(\cos(4\,d\,x)-\sin(4\,d\,x)\,1\mathrm{i})\,(\cos(4\,c)-\sin(4\,c)\,1\mathrm{i})}{16\,(4\,d+b\,1\mathrm{i})}$$
$$-\frac{e^{a+b\,x}\,(\cos(2\,d\,x)+\sin(2\,d\,x)\,1\mathrm{i})\,(\cos(2\,c)+\sin(2\,c)\,1\mathrm{i})\,1\mathrm{i}}{8\,(b+d\,2\mathrm{i})}$$
$$-\frac{e^{a+b\,x}\,(\cos(4\,d\,x)+\sin(4\,d\,x)\,1\mathrm{i})\,(\cos(4\,c)+\sin(4\,c)\,1\mathrm{i})\,1\mathrm{i}}{16\,(b+d\,4\mathrm{i})}$$

input
```
int(cos(c + d*x)^3*exp(a + b*x)*sin(c + d*x),x)
```

output
```
- (exp(a + b*x)*(cos(2*d*x) - sin(2*d*x)*1i)*(cos(2*c) - sin(2*c)*1i))/(8*
(b*1i + 2*d)) - (exp(a + b*x)*(cos(4*d*x) - sin(4*d*x)*1i)*(cos(4*c) - sin
(4*c)*1i))/(16*(b*1i + 4*d)) - (exp(a + b*x)*(cos(2*d*x) + sin(2*d*x)*1i)*
(cos(2*c) + sin(2*c)*1i)*1i)/(8*(b + d*2i)) - (exp(a + b*x)*(cos(4*d*x) +
sin(4*d*x)*1i)*(cos(4*c) + sin(4*c)*1i)*1i)/(16*(b + d*4i))
```

3.45 $\int e^{a+bx}\cos^3(c+dx)\sin^2(c+dx)\,dx$

3.45.1 Optimal result

Integrand size = 24, antiderivative size = 183

$$\int e^{a+bx}\cos^3(c+dx)\sin^2(c+dx)\,dx = \frac{be^{a+bx}\cos(c+dx)}{8(b^2+d^2)} - \frac{be^{a+bx}\cos(3c+3dx)}{16(b^2+9d^2)} - \frac{be^{a+bx}\cos(5c+5dx)}{16(b^2+25d^2)} + \frac{de^{a+bx}\sin(c+dx)}{8(b^2+d^2)} - \frac{3de^{a+bx}\sin(3c+3dx)}{16(b^2+9d^2)} - \frac{5de^{a+bx}\sin(5c+5dx)}{16(b^2+25d^2)}$$

output
```
1/8*b*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-1/16*b*exp(b*x+a)*cos(3*d*x+3*c)/(b^
2+9*d^2)-1/16*b*exp(b*x+a)*cos(5*d*x+5*c)/(b^2+25*d^2)+1/8*d*exp(b*x+a)*si
n(d*x+c)/(b^2+d^2)-3/16*d*exp(b*x+a)*sin(3*d*x+3*c)/(b^2+9*d^2)-5/16*d*exp
(b*x+a)*sin(5*d*x+5*c)/(b^2+25*d^2)
```

3.45.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.60

$$\int e^{a+bx}\cos^3(c+dx)\sin^2(c+dx)\,dx = \frac{1}{16}e^{a+bx}\left(\frac{2(b\cos(c+dx)+d\sin(c+dx))}{b^2+d^2} - \frac{b\cos(3(c+dx))+3d\sin(3(c+dx))}{b^2+9d^2} - \frac{b\cos(5(c+dx))+5d\sin(5(c+dx))}{b^2+25d^2}\right)$$

input
```
Integrate[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x]^2,x]
```

output
```
(E^(a + b*x)*((2*(b*Cos[c + d*x] + d*Sin[c + d*x]))/(b^2 + d^2) - (b*Cos[3
*(c + d*x)] + 3*d*Sin[3*(c + d*x)])/(b^2 + 9*d^2) - (b*Cos[5*(c + d*x)] +
5*d*Sin[5*(c + d*x)])/(b^2 + 25*d^2)))/16
```

3.45.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4972, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sin^2(c+dx)\cos^3(c+dx)\,dx$$

$$\Big\downarrow \; 4972$$

$$\int \left(\frac{1}{8}e^{a+bx}\cos(c+dx)-\frac{1}{16}e^{a+bx}\cos(3c+3dx)-\frac{1}{16}e^{a+bx}\cos(5c+5dx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{de^{a+bx}\sin(c+dx)}{8\left(b^2+d^2\right)}-\frac{3de^{a+bx}\sin(3c+3dx)}{16\left(b^2+9d^2\right)}-\frac{5de^{a+bx}\sin(5c+5dx)}{16\left(b^2+25d^2\right)}+\frac{be^{a+bx}\cos(c+dx)}{8\left(b^2+d^2\right)}-\frac{be^{a+bx}\cos(3c+3dx)}{16\left(b^2+9d^2\right)}-\frac{be^{a+bx}\cos(5c+5dx)}{16\left(b^2+25d^2\right)}$$

input
```
Int[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x]^2,x]
```

output
```
(b*E^(a + b*x)*Cos[c + d*x])/(8*(b^2 + d^2)) - (b*E^(a + b*x)*Cos[3*c + 3*
d*x])/(16*(b^2 + 9*d^2)) - (b*E^(a + b*x)*Cos[5*c + 5*d*x])/(16*(b^2 + 25*
d^2)) + (d*E^(a + b*x)*Sin[c + d*x])/(8*(b^2 + d^2)) - (3*d*E^(a + b*x)*Si
n[3*c + 3*d*x])/(16*(b^2 + 9*d^2)) - (5*d*E^(a + b*x)*Sin[5*c + 5*d*x])/(1
6*(b^2 + 25*d^2))
```

3.45.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

3.45.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.91

method	result
default	$\frac{b\,\mathrm{e}^{xb+a}\cos(dx+c)}{8b^2+8d^2}-\frac{b\,\mathrm{e}^{xb+a}\cos(3dx+3c)}{16(b^2+9d^2)}-\frac{b\,\mathrm{e}^{xb+a}\cos(5dx+5c)}{16(b^2+25d^2)}+\frac{d\,\mathrm{e}^{xb+a}\sin(dx+c)}{8b^2+8d^2}-\frac{3d\,\mathrm{e}^{xb+a}\sin(3dx+3c)}{16(b^2+9d^2)}-\frac{5d\,\mathrm{e}}{}$
parallelrisch	$\frac{\mathrm{e}^{xb+a}\left(\left(-\frac{1}{2}b^5-13d^2b^3-\frac{25}{2}d^4b\right)\cos(3dx+3c)+\left(-\frac{1}{2}b^5-5d^2b^3-\frac{9}{2}d^4b\right)\cos(5dx+5c)+3\left(-\frac{1}{2}b^4d-13d^3b^2-\frac{25}{2}d^5\right)\sin(3dx+3c)+(\right.}{8b^6+280b^4d^2+2072b^2d^4+1800d^6}$
risch	$-\frac{\mathrm{e}^{xb+a}\left(4d\left(b^4+34b^2d^2+225d^4\right)\sin(dx+c)+\left(4b^5+136d^2b^3+900d^4b\right)\cos(dx+c)+\left(-2b^5-20d^2b^3-18d^4b\right)\cos(5dx+5c)-10d(b\right.}{32(5id+b)(3id+b)(id+b)(id-b)(}$

input
```
int(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*b*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-1/16*b*exp(b*x+a)*cos(3*d*x+3*c)/(b^
2+9*d^2)-1/16*b*exp(b*x+a)*cos(5*d*x+5*c)/(b^2+25*d^2)+1/8*d*exp(b*x+a)*si
n(d*x+c)/(b^2+d^2)-3/16*d*exp(b*x+a)*sin(3*d*x+3*c)/(b^2+9*d^2)-5/16*d*exp
(b*x+a)*sin(5*d*x+5*c)/(b^2+25*d^2)
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.09

$$\int e^{a+bx}\cos^3(c+dx)\sin^2(c+dx)\,dx$$
$$=\frac{\left(6\,b^2d^3+30\,d^5-5\,(b^4d+10\,b^2d^3+9\,d^5)\cos{(dx+c)}^4+3\,(b^4d+6\,b^2d^3+5\,d^5)\cos{(dx+c)}^2\right)e^{(bx+a)}\sin{(d}}{b^6+35\,b}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
((6*b^2*d^3 + 30*d^5 - 5*(b^4*d + 10*b^2*d^3 + 9*d^5)*cos(d*x + c)^4 + 3*(
b^4*d + 6*b^2*d^3 + 5*d^5)*cos(d*x + c)^2)*e^(b*x + a)*sin(d*x + c) - ((b^
5 + 10*b^3*d^2 + 9*b*d^4)*cos(d*x + c)^5 - (b^5 + 6*b^3*d^2 + 5*b*d^4)*cos
(d*x + c)^3 - 6*(b^3*d^2 + 5*b*d^4)*cos(d*x + c))*e^(b*x + a))/(b^6 + 35*b
^4*d^2 + 259*b^2*d^4 + 225*d^6)
```

3.45.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.94 (sec) , antiderivative size = 2751, normalized size of antiderivative = 15.03

$$\int e^{a+bx}\cos^3(c+dx)\sin^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)**3*sin(d*x+c)**2,x)
```

output
```
Piecewise((x*exp(a)*sin(c)**2*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-I*x*exp(a
)*exp(-5*I*d*x)*sin(c + d*x)**5/32 - 5*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)
**4*cos(c + d*x)/32 + 5*I*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**3*cos(c + d
*x)**2/16 + 5*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/16 -
5*I*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/32 - x*exp(a)*exp(
-5*I*d*x)*cos(c + d*x)**5/32 - 31*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**5/(96
0*d) + 25*I*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/(192*d) + ex
p(a)*exp(-5*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(6*d) - 3*exp(a)*exp(-5
*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/(64*d) + I*exp(a)*exp(-5*I*d*x)*cos(c
 + d*x)**5/(64*d), Eq(b, -5*I*d)), (I*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*
*5/32 + 3*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/32 - I*x*exp
(a)*exp(-3*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/16 + x*exp(a)*exp(-3*I*d
*x)*sin(c + d*x)**2*cos(c + d*x)**3/16 - 3*I*x*exp(a)*exp(-3*I*d*x)*sin(c
+ d*x)*cos(c + d*x)**4/32 - x*exp(a)*exp(-3*I*d*x)*cos(c + d*x)**5/32 - 7*
exp(a)*exp(-3*I*d*x)*sin(c + d*x)**5/(192*d) + 9*I*exp(a)*exp(-3*I*d*x)*si
n(c + d*x)**4*cos(c + d*x)/(64*d) + exp(a)*exp(-3*I*d*x)*sin(c + d*x)**3*c
os(c + d*x)**2/(6*d) - 7*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)**4
/(64*d) + 3*I*exp(a)*exp(-3*I*d*x)*cos(c + d*x)**5/(64*d), Eq(b, -3*I*d)),
 (I*x*exp(a)*exp(-I*d*x)*sin(c + d*x)**5/16 + x*exp(a)*exp(-I*d*x)*sin(c +
 d*x)**4*cos(c + d*x)/16 + I*x*exp(a)*exp(-I*d*x)*sin(c + d*x)**3*cos(c...
```

3.45.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1144 vs. $2(165) = 330$.

Time = 0.25 (sec) , antiderivative size = 1144, normalized size of antiderivative = 6.25

$$\int e^{a+bx} \cos^3(c+dx) \sin^2(c+dx)\, dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/32*((b^5*cos(5*c)*e^a + 10*b^3*d^2*cos(5*c)*e^a + 9*b*d^4*cos(5*c)*e^a
+ 5*b^4*d*e^a*sin(5*c) + 50*b^2*d^3*e^a*sin(5*c) + 45*d^5*e^a*sin(5*c))*co
s(5*d*x)*e^(b*x) + (b^5*cos(5*c)*e^a + 10*b^3*d^2*cos(5*c)*e^a + 9*b*d^4*c
os(5*c)*e^a - 5*b^4*d*e^a*sin(5*c) - 50*b^2*d^3*e^a*sin(5*c) - 45*d^5*e^a*
sin(5*c))*cos(5*d*x + 10*c)*e^(b*x) + (b^5*cos(5*c)*e^a + 26*b^3*d^2*cos(5
*c)*e^a + 25*b*d^4*cos(5*c)*e^a - 3*b^4*d*e^a*sin(5*c) - 78*b^2*d^3*e^a*si
n(5*c) - 75*d^5*e^a*sin(5*c))*cos(3*d*x + 8*c)*e^(b*x) + (b^5*cos(5*c)*e^a
 + 26*b^3*d^2*cos(5*c)*e^a + 25*b*d^4*cos(5*c)*e^a + 3*b^4*d*e^a*sin(5*c)
+ 78*b^2*d^3*e^a*sin(5*c) + 75*d^5*e^a*sin(5*c))*cos(3*d*x - 2*c)*e^(b*x)
- 2*(b^5*cos(5*c)*e^a + 34*b^3*d^2*cos(5*c)*e^a + 225*b*d^4*cos(5*c)*e^a -
 b^4*d*e^a*sin(5*c) - 34*b^2*d^3*e^a*sin(5*c) - 225*d^5*e^a*sin(5*c))*cos(
d*x + 6*c)*e^(b*x) - 2*(b^5*cos(5*c)*e^a + 34*b^3*d^2*cos(5*c)*e^a + 225*b
*d^4*cos(5*c)*e^a + b^4*d*e^a*sin(5*c) + 34*b^2*d^3*e^a*sin(5*c) + 225*d^5
*e^a*sin(5*c))*cos(d*x - 4*c)*e^(b*x) + (5*b^4*d*cos(5*c)*e^a + 50*b^2*d^3
*cos(5*c)*e^a + 45*d^5*cos(5*c)*e^a - b^5*e^a*sin(5*c) - 10*b^3*d^2*e^a*si
n(5*c) - 9*b*d^4*e^a*sin(5*c))*e^(b*x)*sin(5*d*x) + (5*b^4*d*cos(5*c)*e^a
+ 50*b^2*d^3*cos(5*c)*e^a + 45*d^5*cos(5*c)*e^a + b^5*e^a*sin(5*c) + 10*b^
3*d^2*e^a*sin(5*c) + 9*b*d^4*e^a*sin(5*c))*e^(b*x)*sin(5*d*x + 10*c) + (3*
b^4*d*cos(5*c)*e^a + 78*b^2*d^3*cos(5*c)*e^a + 75*d^5*cos(5*c)*e^a + b^5*e
^a*sin(5*c) + 26*b^3*d^2*e^a*sin(5*c) + 25*b*d^4*e^a*sin(5*c))*e^(b*x)*...
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.83

$$\begin{aligned}\int e^{a+bx} \cos^3(c+dx) \sin^2(c+dx)\, dx = &-\frac{1}{16}\left(\frac{b\cos(5\,dx+5\,c)}{b^2+25\,d^2}+\frac{5\,d\sin(5\,dx+5\,c)}{b^2+25\,d^2}\right)e^{(bx+a)} \\ &-\frac{1}{16}\left(\frac{b\cos(3\,dx+3\,c)}{b^2+9\,d^2}+\frac{3\,d\sin(3\,dx+3\,c)}{b^2+9\,d^2}\right)e^{(bx+a)} \\ &+\frac{1}{8}\left(\frac{b\cos(dx+c)}{b^2+d^2}+\frac{d\sin(dx+c)}{b^2+d^2}\right)e^{(bx+a)}\end{aligned}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/16*(b*cos(5*d*x + 5*c)/(b^2 + 25*d^2) + 5*d*sin(5*d*x + 5*c)/(b^2 + 25*
d^2))*e^(b*x + a) - 1/16*(b*cos(3*d*x + 3*c)/(b^2 + 9*d^2) + 3*d*sin(3*d*x
 + 3*c)/(b^2 + 9*d^2))*e^(b*x + a) + 1/8*(b*cos(d*x + c)/(b^2 + d^2) + d*s
in(d*x + c)/(b^2 + d^2))*e^(b*x + a)
```

3.45.9 Mupad [B] (verification not implemented)

Time = 28.85 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.39

$$\begin{aligned}\int e^{a+bx}\cos^3(c+dx)\sin^2(c+dx)\,dx &= \frac{e^{a+b\,x}\,(\cos(d\,x)-\sin(d\,x)\;1\mathrm{i})\;(\cos(c)-\sin(c)\;1\mathrm{i})}{16\;(b-d\,1\mathrm{i})}\\&\quad-\frac{e^{a+b\,x}\,(\cos(3\,d\,x)+\sin(3\,d\,x)\;1\mathrm{i})\;(\cos(3\,c)+\sin(3\,c)\;1\mathrm{i})\;1\mathrm{i}}{32\;(-3\,d+b\,1\mathrm{i})}\\&\quad-\frac{e^{a+b\,x}\,(\cos(5\,d\,x)+\sin(5\,d\,x)\;1\mathrm{i})\;(\cos(5\,c)+\sin(5\,c)\;1\mathrm{i})\;1\mathrm{i}}{32\;(-5\,d+b\,1\mathrm{i})}\\&\quad+\frac{e^{a+b\,x}\,(\cos(d\,x)+\sin(d\,x)\;1\mathrm{i})\;(\cos(c)+\sin(c)\;1\mathrm{i})\;1\mathrm{i}}{16\;(-d+b\,1\mathrm{i})}\\&\quad-\frac{e^{a+b\,x}\,(\cos(3\,d\,x)-\sin(3\,d\,x)\;1\mathrm{i})\;(\cos(3\,c)-\sin(3\,c)\;1\mathrm{i})}{32\;(b-d\,3\mathrm{i})}\\&\quad-\frac{e^{a+b\,x}\,(\cos(5\,d\,x)-\sin(5\,d\,x)\;1\mathrm{i})\;(\cos(5\,c)-\sin(5\,c)\;1\mathrm{i})}{32\;(b-d\,5\mathrm{i})}\end{aligned}$$

input
```
int(cos(c + d*x)^3*exp(a + b*x)*sin(c + d*x)^2,x)
```

output
```
(exp(a + b*x)*(cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i))/(16*(b - d*1i
)) - (exp(a + b*x)*(cos(3*d*x) + sin(3*d*x)*1i)*(cos(3*c) + sin(3*c)*1i)*1
i)/(32*(b*1i - 3*d)) - (exp(a + b*x)*(cos(5*d*x) + sin(5*d*x)*1i)*(cos(5*c
) + sin(5*c)*1i)*1i)/(32*(b*1i - 5*d)) + (exp(a + b*x)*(cos(d*x) + sin(d*x
)*1i)*(cos(c) + sin(c)*1i)*1i)/(16*(b*1i - d)) - (exp(a + b*x)*(cos(3*d*x)
 - sin(3*d*x)*1i)*(cos(3*c) - sin(3*c)*1i))/(32*(b - d*3i)) - (exp(a + b*x
)*(cos(5*d*x) - sin(5*d*x)*1i)*(cos(5*c) - sin(5*c)*1i))/(32*(b - d*5i))
```

3.46 $\int e^{a+bx}\cos^3(c+dx)\sin^3(c+dx)\,dx$

3.46.1 Optimal result

Integrand size = 24, antiderivative size = 129

$$\int e^{a+bx}\cos^3(c+dx)\sin^3(c+dx)\,dx = -\frac{3de^{a+bx}\cos(2c+2dx)}{16(b^2+4d^2)}+\frac{3de^{a+bx}\cos(6c+6dx)}{16(b^2+36d^2)} + \frac{3be^{a+bx}\sin(2c+2dx)}{32(b^2+4d^2)}-\frac{be^{a+bx}\sin(6c+6dx)}{32(b^2+36d^2)}$$

output
```
-3/16*d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)+3/16*d*exp(b*x+a)*cos(6*d*x+
6*c)/(b^2+36*d^2)+3/32*b*exp(b*x+a)*sin(2*d*x+2*c)/(b^2+4*d^2)-1/32*b*exp(
b*x+a)*sin(6*d*x+6*c)/(b^2+36*d^2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

$$\int e^{a+bx}\cos^3(c+dx)\sin^3(c+dx)\,dx = \frac{e^{a+bx}(-6d(b^2+36d^2)\cos(2(c+dx))+6d(b^2+4d^2)\cos(6(c+dx))-2b(-b^2-52d^2+(b^2+4d^2)\cos(4(c}{32(b^4+40b^2d^2+144d^4)}$$

input
```
Integrate[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x]^3,x]
```

output
```
(E^(a + b*x)*(-6*d*(b^2 + 36*d^2)*Cos[2*(c + d*x)] + 6*d*(b^2 + 4*d^2)*Cos
[6*(c + d*x)] - 2*b*(-b^2 - 52*d^2 + (b^2 + 4*d^2)*Cos[4*(c + d*x)])*Sin[2
*(c + d*x)]))/(32*(b^4 + 40*b^2*d^2 + 144*d^4))
```

3.46.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4972, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sin^3(c+dx)\cos^3(c+dx)\,dx$$

$$\Big\downarrow \text{4972}$$

$$\int \left(\frac{3}{32}e^{a+bx}\sin(2c+2dx)-\frac{1}{32}e^{a+bx}\sin(6c+6dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3be^{a+bx}\sin(2c+2dx)}{32\,(b^2+4d^2)}-\frac{be^{a+bx}\sin(6c+6dx)}{32\,(b^2+36d^2)}-\frac{3de^{a+bx}\cos(2c+2dx)}{16\,(b^2+4d^2)}+\frac{3de^{a+bx}\cos(6c+6dx)}{16\,(b^2+36d^2)}$$

input
```
Int[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x]^3,x]
```

output
```
(-3*d*E^(a + b*x)*Cos[2*c + 2*d*x])/(16*(b^2 + 4*d^2)) + (3*d*E^(a + b*x)*
Cos[6*c + 6*d*x])/(16*(b^2 + 36*d^2)) + (3*b*E^(a + b*x)*Sin[2*c + 2*d*x])
/(32*(b^2 + 4*d^2)) - (b*E^(a + b*x)*Sin[6*c + 6*d*x])/(32*(b^2 + 36*d^2))
```

3.46.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

3.46.4 Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

method	result
parallelrisch	$-\frac{\left(\left(b^{3}+4 b\, d^{2}\right) \sin \left(6 d x +6 c \right)+\left(-6 b^{2} d -24 d^{3}\right) \cos \left(6 d x +6 c \right)-3 \left(b^{2}+36 d^{2}\right) \left(b \sin \left(2 d x +2 c \right)-2 d \cos \left(2 d x +2 c \right)\right)\right) {\mathrm e}^{x b +a}}{32 b^{4}+1280 b^{2} d^{2}+4608 d^{4}}$
default	$-\frac{3 d\, {\mathrm e}^{x b +a} \cos \left(2 d x +2 c \right)}{16 \left(b^{2}+4 d^{2}\right)}+\frac{3 d\, {\mathrm e}^{x b +a} \cos \left(6 d x +6 c \right)}{16 \left(b^{2}+36 d^{2}\right)}+\frac{3 b\, {\mathrm e}^{x b +a} \sin \left(2 d x +2 c \right)}{32 \left(b^{2}+4 d^{2}\right)}-\frac{b\, {\mathrm e}^{x b +a} \sin \left(6 d x +6 c \right)}{32 \left(b^{2}+36 d^{2}\right)}$
risch	$\frac{i {\mathrm e}^{x b +a} \left(-12 i d \left(b^{2}+4 d^{2}\right) \cos \left(6 d x +6 c \right)-i \left(-2 b^{3}-8 b\, d^{2}\right) \sin \left(6 d x +6 c \right)+12 i d \left(b^{2}+36 d^{2}\right) \cos \left(2 d x +2 c \right)-i \left(6 b^{3}+216 b\, d^{2}\right) \sin \left(2 d x +2 c \right) \ldots}{64 \left(6 i d +b \right) \left(2 i d +b \right) \left(2 i d -b \right) \left(6 i d -b \right)}$

input
```
int(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-((b^3+4*b*d^2)*sin(6*d*x+6*c)+(-6*b^2*d-24*d^3)*cos(6*d*x+6*c)-3*(b^2+36*
d^2)*(b*sin(2*d*x+2*c)-2*d*cos(2*d*x+2*c)))*exp(b*x+a)/(32*b^4+1280*b^2*d^
2+4608*d^4)
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.21

$$\int e^{a+bx} \cos^3(c+dx) \sin^3(c+dx)\, dx = -\frac{\left(\left(b^{3} + 4\, b d^{2}\right) \cos\left(d x + c\right)^{5} - 6\, b d^{2} \cos\left(d x + c\right) - \left(b^{3} + 4\, b d^{2}\right) \cos\left(d x + c\right)^{3}\right) e^{\left(b x + a\right)} \sin\left(d x + c\right) - 3\, \left(2\, \left(b \ldots\right.\right.}{b^{4} + 40\, b^{2} d^{2} + 144 \ldots}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^3,x, algorithm="fricas")
```

output
```
-(((b^3 + 4*b*d^2)*cos(d*x + c)^5 - 6*b*d^2*cos(d*x + c) - (b^3 + 4*b*d^2)
*cos(d*x + c)^3)*e^(b*x + a)*sin(d*x + c) - 3*(2*(b^2*d + 4*d^3)*cos(d*x +
 c)^6 + b^2*d*cos(d*x + c)^2 - 3*(b^2*d + 4*d^3)*cos(d*x + c)^4 + 2*d^3)*e
^(b*x + a))/(b^4 + 40*b^2*d^2 + 144*d^4)
```

3.46.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 71.62 (sec) , antiderivative size = 1991, normalized size of antiderivative = 15.43

$$\int e^{a+bx}\cos^3(c+dx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)**3*sin(d*x+c)**3,x)
```

output
```
Piecewise((x*exp(a)*sin(c)**3*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-I*x*exp(a
)*exp(-6*I*d*x)*sin(c + d*x)**6/64 - 3*x*exp(a)*exp(-6*I*d*x)*sin(c + d*x)
**5*cos(c + d*x)/32 + 15*I*x*exp(a)*exp(-6*I*d*x)*sin(c + d*x)**4*cos(c +
d*x)**2/64 + 5*x*exp(a)*exp(-6*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**3/16 -
 15*I*x*exp(a)*exp(-6*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**4/64 - 3*x*exp(
a)*exp(-6*I*d*x)*sin(c + d*x)*cos(c + d*x)**5/32 + I*x*exp(a)*exp(-6*I*d*x
)*cos(c + d*x)**6/64 - exp(a)*exp(-6*I*d*x)*sin(c + d*x)**6/(160*d) + 7*I*
exp(a)*exp(-6*I*d*x)*sin(c + d*x)**5*cos(c + d*x)/(320*d) + 11*I*exp(a)*ex
p(-6*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**3/(96*d) + 7*I*exp(a)*exp(-6*I*d
*x)*sin(c + d*x)*cos(c + d*x)**5/(320*d) + exp(a)*exp(-6*I*d*x)*cos(c + d*
x)**6/(160*d), Eq(b, -6*I*d)), (3*I*x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**6
/64 + 3*x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**5*cos(c + d*x)/32 + 3*I*x*exp
(a)*exp(-2*I*d*x)*sin(c + d*x)**4*cos(c + d*x)**2/64 + 3*x*exp(a)*exp(-2*I
*d*x)*sin(c + d*x)**3*cos(c + d*x)**3/16 - 3*I*x*exp(a)*exp(-2*I*d*x)*sin(
c + d*x)**2*cos(c + d*x)**4/64 + 3*x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)*cos
(c + d*x)**5/32 - 3*I*x*exp(a)*exp(-2*I*d*x)*cos(c + d*x)**6/64 - 3*exp(a)
*exp(-2*I*d*x)*sin(c + d*x)**6/(32*d) + 15*I*exp(a)*exp(-2*I*d*x)*sin(c +
d*x)**5*cos(c + d*x)/(64*d) + 13*I*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**3*co
s(c + d*x)**3/(32*d) + 15*I*exp(a)*exp(-2*I*d*x)*sin(c + d*x)*cos(c + d*x)
**5/(64*d) + 3*exp(a)*exp(-2*I*d*x)*cos(c + d*x)**6/(32*d), Eq(b, -2*I*...
```

3.46.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(117) = 234.

Time = 0.22 (sec) , antiderivative size = 550, normalized size of antiderivative = 4.26

$$\int e^{a+bx}\cos^3(c+dx)\sin^3(c+dx)\,dx$$
$$= \frac{\left(6\,b^2d\cos(6\,c)\,e^a + 24\,d^3\cos(6\,c)\,e^a - b^3e^a\sin(6\,c) - 4\,bd^2e^a\sin(6\,c)\right)\cos(6\,dx)\,e^{(bx)} + \left(6\,b^2d\cos(6\,c)\,e^a + \cdots\right.}{}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/64*((6*b^2*d*cos(6*c)*e^a + 24*d^3*cos(6*c)*e^a - b^3*e^a*sin(6*c) - 4*b
*d^2*e^a*sin(6*c))*cos(6*d*x)*e^(b*x) + (6*b^2*d*cos(6*c)*e^a + 24*d^3*cos
(6*c)*e^a + b^3*e^a*sin(6*c) + 4*b*d^2*e^a*sin(6*c))*cos(6*d*x + 12*c)*e^(
b*x) - 3*(2*b^2*d*cos(6*c)*e^a + 72*d^3*cos(6*c)*e^a + b^3*e^a*sin(6*c) +
36*b*d^2*e^a*sin(6*c))*cos(2*d*x + 8*c)*e^(b*x) - 3*(2*b^2*d*cos(6*c)*e^a
+ 72*d^3*cos(6*c)*e^a - b^3*e^a*sin(6*c) - 36*b*d^2*e^a*sin(6*c))*cos(2*d*
x - 4*c)*e^(b*x) - (b^3*cos(6*c)*e^a + 4*b*d^2*cos(6*c)*e^a + 6*b^2*d*e^a*
sin(6*c) + 24*d^3*e^a*sin(6*c))*e^(b*x)*sin(6*d*x) - (b^3*cos(6*c)*e^a + 4
*b*d^2*cos(6*c)*e^a - 6*b^2*d*e^a*sin(6*c) - 24*d^3*e^a*sin(6*c))*e^(b*x)*
sin(6*d*x + 12*c) + 3*(b^3*cos(6*c)*e^a + 36*b*d^2*cos(6*c)*e^a - 2*b^2*d*
e^a*sin(6*c) - 72*d^3*e^a*sin(6*c))*e^(b*x)*sin(2*d*x + 8*c) + 3*(b^3*cos(
6*c)*e^a + 36*b*d^2*cos(6*c)*e^a + 2*b^2*d*e^a*sin(6*c) + 72*d^3*e^a*sin(6
*c))*e^(b*x)*sin(2*d*x - 4*c))/(b^4*cos(6*c)^2 + b^4*sin(6*c)^2 + 144*(cos
(6*c)^2 + sin(6*c)^2)*d^4 + 40*(b^2*cos(6*c)^2 + b^2*sin(6*c)^2)*d^2)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

$$\int e^{a+bx}\cos^3(c+dx)\sin^3(c+dx)\,dx = \frac{1}{32}\left(\frac{6\,d\cos\left(6\,dx+6\,c\right)}{b^2+36\,d^2}-\frac{b\sin\left(6\,dx+6\,c\right)}{b^2+36\,d^2}\right)e^{(bx+a)} - \frac{3}{32}\left(\frac{2\,d\cos\left(2\,dx+2\,c\right)}{b^2+4\,d^2}-\frac{b\sin\left(2\,dx+2\,c\right)}{b^2+4\,d^2}\right)e^{(bx+a)}$$

input
```
integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^3,x, algorithm="giac")
```

output
```
1/32*(6*d*cos(6*d*x + 6*c)/(b^2 + 36*d^2) - b*sin(6*d*x + 6*c)/(b^2 + 36*d
^2))*e^(b*x + a) - 3/32*(2*d*cos(2*d*x + 2*c)/(b^2 + 4*d^2) - b*sin(2*d*x
+ 2*c)/(b^2 + 4*d^2))*e^(b*x + a)
```

3.46.9 Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.38

$$\int e^{a+bx}\cos^3(c+dx)\sin^3(c+dx)\,dx$$
$$= -\frac{3\,e^{a+b\,x}\,(\cos(2\,d\,x)-\sin(2\,d\,x)\;1\mathrm{i})\;(\cos(2\,c)-\sin(2\,c)\;1\mathrm{i})}{64\;(2\,d+b\,1\mathrm{i})}$$
$$+\frac{e^{a+b\,x}\,(\cos(6\,d\,x)-\sin(6\,d\,x)\;1\mathrm{i})\;(\cos(6\,c)-\sin(6\,c)\;1\mathrm{i})}{64\;(6\,d+b\,1\mathrm{i})}$$
$$-\frac{e^{a+b\,x}\,(\cos(2\,d\,x)+\sin(2\,d\,x)\;1\mathrm{i})\;(\cos(2\,c)+\sin(2\,c)\;1\mathrm{i})\;3\mathrm{i}}{64\;(b+d\,2\mathrm{i})}$$
$$+\frac{e^{a+b\,x}\,(\cos(6\,d\,x)+\sin(6\,d\,x)\;1\mathrm{i})\;(\cos(6\,c)+\sin(6\,c)\;1\mathrm{i})\;1\mathrm{i}}{64\;(b+d\,6\mathrm{i})}$$

input
```
int(cos(c + d*x)^3*exp(a + b*x)*sin(c + d*x)^3,x)
```

output
```
(exp(a + b*x)*(cos(6*d*x) - sin(6*d*x)*1i)*(cos(6*c) - sin(6*c)*1i))/(64*(
b*1i + 6*d)) - (3*exp(a + b*x)*(cos(2*d*x) - sin(2*d*x)*1i)*(cos(2*c) - si
n(2*c)*1i))/(64*(b*1i + 2*d)) - (exp(a + b*x)*(cos(2*d*x) + sin(2*d*x)*1i)
*(cos(2*c) + sin(2*c)*1i)*3i)/(64*(b + d*2i)) + (exp(a + b*x)*(cos(6*d*x)
+ sin(6*d*x)*1i)*(cos(6*c) + sin(6*c)*1i)*1i)/(64*(b + d*6i))
```

3.47 $\int e^x x \sin(x)\,dx$

3.47.1 Optimal result

Integrand size = 7, antiderivative size = 30

$$\int e^x x \sin(x)\,dx = \frac{1}{2}e^x \cos(x) - \frac{1}{2}e^x x \cos(x) + \frac{1}{2}e^x x \sin(x)$$

output
```
1/2*exp(x)*cos(x)-1/2*exp(x)*x*cos(x)+1/2*exp(x)*x*sin(x)
```

3.47.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

$$\int e^x x \sin(x)\,dx = \frac{1}{2}e^x(\cos(x) - x\cos(x) + x\sin(x))$$

input
```
Integrate[E^x*x*Sin[x],x]
```

output
```
(E^x*(Cos[x] - x*Cos[x] + x*Sin[x]))/2
```

3.47.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4968, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x x \sin(x)\,dx$$

$$\Big\downarrow \text{ 4968}$$

$$-\int \left(\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)\right)dx + \frac{1}{2}e^x x \sin(x) - \frac{1}{2}e^x x \cos(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x \cos(x) - \frac{1}{2}e^x x \cos(x)$$

input
```
Int[E^x*x*Sin[x],x]
```

output
```
(E^x*Cos[x])/2 - (E^x*x*Cos[x])/2 + (E^x*x*Sin[x])/2
```

3.47.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4968
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

3.47.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

method	result	size
parallelrisch	$-\frac{((x-1)\cos(x)-x\sin(x))\mathrm{e}^{x}}{2}$	17
default	$\left(-\frac{x}{2}+\frac{1}{2}\right)\mathrm{e}^{x}\cos\left(x\right)+\frac{\mathrm{e}^{x}x\sin(x)}{2}$	19
risch	$\left(-\frac{1}{8}-\frac{i}{8}\right)\left(-1+i+2x\right)\mathrm{e}^{(1+i)x}+\left(-\frac{1}{8}+\frac{i}{8}\right)\left(-1-i+2x\right)\mathrm{e}^{(1-i)x}$	36
norman	$\frac{\mathrm{e}^{x}x\tan\left(\frac{x}{2}\right)-\frac{\mathrm{e}^{x}x}{2}-\frac{\mathrm{e}^{x}\tan\left(\frac{x}{2}\right)^{2}}{2}+\frac{\mathrm{e}^{x}x\tan\left(\frac{x}{2}\right)^{2}}{2}+\frac{\mathrm{e}^{x}}{2}}{1+\tan\left(\frac{x}{2}\right)^{2}}$	51

input
```
int(exp(x)*x*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((x-1)*cos(x)-x*sin(x))*exp(x)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^{x}x\sin(x)\,dx=-\frac{1}{2}\left(x-1\right)\cos\left(x\right)e^{x}+\frac{1}{2}\,xe^{x}\sin\left(x\right)$$

input
```
integrate(exp(x)*x*sin(x),x, algorithm="fricas")
```

output
```
-1/2*(x - 1)*cos(x)*e^x + 1/2*x*e^x*sin(x)
```

3.47.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int e^{x}x\sin(x)\,dx=\frac{xe^{x}\sin\left(x\right)}{2}-\frac{xe^{x}\cos\left(x\right)}{2}+\frac{e^{x}\cos\left(x\right)}{2}$$

input
```
integrate(exp(x)*x*sin(x),x)
```

output
```
x*exp(x)*sin(x)/2 - x*exp(x)*cos(x)/2 + exp(x)*cos(x)/2
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \sin(x)\, dx = -\frac{1}{2}\,(x-1)\cos{(x)}\,e^x + \frac{1}{2}\,xe^x\sin{(x)}$$

input
```
integrate(exp(x)*x*sin(x),x, algorithm="maxima")
```

output
```
-1/2*(x - 1)*cos(x)*e^x + 1/2*x*e^x*sin(x)
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.53

$$\int e^x x \sin(x)\, dx = -\frac{1}{2}\,((x-1)\cos{(x)} - x\sin{(x)})e^x$$

input
```
integrate(exp(x)*x*sin(x),x, algorithm="giac")
```

output
```
-1/2*((x - 1)*cos(x) - x*sin(x))*e^x
```

3.47.9 Mupad [B] (verification not implemented)

Time = 26.39 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.53

$$\int e^x x \sin(x)\, dx = \frac{\mathrm{e}^x\,(\cos{(x)} - x\,\cos{(x)} + x\,\sin{(x)})}{2}$$

input
```
int(x*exp(x)*sin(x),x)
```

output
```
(exp(x)*(cos(x) - x*cos(x) + x*sin(x)))/2
```

3.48 $\int e^x x^2 \sin(x)\,dx$

3.48.1 Optimal result

Integrand size = 9, antiderivative size = 50

$$\int e^x x^2 \sin(x)\,dx = -\frac{1}{2}e^x\cos(x) + e^x x\cos(x) - \frac{1}{2}e^x x^2\cos(x) - \frac{1}{2}e^x\sin(x) + \frac{1}{2}e^x x^2\sin(x)$$

output
```
-1/2*exp(x)*cos(x)+exp(x)*x*cos(x)-1/2*exp(x)*x^2*cos(x)-1/2*exp(x)*sin(x)
+1/2*exp(x)*x^2*sin(x)
```

3.48.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.50

$$\int e^x x^2 \sin(x)\,dx = \frac{1}{2}e^x\left(-(-1+x)^2\cos(x) + \left(-1+x^2\right)\sin(x)\right)$$

input
```
Integrate[E^x*x^2*Sin[x],x]
```

output
```
(E^x*(-((-1 + x)^2*Cos[x]) + (-1 + x^2)*Sin[x]))/2
```

3.48.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4968, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x x^2 \sin(x) \, dx$$

$$\Big\downarrow \text{4968}$$

$$-2 \int -\frac{1}{2} x (e^x \cos(x) - e^x \sin(x)) \, dx + \frac{1}{2} e^x x^2 \sin(x) - \frac{1}{2} e^x x^2 \cos(x)$$

$$\Big\downarrow \text{27}$$

$$\int x (e^x \cos(x) - e^x \sin(x)) \, dx + \frac{1}{2} e^x x^2 \sin(x) - \frac{1}{2} e^x x^2 \cos(x)$$

$$\Big\downarrow \text{2010}$$

$$\int (e^x x \cos(x) - e^x x \sin(x)) \, dx + \frac{1}{2} e^x x^2 \sin(x) - \frac{1}{2} e^x x^2 \cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} e^x x^2 \sin(x) - \frac{1}{2} e^x x^2 \cos(x) - \frac{1}{2} e^x \sin(x) + e^x x \cos(x) - \frac{1}{2} e^x \cos(x)$$

input
```
Int[E^x*x^2*Sin[x],x]
```

output
```
-1/2*(E^x*Cos[x]) + E^x*x*Cos[x] - (E^x*x^2*Cos[x])/2 - (E^x*Sin[x])/2 + (
E^x*x^2*Sin[x])/2
```

3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4968
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

3.48.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.46

method	result	size
parallelrisch	$-\frac{(x-1)\,\mathrm{e}^{x}\left((-x-1)\sin\left(x\right)+(x-1)\cos\left(x\right)\right)}{2}$	23
default	$\left(-\frac{1}{2}x^{2}+x-\frac{1}{2}\right)\mathrm{e}^{x}\cos\left(x\right)+\left(\frac{x^{2}}{2}-\frac{1}{2}\right)\mathrm{e}^{x}\sin\left(x\right)$	27
risch	$\left(-\frac{1}{4}-\frac{i}{4}\right)\left(x^{2}+ix-x-i\right)\mathrm{e}^{(1+i)x}+\left(-\frac{1}{4}+\frac{i}{4}\right)\left(x^{2}-ix-x+i\right)\mathrm{e}^{(1-i)x}$	48
norman	$\frac{\mathrm{e}^{x}x+\mathrm{e}^{x}x^{2}\tan\left(\frac{x}{2}\right)-\frac{\mathrm{e}^{x}x^{2}}{2}-\mathrm{e}^{x}\tan\left(\frac{x}{2}\right)+\frac{\mathrm{e}^{x}\tan\left(\frac{x}{2}\right)^{2}}{2}-\mathrm{e}^{x}x\tan\left(\frac{x}{2}\right)^{2}+\frac{\mathrm{e}^{x}x^{2}\tan\left(\frac{x}{2}\right)^{2}}{2}-\frac{\mathrm{e}^{x}}{2}}{1+\tan\left(\frac{x}{2}\right)^{2}}$	80

input
```
int(exp(x)*x^2*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x-1)*exp(x)*((-x-1)*sin(x)+(x-1)*cos(x))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int e^{x}x^{2}\sin(x)\,dx=-\frac{1}{2}\left(x^{2}-2\,x+1\right)\cos\left(x\right)e^{x}+\frac{1}{2}\left(x^{2}-1\right)e^{x}\sin\left(x\right)$$

input
```
integrate(exp(x)*x^2*sin(x),x, algorithm="fricas")
```

output
```
-1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

3.48.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int e^x x^2 \sin(x)\,dx = \frac{x^2 e^x \sin(x)}{2} - \frac{x^2 e^x \cos(x)}{2} + x e^x \cos(x) - \frac{e^x \sin(x)}{2} - \frac{e^x \cos(x)}{2}$$

input
```
integrate(exp(x)*x**2*sin(x),x)
```

output
```
x**2*exp(x)*sin(x)/2 - x**2*exp(x)*cos(x)/2 + x*exp(x)*cos(x) - exp(x)*sin
(x)/2 - exp(x)*cos(x)/2
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int e^x x^2 \sin(x)\,dx = -\frac{1}{2}\left(x^2 - 2x + 1\right)\cos(x)\,e^x + \frac{1}{2}\left(x^2 - 1\right)e^x \sin(x)$$

input
```
integrate(exp(x)*x^2*sin(x),x, algorithm="maxima")
```

output
```
-1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.50

$$\int e^x x^2 \sin(x)\,dx = -\frac{1}{2}\left(\left(x^2 - 2x + 1\right)\cos(x) - \left(x^2 - 1\right)\sin(x)\right)e^x$$

input
```
integrate(exp(x)*x^2*sin(x),x, algorithm="giac")
```

output
```
-1/2*((x^2 - 2*x + 1)*cos(x) - (x^2 - 1)*sin(x))*e^x
```

3.48.9 Mupad [B] (verification not implemented)

Time = 27.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.42

$$\int e^x x^2 \sin(x)\,dx = \frac{e^x\,(x-1)\,\left(\cos\left(x\right)+\sin\left(x\right)-x\,\cos\left(x\right)+x\,\sin\left(x\right)\right)}{2}$$

input
```
int(x^2*exp(x)*sin(x),x)
```

output
```
(exp(x)*(x - 1)*(cos(x) + sin(x) - x*cos(x) + x*sin(x)))/2
```

3.49 $\int e^x x \cos(x)\, dx$

3.49.1 Optimal result

Integrand size = 7, antiderivative size = 30

$$\int e^x x \cos(x)\, dx = \frac{1}{2}e^x x \cos(x) - \frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x)$$

output
```
1/2*exp(x)*x*cos(x)-1/2*exp(x)*sin(x)+1/2*exp(x)*x*sin(x)
```

3.49.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int e^x x \cos(x)\, dx = \frac{1}{2}e^x(x \cos(x) + (-1 + x) \sin(x))$$

input
```
Integrate[E^x*x*Cos[x],x]
```

output
```
(E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2
```

3.49.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4969, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x x \cos(x) \, dx$$

$$\Big\downarrow \text{4969}$$

$$-\int \left(\frac{1}{2}e^x \cos(x)+\frac{1}{2}e^x \sin(x)\right)dx+\frac{1}{2}e^x x \sin(x)+\frac{1}{2}e^x x \cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}e^x \sin(x)+\frac{1}{2}e^x x \sin(x)+\frac{1}{2}e^x x \cos(x)$$

input
```
Int[E^x*x*Cos[x],x]
```

output
```
(E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

3.49.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4969
```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

3.49.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.53

method	result	size
parallelrisch	$\frac{e^{x}\left(\left(x-1\right)\sin\left(x\right)+x\cos\left(x\right)\right)}{2}$	16
default	$\frac{e^{x}x\cos\left(x\right)}{2}-\left(-\frac{x}{2}+\frac{1}{2}\right)e^{x}\sin\left(x\right)$	20
risch	$\left(\frac{1}{8}-\frac{i}{8}\right)\left(-1+i+2x\right)e^{\left(1+i\right)x}+\left(\frac{1}{8}+\frac{i}{8}\right)\left(-1-i+2x\right)e^{\left(1-i\right)x}$	36
norman	$\frac{e^{x}x\tan\left(\frac{x}{2}\right)+\frac{e^{x}x}{2}-e^{x}\tan\left(\frac{x}{2}\right)-\frac{e^{x}x\tan\left(\frac{x}{2}\right)^{2}}{2}}{1+\tan\left(\frac{x}{2}\right)^{2}}$	45

input
```
int(exp(x)*x*cos(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)*((x-1)*sin(x)+x*cos(x))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^{x}x\cos(x)\,dx=\frac{1}{2}\,x\cos\left(x\right)e^{x}+\frac{1}{2}\,(x-1)e^{x}\sin\left(x\right)$$

input
```
integrate(exp(x)*x*cos(x),x, algorithm="fricas")
```

output
```
1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

3.49.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int e^{x}x\cos(x)\,dx=\frac{xe^{x}\sin\left(x\right)}{2}+\frac{xe^{x}\cos\left(x\right)}{2}-\frac{e^{x}\sin\left(x\right)}{2}$$

input
```
integrate(exp(x)*x*cos(x),x)
```

output
```
x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^{x}x\cos(x)\,dx = \frac{1}{2}\,x\cos\left(x\right)e^{x} + \frac{1}{2}\,(x-1)e^{x}\sin\left(x\right)$$

input
```
integrate(exp(x)*x*cos(x),x, algorithm="maxima")
```

output
```
1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.50

$$\int e^{x}x\cos(x)\,dx = \frac{1}{2}\,(x\cos\left(x\right) + (x-1)\sin\left(x\right))e^{x}$$

input
```
integrate(exp(x)*x*cos(x),x, algorithm="giac")
```

output
```
1/2*(x*cos(x) + (x - 1)*sin(x))*e^x
```

3.49.9 Mupad [B] (verification not implemented)

Time = 27.90 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^{x}x\cos(x)\,dx = \frac{\mathrm{e}^{x}\,\left(x\,\cos\left(x\right)-\sin\left(x\right)+x\,\sin\left(x\right)\right)}{2}$$

input
```
int(x*exp(x)*cos(x),x)
```

output
```
(exp(x)*(x*cos(x) - sin(x) + x*sin(x)))/2
```

3.50 $\int e^x x^2 \cos(x) \, dx$

3.50.1 Optimal result

Integrand size = 9, antiderivative size = 51

$$\int e^x x^2 \cos(x) \, dx = -\frac{1}{2} e^x \cos(x) + \frac{1}{2} e^x x^2 \cos(x) + \frac{1}{2} e^x \sin(x) - e^x x \sin(x) + \frac{1}{2} e^x x^2 \sin(x)$$

output
```
-1/2*exp(x)*cos(x)+1/2*exp(x)*x^2*cos(x)+1/2*exp(x)*sin(x)-exp(x)*x*sin(x)
+1/2*exp(x)*x^2*sin(x)
```

3.50.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.45

$$\int e^x x^2 \cos(x) \, dx = \frac{1}{2} e^x (-1 + x)((1 + x) \cos(x) + (-1 + x) \sin(x))$$

input
```
Integrate[E^x*x^2*Cos[x],x]
```

output
```
(E^x*(-1 + x)*((1 + x)*Cos[x] + (-1 + x)*Sin[x]))/2
```

3.50.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4969, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x x^2 \cos(x) \, dx$$

$$\Big\downarrow \text{ 4969}$$

$$-2 \int \frac{1}{2} x (e^x \cos(x) + e^x \sin(x)) \, dx + \frac{1}{2} e^x x^2 \sin(x) + \frac{1}{2} e^x x^2 \cos(x)$$

$$\Big\downarrow \text{ 27}$$

$$-\int x (e^x \cos(x) + e^x \sin(x)) \, dx + \frac{1}{2} e^x x^2 \sin(x) + \frac{1}{2} e^x x^2 \cos(x)$$

$$\Big\downarrow \text{ 2010}$$

$$-\int (e^x x \cos(x) + e^x x \sin(x)) \, dx + \frac{1}{2} e^x x^2 \sin(x) + \frac{1}{2} e^x x^2 \cos(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} e^x x^2 \sin(x) + \frac{1}{2} e^x x^2 \cos(x) - e^x x \sin(x) + \frac{1}{2} e^x \sin(x) - \frac{1}{2} e^x \cos(x)$$

input
```
Int[E^x*x^2*Cos[x],x]
```

output
```
-1/2*(E^x*Cos[x]) + (E^x*x^2*Cos[x])/2 + (E^x*Sin[x])/2 - E^x*x*Sin[x] + (
E^x*x^2*Sin[x])/2
```

3.50.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4969
```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

3.50.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.41

method	result	size
parallelrisch	$\frac{(x-1)((x+1)\cos(x)+(x-1)\sin(x))\mathrm{e}^{x}}{2}$	21
default	$\left(\frac{x^{2}}{2}-\frac{1}{2}\right)\mathrm{e}^{x}\cos\left(x\right)-\left(-\frac{1}{2}x^{2}+x-\frac{1}{2}\right)\mathrm{e}^{x}\sin\left(x\right)$	28
risch	$\left(\frac{1}{4}-\frac{i}{4}\right)\left(x^{2}+ix-x-i\right)\mathrm{e}^{(1+i)x}+\left(\frac{1}{4}+\frac{i}{4}\right)\left(x^{2}-ix-x+i\right)\mathrm{e}^{(1-i)x}$	48
norman	$\frac{\mathrm{e}^{x}\tan\left(\frac{x}{2}\right)+\mathrm{e}^{x}x^{2}\tan\left(\frac{x}{2}\right)+\frac{\mathrm{e}^{x}x^{2}}{2}+\frac{\mathrm{e}^{x}\tan\left(\frac{x}{2}\right)^{2}}{2}-2\,\mathrm{e}^{x}x\tan\left(\frac{x}{2}\right)-\frac{\mathrm{e}^{x}x^{2}\tan\left(\frac{x}{2}\right)^{2}}{2}-\frac{\mathrm{e}^{x}}{2}}{1+\tan\left(\frac{x}{2}\right)^{2}}$	73

input
```
int(exp(x)*x^2*cos(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x-1)*((x+1)*cos(x)+(x-1)*sin(x))*exp(x)
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.51

$$\int e^{x}x^{2}\cos(x)\,dx=\frac{1}{2}\left(x^{2}-1\right)\cos\left(x\right)e^{x}+\frac{1}{2}\left(x^{2}-2\,x+1\right)e^{x}\sin\left(x\right)$$

input
```
integrate(exp(x)*x^2*cos(x),x, algorithm="fricas")
```

output
```
1/2*(x^2 - 1)*cos(x)*e^x + 1/2*(x^2 - 2*x + 1)*e^x*sin(x)
```

3.50.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int e^{x}x^{2}\cos(x)\,dx=\frac{x^{2}e^{x}\sin{(x)}}{2}+\frac{x^{2}e^{x}\cos{(x)}}{2}-xe^{x}\sin{(x)}+\frac{e^{x}\sin{(x)}}{2}-\frac{e^{x}\cos{(x)}}{2}$$

input
```
integrate(exp(x)*x**2*cos(x),x)
```

output
```
x**2*exp(x)*sin(x)/2 + x**2*exp(x)*cos(x)/2 - x*exp(x)*sin(x) + exp(x)*sin
(x)/2 - exp(x)*cos(x)/2
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.51

$$\int e^{x}x^{2}\cos(x)\,dx=\frac{1}{2}\left(x^{2}-1\right)\cos{(x)}\,e^{x}+\frac{1}{2}\left(x^{2}-2\,x+1\right)e^{x}\sin{(x)}$$

input
```
integrate(exp(x)*x^2*cos(x),x, algorithm="maxima")
```

output
```
1/2*(x^2 - 1)*cos(x)*e^x + 1/2*(x^2 - 2*x + 1)*e^x*sin(x)
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.47

$$\int e^{x}x^{2}\cos(x)\,dx=\frac{1}{2}\left(\left(x^{2}-1\right)\cos{(x)}+\left(x^{2}-2\,x+1\right)\sin{(x)}\right)e^{x}$$

input
```
integrate(exp(x)*x^2*cos(x),x, algorithm="giac")
```

output
```
1/2*((x^2 - 1)*cos(x) + (x^2 - 2*x + 1)*sin(x))*e^x
```

3.50.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.43

$$\int e^x x^2 \cos(x)\,dx = \frac{e^x\,(x-1)\,\left(\cos(x)-\sin(x)+x\,\cos(x)+x\,\sin(x)\right)}{2}$$

input
```
int(x^2*exp(x)*cos(x),x)
```

output
```
(exp(x)*(x - 1)*(cos(x) - sin(x) + x*cos(x) + x*sin(x)))/2
```

3.51 $\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx$

3.51.1 Optimal result

Integrand size = 19, antiderivative size = 27

$$\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx=-\frac{23}{25}e^{3x}\cos(4x)-\frac{14}{25}e^{3x}\sin(4x)$$

output
```
-23/25*exp(3*x)*cos(4*x)-14/25*exp(3*x)*sin(4*x)
```

3.51.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx=-\frac{1}{25}e^{3x}(23\cos(4x)+14\sin(4x))$$

input
```
Integrate[E^(3*x)*(-5*Cos[4*x] + 2*Sin[4*x]),x]
```

output
```
-1/25*(E^(3*x)*(23*Cos[4*x] + 14*Sin[4*x]))
```

3.51.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3x}(2\sin(4x)-5\cos(4x))\,dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(2e^{3x}\sin(4x)-5e^{3x}\cos(4x)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{14}{25}e^{3x}\sin(4x)-\frac{23}{25}e^{3x}\cos(4x)$$

input
```
Int[E^(3*x)*(-5*Cos[4*x] + 2*Sin[4*x]),x]
```

output
```
(-23*E^(3*x)*Cos[4*x])/25 - (14*E^(3*x)*Sin[4*x])/25
```

3.51.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.51.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result
parallelrisch	$-\frac{e^{3x}(23\cos(4x)+14\sin(4x))}{25}$
parts	$-\frac{23\,e^{3x}\cos(4x)}{25}-\frac{14\,e^{3x}\sin(4x)}{25}$
risch	$-\frac{23\,e^{(3+4i)x}}{50}+\frac{7i e^{(3+4i)x}}{25}-\frac{23\,e^{(3-4i)x}}{50}-\frac{7i e^{(3-4i)x}}{25}$
norman	$\frac{-\frac{28\,e^{3x}\tan(2x)}{25}+\frac{23\,e^{3x}\tan(2x)^2}{25}-\frac{23\,e^{3x}}{25}}{1+\tan(2x)^2}$
default	$-\frac{8(3\cos(x)+4\sin(x))e^{3x}\cos(x)^3}{5}+\frac{8(3\cos(x)+2\sin(x))e^{3x}\cos(x)}{5}-\frac{3\,e^{3x}}{5}-\frac{8\,e^{3x}\cos(4x)}{25}+\frac{6\,e^{3x}\sin(4x)}{25}-\frac{8\,e^{3x}\,\mathrm{co}}{13}$

input
```
int(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/25*exp(3*x)*(23*cos(4*x)+14*sin(4*x))
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx=-\frac{23}{25}\cos(4x)\,e^{(3x)}-\frac{14}{25}\,e^{(3x)}\sin(4x)$$

input
```
integrate(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x, algorithm="fricas")
```

output
```
-23/25*cos(4*x)*e^(3*x) - 14/25*e^(3*x)*sin(4*x)
```

3.51.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx=-\frac{14e^{3x}\sin(4x)}{25}-\frac{23e^{3x}\cos(4x)}{25}$$

input
```
integrate(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x)
```

output
```
-14*exp(3*x)*sin(4*x)/25 - 23*exp(3*x)*cos(4*x)/25
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx = -\frac{2}{25}\,(4\,\cos(4\,x)-3\,\sin(4\,x))e^{(3\,x)} - \frac{1}{5}\,(3\,\cos(4\,x)+4\,\sin(4\,x))e^{(3\,x)}$$

input
```
integrate(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x, algorithm="maxima")
```

output
```
-2/25*(4*cos(4*x) - 3*sin(4*x))*e^(3*x) - 1/5*(3*cos(4*x) + 4*sin(4*x))*e^
(3*x)
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx = -\frac{2}{25}\,(4\,\cos(4\,x)-3\,\sin(4\,x))e^{(3\,x)} - \frac{1}{5}\,(3\,\cos(4\,x)+4\,\sin(4\,x))e^{(3\,x)}$$

input
```
integrate(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x, algorithm="giac")
```

output
```
-2/25*(4*cos(4*x) - 3*sin(4*x))*e^(3*x) - 1/5*(3*cos(4*x) + 4*sin(4*x))*e^
(3*x)
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx = -\frac{\mathrm{e}^{3\,x}\,(23\,\cos(4\,x)+14\,\sin(4\,x))}{25}$$

input
```
int(-exp(3*x)*(5*cos(4*x) - 2*sin(4*x)),x)
```

output
```
-(exp(3*x)*(23*cos(4*x) + 14*sin(4*x)))/25
```

3.52 $\int \left(e^{-x} \sin(x)+e^x \sin(x)\right) \, dx$

3.52.1 Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \left(e^{-x} \sin(x)+e^x \sin(x)\right) \, dx=-\frac{1}{2} e^{-x} \cos(x)-\frac{1}{2} e^x \cos(x)-\frac{1}{2} e^{-x} \sin(x)+\frac{1}{2} e^x \sin(x)$$

output
```
-1/2*cos(x)/exp(x)-1/2*exp(x)*cos(x)-1/2*sin(x)/exp(x)+1/2*exp(x)*sin(x)
```

3.52.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \left(e^{-x} \sin(x)+e^x \sin(x)\right) \, dx=-\frac{1}{2} e^x \left(1+e^{-2x}\right) \cos(x)-\frac{1}{2} e^x \left(-1+e^{-2x}\right) \sin(x)$$

input
```
Integrate[Sin[x]/E^x + E^x*Sin[x],x]
```

output
```
-1/2*(E^x*(1 + E^(-2*x))*Cos[x]) - (E^x*(-1 + E^(-2*x))*Sin[x])/2
```

3.52.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^{-x}\sin(x)+e^{x}\sin(x)\right)\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{2}e^{-x}\sin(x)+\frac{1}{2}e^{x}\sin(x)-\frac{1}{2}e^{-x}\cos(x)-\frac{1}{2}e^{x}\cos(x)$$

input
```
Int[Sin[x]/E^x + E^x*Sin[x],x]
```

output
```
-1/2*Cos[x]/E^x - (E^x*Cos[x])/2 - Sin[x]/(2*E^x) + (E^x*Sin[x])/2
```

3.52.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.52.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{(-\cos(x)-\sin(x))\,\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}(\cos(x)-\sin(x))}{2}$	28
default	$-\frac{\mathrm{e}^{x}\cos(x)}{2}+\frac{\mathrm{e}^{x}\sin(x)}{2}-\frac{\mathrm{e}^{-x}\cos(x)}{2}-\frac{\mathrm{e}^{-x}\sin(x)}{2}$	30
parts	$-\frac{\mathrm{e}^{x}\cos(x)}{2}+\frac{\mathrm{e}^{x}\sin(x)}{2}-\frac{\mathrm{e}^{-x}\cos(x)}{2}-\frac{\mathrm{e}^{-x}\sin(x)}{2}$	30
norman	$\frac{\left(-\frac{1}{2}+\mathrm{e}^{2x}\tan\left(\frac{x}{2}\right)-\frac{\mathrm{e}^{2x}}{2}+\frac{\tan\left(\frac{x}{2}\right)^{2}}{2}+\frac{\mathrm{e}^{2x}\tan\left(\frac{x}{2}\right)^{2}}{2}-\tan\left(\frac{x}{2}\right)\right)\mathrm{e}^{-x}}{1+\tan\left(\frac{x}{2}\right)^{2}}$	59
risch	$-\frac{\mathrm{e}^{(-1+i)x}}{4}+\frac{i\mathrm{e}^{(-1+i)x}}{4}-\frac{\mathrm{e}^{(-1-i)x}}{4}-\frac{i\mathrm{e}^{(-1-i)x}}{4}-\frac{\mathrm{e}^{(1+i)x}}{4}-\frac{i\mathrm{e}^{(1+i)x}}{4}-\frac{\mathrm{e}^{(1-i)x}}{4}+\frac{i\mathrm{e}^{(1-i)x}}{4}$	70

input
```
int(sin(x)/exp(x)+exp(x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-cos(x)-sin(x))*exp(-x)-1/2*exp(x)*(cos(x)-sin(x))
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \left(e^{-x}\sin(x)+e^{x}\sin(x)\right)\,dx=-\frac{1}{2}\left(\cos\left(x\right)e^{\left(2\,x\right)}-\left(e^{\left(2\,x\right)}-1\right)\sin\left(x\right)+\cos\left(x\right)\right)e^{\left(-x\right)}$$

input
```
integrate(sin(x)/exp(x)+exp(x)*sin(x),x, algorithm="fricas")
```

output
```
-1/2*(cos(x)*e^(2*x) - (e^(2*x) - 1)*sin(x) + cos(x))*e^(-x)
```

3.52.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \left(e^{-x}\sin(x)+e^{x}\sin(x)\right)\,dx=\frac{e^{x}\sin\left(x\right)}{2}-\frac{e^{x}\cos\left(x\right)}{2}-\frac{e^{-x}\sin\left(x\right)}{2}-\frac{e^{-x}\cos\left(x\right)}{2}$$

input
```
integrate(sin(x)/exp(x)+exp(x)*sin(x),x)
```

output
```
exp(x)*sin(x)/2 - exp(x)*cos(x)/2 - exp(-x)*sin(x)/2 - exp(-x)*cos(x)/2
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \left(e^{-x}\sin(x)+e^{x}\sin(x)\right)\,dx=-\frac{1}{2}\left(\cos\left(x\right)+\sin\left(x\right)\right)e^{\left(-x\right)}-\frac{1}{2}\left(\cos\left(x\right)-\sin\left(x\right)\right)e^{x}$$

input
```
integrate(sin(x)/exp(x)+exp(x)*sin(x),x, algorithm="maxima")
```

output
```
-1/2*(cos(x) + sin(x))*e^(-x) - 1/2*(cos(x) - sin(x))*e^x
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \left(e^{-x}\sin(x)+e^{x}\sin(x)\right)\,dx=-\frac{1}{2}\left(\cos\left(x\right)+\sin\left(x\right)\right)e^{(-x)}-\frac{1}{2}\left(\cos\left(x\right)-\sin\left(x\right)\right)e^{x}$$

input
```
integrate(sin(x)/exp(x)+exp(x)*sin(x),x, algorithm="giac")
```

output
```
-1/2*(cos(x) + sin(x))*e^(-x) - 1/2*(cos(x) - sin(x))*e^x
```

3.52.9 Mupad [B] (verification not implemented)

Time = 27.97 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \left(e^{-x}\sin(x)+e^{x}\sin(x)\right)\,dx=-\mathrm{e}^{-x}\left(\frac{\cos\left(x\right)}{2}+\frac{\sin\left(x\right)}{2}+\frac{\mathrm{e}^{2\,x}\,\cos\left(x\right)}{2}-\frac{\mathrm{e}^{2\,x}\,\sin\left(x\right)}{2}\right)$$

input
```
int(exp(x)*sin(x) + exp(-x)*sin(x),x)
```

output
```
-exp(-x)*(cos(x)/2 + sin(x)/2 + (exp(2*x)*cos(x))/2 - (exp(2*x)*sin(x))/2)
```

3.53 $\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx$

3.53.1 Optimal result

Integrand size = 26, antiderivative size = 82

$$\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx = \frac{iF^{a+bx}}{be\log(F)} - \frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},ie^{i(c+dx)}\right)}{be\log(F)}$$

output
```
I*F^(b*x+a)/b/e/ln(F)-2*I*F^(b*x+a)*hypergeom([1, -I*b*ln(F)/d],[1-I*b*ln(
F)/d],I*exp(I*(d*x+c)))/b/e/ln(F)
```

3.53.2 Mathematica [A] (verified)

Time = 3.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx = -\frac{iF^{a+bx}\left(-1+2\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},ie^{i(c+dx)}\right)\right)}{be\log(F)}$$

input
```
Integrate[(F^(a + b*x)*Cos[c + d*x])/(e + e*Sin[c + d*x]),x]
```

output
```
((-I)*F^(a + b*x)*(-1 + 2*Hypergeometric2F1[1, ((-I)*b*Log[F])/d, 1 - (I*b
*Log[F])/d, I*E^(I*(c + d*x))]))/(b*e*Log[F])
```

3.53.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {4962, 4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+bx}\cos(c+dx)}{e\sin(c+dx)+e}dx$$

$$\Big\downarrow \text{ 4962}$$

$$\frac{\int F^{a+bx}\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{e}$$

$$\Big\downarrow \text{ 4943}$$

$$-\frac{i\int \left(\frac{2F^{a+bx}}{1-e^{\frac{1}{2}i(2c+2dx+\pi)}}-F^{a+bx}\right)dx}{e}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{i\left(-\frac{F^{a+bx}}{b\log(F)}+\frac{2F^{a+bx}\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},e^{\frac{1}{2}i(2c+2dx+\pi)}\right)}{b\log(F)}\right)}{e}$$

input
```
Int[(F^(a + b*x)*Cos[c + d*x])/(e + e*Sin[c + d*x]),x]
```

output
```
((-I)*(-(F^(a + b*x)/(b*Log[F])) + (2*F^(a + b*x)*Hypergeometric2F1[1, ((-
I)*b*Log[F])/d, 1 - (I*b*Log[F])/d, E^((I/2)*(2*c + Pi + 2*d*x))])/(b*Log[
F])))/e
```

3.53.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

rule 4962
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Simp[g^n   Int[F^(c*(a
+ b*x))*Tan[f*(Pi/(4*g)) - d/2 - e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d
, e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

3.53.4 Maple [F]

$$\int \frac{F^{xb+a}\cos(dx+c)}{e+e\sin(dx+c)}dx$$

input
```
int(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x)
```

output
```
int(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x)
```

3.53.5 Fricas [F]

$$\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx = \int \frac{F^{bx+a}\cos(dx+c)}{e\sin(dx+c)+e}\,dx$$

input
```
integrate(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(F^(b*x + a)*cos(d*x + c)/(e*sin(d*x + c) + e), x)
```

3.53.6 Sympy [F]

$$\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx = \frac{\int \frac{F^{a+bx}\cos(c+dx)}{\sin(c+dx)+1}\,dx}{e}$$

input
```
integrate(F**(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x)
```

output
```
Integral(F**(a + b*x)*cos(c + d*x)/(sin(c + d*x) + 1), x)/e
```

3.53.7 Maxima [F]

$$\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx = \int \frac{F^{bx+a}\cos{(dx+c)}}{e\sin{(dx+c)}+e}\,dx$$

input
```
integrate(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-(2*F^(b*x)*F^a*b*d*cos(d*x + c)*log(F) + 2*F^(b*x)*F^a*d^2*sin(d*x + c) +
 (F^a*b^2*log(F)^2 + F^a*d^2)*F^(b*x)*cos(d*x + c)^2 + (F^a*b^2*log(F)^2 +
 F^a*d^2)*F^(b*x)*sin(d*x + c)^2 - (F^a*b^2*log(F)^2 - F^a*d^2)*F^(b*x) -
2*((F^a*b^3*d*log(F)^3 + F^a*b*d^3*log(F))*e*cos(d*x + c)^2 + (F^a*b^3*d*l
og(F)^3 + F^a*b*d^3*log(F))*e*sin(d*x + c)^2 + 2*(F^a*b^3*d*log(F)^3 + F^a
*b*d^3*log(F))*e*sin(d*x + c) + (F^a*b^3*d*log(F)^3 + F^a*b*d^3*log(F))*e)
*integrate((2*F^(b*x)*b*cos(d*x + c)*log(F) + F^(b*x)*b*log(F)*sin(2*d*x +
 2*c) - F^(b*x)*d*cos(2*d*x + 2*c) + 2*F^(b*x)*d*sin(d*x + c) + F^(b*x)*d)
/((b^2*log(F)^2 + d^2)*e*cos(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*cos
(d*x + c)^2 + 4*(b^2*log(F)^2 + d^2)*e*cos(d*x + c)*sin(2*d*x + 2*c) + (b^
2*log(F)^2 + d^2)*e*sin(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*sin(d*x
+ c)^2 + 4*(b^2*log(F)^2 + d^2)*e*sin(d*x + c) + (b^2*log(F)^2 + d^2)*e -
2*(2*(b^2*log(F)^2 + d^2)*e*sin(d*x + c) + (b^2*log(F)^2 + d^2)*e)*cos(2*d
*x + 2*c)), x))/((b^3*log(F)^3 + b*d^2*log(F))*e*cos(d*x + c)^2 + (b^3*log
(F)^3 + b*d^2*log(F))*e*sin(d*x + c)^2 + 2*(b^3*log(F)^3 + b*d^2*log(F))*e
*sin(d*x + c) + (b^3*log(F)^3 + b*d^2*log(F))*e)
```

3.53.8 Giac [F]

$$\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx = \int \frac{F^{bx+a}\cos{(dx+c)}}{e\sin{(dx+c)}+e}\,dx$$

input
```
integrate(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate(F^(b*x + a)*cos(d*x + c)/(e*sin(d*x + c) + e), x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx = \int \frac{F^{a+b\,x}\,\cos\left(c+d\,x\right)}{e+e\,\sin\left(c+d\,x\right)}\,dx$$

input
```
int((F^(a + b*x)*cos(c + d*x))/(e + e*sin(c + d*x)),x)
```

output
```
int((F^(a + b*x)*cos(c + d*x))/(e + e*sin(c + d*x)), x)
```

3.54 $\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx$

3.54.1 Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx = -\frac{iF^{a+bx}}{be\log(F)}+\frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-ie^{i(c+dx)}\right)}{be\log(F)}$$

output
```
-I*F^(b*x+a)/b/e/ln(F)+2*I*F^(b*x+a)*hypergeom([1, -I*b*ln(F)/d],[1-I*b*ln
(F)/d],-I*exp(I*(d*x+c)))/b/e/ln(F)
```

3.54.2 Mathematica [A] (verified)

Time = 3.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx = \frac{iF^{a+bx}\left(-1+2\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-ie^{i(c+dx)}\right)\right)}{be\log(F)}$$

input
```
Integrate[(F^(a + b*x)*Cos[c + d*x])/(e - e*Sin[c + d*x]),x]
```

output
```
(I*F^(a + b*x)*(-1 + 2*Hypergeometric2F1[1, ((-I)*b*Log[F])/d, 1 - (I*b*Lo
g[F])/d, (-I)*E^(I*(c + d*x))]))/(b*e*Log[F])
```

3.54.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4962, 25, 4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 4962}$$

$$-\frac{\int -F^{a+bx}\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{e}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int F^{a+bx}\tan\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx}{e}$$

$$\Big\downarrow \text{ 4942}$$

$$\frac{i\int \left(\frac{2F^{a+bx}}{1+e^{\frac{1}{2}i(2c+2dx+\pi)}}-F^{a+bx}\right)dx}{e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i\left(-\frac{F^{a+bx}}{b\log(F)}+\frac{2F^{a+bx}\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-ie^{i(c+dx)}\right)}{b\log(F)}\right)}{e}$$

input
```
Int[(F^(a + b*x)*Cos[c + d*x])/(e - e*Sin[c + d*x]),x]
```

output
```
(I*(-(F^(a + b*x)/(b*Log[F])) + (2*F^(a + b*x)*Hypergeometric2F1[1, ((-I)*
b*Log[F])/d, 1 - (I*b*Log[F])/d, (-I)*E^(I*(c + d*x))])/(b*Log[F])))/e
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

rule 4962
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Simp[g^n   Int[F^(c*(a
+ b*x))*Tan[f*(Pi/(4*g)) - d/2 - e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d
, e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

3.54.4 Maple [F]

$$\int \frac{F^{xb+a}\cos(dx+c)}{e-e\sin(dx+c)}dx$$

input
```
int(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x)
```

output
```
int(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x)
```

3.54.5 Fricas [F]

$$\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx = \int -\frac{F^{bx+a}\cos(dx+c)}{e\sin(dx+c)-e}\,dx$$

input
```
integrate(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x, algorithm="fricas")
```

output
```
integral(-F^(b*x + a)*cos(d*x + c)/(e*sin(d*x + c) - e), x)
```

3.54.6 Sympy [F]

$$\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx = -\frac{\int \frac{F^{a+bx}\cos(c+dx)}{\sin(c+dx)-1}\,dx}{e}$$

input
```
integrate(F**(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x)
```

output
```
-Integral(F**(a + b*x)*cos(c + d*x)/(sin(c + d*x) - 1), x)/e
```

3.54.7 Maxima [F]

$$\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx = \int -\frac{F^{bx+a}\cos(dx+c)}{e\sin(dx+c)-e}\,dx$$

input
```
integrate(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-(2*F^(b*x)*F^a*b*d*cos(d*x + c)*log(F) + 2*F^(b*x)*F^a*d^2*sin(d*x + c) -
 (F^a*b^2*log(F)^2 + F^a*d^2)*F^(b*x)*cos(d*x + c)^2 - (F^a*b^2*log(F)^2 +
 F^a*d^2)*F^(b*x)*sin(d*x + c)^2 + (F^a*b^2*log(F)^2 - F^a*d^2)*F^(b*x) +
2*((F^a*b^3*d*log(F)^3 + F^a*b*d^3*log(F))*e*cos(d*x + c)^2 + (F^a*b^3*d*l
og(F)^3 + F^a*b*d^3*log(F))*e*sin(d*x + c)^2 - 2*(F^a*b^3*d*log(F)^3 + F^a
*b*d^3*log(F))*e*sin(d*x + c) + (F^a*b^3*d*log(F)^3 + F^a*b*d^3*log(F))*e)
*integrate(-(2*F^(b*x)*b*cos(d*x + c)*log(F) - F^(b*x)*b*log(F)*sin(2*d*x
+ 2*c) + F^(b*x)*d*cos(2*d*x + 2*c) + 2*F^(b*x)*d*sin(d*x + c) - F^(b*x)*d
)/((b^2*log(F)^2 + d^2)*e*cos(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*co
s(d*x + c)^2 - 4*(b^2*log(F)^2 + d^2)*e*cos(d*x + c)*sin(2*d*x + 2*c) + (b
^2*log(F)^2 + d^2)*e*sin(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*sin(d*x
 + c)^2 - 4*(b^2*log(F)^2 + d^2)*e*sin(d*x + c) + (b^2*log(F)^2 + d^2)*e +
 2*(2*(b^2*log(F)^2 + d^2)*e*sin(d*x + c) - (b^2*log(F)^2 + d^2)*e)*cos(2*
d*x + 2*c)), x))/((b^3*log(F)^3 + b*d^2*log(F))*e*cos(d*x + c)^2 + (b^3*lo
g(F)^3 + b*d^2*log(F))*e*sin(d*x + c)^2 - 2*(b^3*log(F)^3 + b*d^2*log(F))*
e*sin(d*x + c) + (b^3*log(F)^3 + b*d^2*log(F))*e)
```

3.54.8 Giac [F]

$$\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx = \int -\frac{F^{bx+a}\cos(dx+c)}{e\sin(dx+c)-e}\,dx$$

input
```
integrate(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x, algorithm="giac")
```

output
```
integrate(-F^(b*x + a)*cos(d*x + c)/(e*sin(d*x + c) - e), x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx = \int \frac{F^{a+b\,x}\cos(c+d\,x)}{e-e\,\sin(c+d\,x)}\,dx$$

input
```
int((F^(a + b*x)*cos(c + d*x))/(e - e*sin(c + d*x)),x)
```

output
```
int((F^(a + b*x)*cos(c + d*x))/(e - e*sin(c + d*x)), x)
```

3.55 $\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx$

3.55.1 Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx = -\frac{iF^{a+bx}}{be\log(F)}+\frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-e^{i(c+dx)}\right)}{be\log(F)}$$

output
```
-I*F^(b*x+a)/b/e/ln(F)+2*I*F^(b*x+a)*hypergeom([1, -I*b*ln(F)/d],[1-I*b*ln
(F)/d],-exp(I*(d*x+c)))/b/e/ln(F)
```

3.55.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx = \frac{iF^{a+bx}\left(-1+2\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-\cos(c+dx)-i\sin(c+dx)\right)\right)}{be\log(F)}$$

input
```
Integrate[(F^(a + b*x)*Sin[c + d*x])/(e + e*Cos[c + d*x]),x]
```

output
```
(I*F^(a + b*x)*(-1 + 2*Hypergeometric2F1[1, ((-I)*b*Log[F])/d, 1 - (I*b*Lo
g[F])/d, -Cos[c + d*x] - I*Sin[c + d*x]]))/(b*e*Log[F])
```

3.55.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {4963, 4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+bx}\sin(c+dx)}{e\cos(c+dx)+e}dx$$

$$\Big\downarrow \text{4963}$$

$$\frac{\int F^{a+bx}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{e}$$

$$\Big\downarrow \text{4942}$$

$$\frac{i\int \left(\frac{2F^{a+bx}}{1+e^{i(c+dx)}}-F^{a+bx}\right)dx}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{F^{a+bx}}{b\log(F)}+\frac{2F^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-e^{i(c+dx)}\right)}{b\log(F)}\right)}{e}$$

input
```
Int[(F^(a + b*x)*Sin[c + d*x])/(e + e*Cos[c + d*x]),x]
```

output
```
(I*(-(F^(a + b*x)/(b*Log[F])) + (2*F^(a + b*x)*Hypergeometric2F1[1, ((-I)*
b*Log[F])/d, 1 - (I*b*Log[F])/d, -E^(I*(c + d*x))])/(b*Log[F])))/e
```

3.55.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

rule 4963
```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Simp[f^n   Int[F^(c*(a
+ b*x))*Tan[d/2 + e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
&& EqQ[f - g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

3.55.4 Maple [F]

$$\int \frac{F^{xb+a} \sin \left(d x +c \right)}{e +e \cos \left(d x +c \right)}d x$$

input
```
int(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x)
```

output
```
int(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x)
```

3.55.5 Fricas [F]

$$\int \frac{F^{a+bx} \sin(c+dx)}{e+e\cos(c+dx)}\,dx = \int \frac{F^{bx+a} \sin\left(dx+c\right)}{e\cos\left(dx+c\right)+e}\,dx$$

input
```
integrate(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x, algorithm="fricas")
```

output
```
integral(F^(b*x + a)*sin(d*x + c)/(e*cos(d*x + c) + e), x)
```

3.55.6 Sympy [F]

$$\int \frac{F^{a+bx} \sin(c+dx)}{e+e\cos(c+dx)}\,dx = \frac{\int \frac{F^{a+bx} \sin\left(c+dx\right)}{\cos\left(c+dx\right)+1}\,dx}{e}$$

input
```
integrate(F**(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x)
```

output
```
Integral(F**(a + b*x)*sin(c + d*x)/(cos(c + d*x) + 1), x)/e
```

3.55.7 Maxima [F]

$$\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx = \int \frac{F^{bx+a}\sin(dx+c)}{e\cos(dx+c)+e}\,dx$$

input
```
integrate(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x, algorithm="maxima")
```

output
```
2*(F^(b*x)*F^a*b*log(F)*sin(d*x + c) - F^(b*x)*F^a*d*cos(d*x + c) - F^(b*x
)*F^a*d + ((F^a*b^2*d*log(F)^2 + F^a*d^3)*e*cos(d*x + c)^2 + (F^a*b^2*d*lo
g(F)^2 + F^a*d^3)*e*sin(d*x + c)^2 + 2*(F^a*b^2*d*log(F)^2 + F^a*d^3)*e*co
s(d*x + c) + (F^a*b^2*d*log(F)^2 + F^a*d^3)*e)*integrate((F^(b*x)*b*cos(2*
d*x + 2*c)*log(F) + 2*F^(b*x)*b*cos(d*x + c)*log(F) + F^(b*x)*b*log(F) + F
^(b*x)*d*sin(2*d*x + 2*c) + 2*F^(b*x)*d*sin(d*x + c))/((b^2*log(F)^2 + d^2
)*e*cos(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*cos(d*x + c)^2 + (b^2*lo
g(F)^2 + d^2)*e*sin(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*sin(2*d*x +
2*c)*sin(d*x + c) + 4*(b^2*log(F)^2 + d^2)*e*sin(d*x + c)^2 + 4*(b^2*log(F
)^2 + d^2)*e*cos(d*x + c) + (b^2*log(F)^2 + d^2)*e + 2*(2*(b^2*log(F)^2 +
d^2)*e*cos(d*x + c) + (b^2*log(F)^2 + d^2)*e)*cos(2*d*x + 2*c)), x))/((b^2
*log(F)^2 + d^2)*e*cos(d*x + c)^2 + (b^2*log(F)^2 + d^2)*e*sin(d*x + c)^2
+ 2*(b^2*log(F)^2 + d^2)*e*cos(d*x + c) + (b^2*log(F)^2 + d^2)*e)
```

3.55.8 Giac [F]

$$\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx = \int \frac{F^{bx+a}\sin(dx+c)}{e\cos(dx+c)+e}\,dx$$

input
```
integrate(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(F^(b*x + a)*sin(d*x + c)/(e*cos(d*x + c) + e), x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx = \int \frac{F^{a+b\,x}\,\sin\left(c+d\,x\right)}{e+e\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((F^(a + b*x)*sin(c + d*x))/(e + e*cos(c + d*x)),x)
```

output
```
int((F^(a + b*x)*sin(c + d*x))/(e + e*cos(c + d*x)), x)
```

3.56 $\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx$

3.56.1 Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx = \frac{iF^{a+bx}}{be\log(F)} - \frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},e^{i(c+dx)}\right)}{be\log(F)}$$

output
```
I*F^(b*x+a)/b/e/ln(F)-2*I*F^(b*x+a)*hypergeom([1, -I*b*ln(F)/d],[1-I*b*ln(
F)/d],exp(I*(d*x+c)))/b/e/ln(F)
```

3.56.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx = -\frac{iF^{a+bx}\left(-1+2\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},\cos(c+dx)+i\sin(c+dx)\right)\right)}{be\log(F)}$$

input
```
Integrate[(F^(a + b*x)*Sin[c + d*x])/(e - e*Cos[c + d*x]),x]
```

output
```
((-I)*F^(a + b*x)*(-1 + 2*Hypergeometric2F1[1, ((-I)*b*Log[F])/d, 1 - (I*b
*Log[F])/d, Cos[c + d*x] + I*Sin[c + d*x]]))/(b*e*Log[F])
```

3.56.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4964, 4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx$$

$$\Big\downarrow \ \text{4964}$$

$$\frac{\int F^{a+bx}\cot\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{e}$$

$$\Big\downarrow \ \text{4943}$$

$$-\frac{i\int \left(\frac{2F^{a+bx}}{1-e^{i(c+dx)}}-F^{a+bx}\right)dx}{e}$$

$$\Big\downarrow \ \text{2009}$$

$$-\frac{i\left(-\frac{F^{a+bx}}{b\log(F)}+\frac{2F^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},e^{i(c+dx)}\right)}{b\log(F)}\right)}{e}$$

input
```
Int[(F^(a + b*x)*Sin[c + d*x])/(e - e*Cos[c + d*x]),x]
```

output
```
((-I)*(-(F^(a + b*x)/(b*Log[F])) + (2*F^(a + b*x)*Hypergeometric2F1[1, ((-
I)*b*Log[F])/d, 1 - (I*b*Log[F])/d, E^(I*(c + d*x))])/(b*Log[F])))/e
```

3.56.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

rule 4964
```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Simp[f^n   Int[F^(c*(a
+ b*x))*Cot[d/2 + e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
&& EqQ[f + g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

3.56.4 Maple [F]

$$\int \frac{F^{xb+a} \sin \left(d x+c\right)}{e-e \cos \left(d x+c\right)}dx$$

input
```
int(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x)
```

output
```
int(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x)
```

3.56.5 Fricas [F]

$$\int \frac{F^{a+bx} \sin(c+dx)}{e-e\cos(c+dx)}\,dx = \int -\frac{F^{bx+a} \sin \left(dx+c\right)}{e \cos \left(dx+c\right)-e}\,dx$$

input
```
integrate(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x, algorithm="fricas")
```

output
```
integral(-F^(b*x + a)*sin(d*x + c)/(e*cos(d*x + c) - e), x)
```

3.56.6 Sympy [F]

$$\int \frac{F^{a+bx} \sin(c+dx)}{e-e\cos(c+dx)}\,dx = -\frac{\int \frac{F^{a+bx} \sin \left(c+dx\right)}{\cos \left(c+dx\right)-1}\,dx}{e}$$

input
```
integrate(F**(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x)
```

output
```
-Integral(F**(a + b*x)*sin(c + d*x)/(cos(c + d*x) - 1), x)/e
```

3.56.7 Maxima [F]

$$\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx = \int -\frac{F^{bx+a}\sin\left(dx+c\right)}{e\cos\left(dx+c\right)-e}\,dx$$

input
```
integrate(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x, algorithm="maxima")
```

output
```
2*(F^(b*x)*F^a*b*log(F)*sin(d*x + c) - F^(b*x)*F^a*d*cos(d*x + c) + F^(b*x
)*F^a*d - ((F^a*b^2*d*log(F)^2 + F^a*d^3)*e*cos(d*x + c)^2 + (F^a*b^2*d*lo
g(F)^2 + F^a*d^3)*e*sin(d*x + c)^2 - 2*(F^a*b^2*d*log(F)^2 + F^a*d^3)*e*co
s(d*x + c) + (F^a*b^2*d*log(F)^2 + F^a*d^3)*e)*integrate((F^(b*x)*b*cos(2*
d*x + 2*c)*log(F) - 2*F^(b*x)*b*cos(d*x + c)*log(F) + F^(b*x)*b*log(F) + F
^(b*x)*d*sin(2*d*x + 2*c) - 2*F^(b*x)*d*sin(d*x + c))/((b^2*log(F)^2 + d^2
)*e*cos(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*cos(d*x + c)^2 + (b^2*lo
g(F)^2 + d^2)*e*sin(2*d*x + 2*c)^2 - 4*(b^2*log(F)^2 + d^2)*e*sin(2*d*x +
2*c)*sin(d*x + c) + 4*(b^2*log(F)^2 + d^2)*e*sin(d*x + c)^2 - 4*(b^2*log(F
)^2 + d^2)*e*cos(d*x + c) + (b^2*log(F)^2 + d^2)*e - 2*(2*(b^2*log(F)^2 +
d^2)*e*cos(d*x + c) - (b^2*log(F)^2 + d^2)*e)*cos(2*d*x + 2*c)), x))/((b^2
*log(F)^2 + d^2)*e*cos(d*x + c)^2 + (b^2*log(F)^2 + d^2)*e*sin(d*x + c)^2
- 2*(b^2*log(F)^2 + d^2)*e*cos(d*x + c) + (b^2*log(F)^2 + d^2)*e)
```

3.56.8 Giac [F]

$$\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx = \int -\frac{F^{bx+a}\sin\left(dx+c\right)}{e\cos\left(dx+c\right)-e}\,dx$$

input
```
integrate(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(-F^(b*x + a)*sin(d*x + c)/(e*cos(d*x + c) - e), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx = \int \frac{F^{a+b\,x}\,\sin\left(c+d\,x\right)}{e-e\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((F^(a + b*x)*sin(c + d*x))/(e - e*cos(c + d*x)),x)
```

output
```
int((F^(a + b*x)*sin(c + d*x))/(e - e*cos(c + d*x)), x)
```

3.57 $\int e^{x^2} \sin(bx)\,dx$

3.57.1 Optimal result

Integrand size = 10, antiderivative size = 69

$$\int e^{x^2} \sin(bx)\,dx = \frac{1}{4} i e^{\frac{b^2}{4}} \sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(-ib+2x)\right) - \frac{1}{4} i e^{\frac{b^2}{4}} \sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(ib+2x)\right)$$

output
```
-1/4*I*exp(1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)-1/4*I*exp(1/4*b^2)*erfi(1/2*I
*b+x)*Pi^(1/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.62

$$\int e^{x^2} \sin(bx)\,dx = \frac{1}{4} e^{\frac{b^2}{4}} \sqrt{\pi} \left(\text{erf}\left(\frac{b}{2} - ix\right) + \text{erf}\left(\frac{b}{2} + ix\right)\right)$$

input
```
Integrate[E^x^2*Sin[b*x],x]
```

output
```
(E^(b^2/4)*Sqrt[Pi]*(Erf[b/2 - I*x] + Erf[b/2 + I*x]))/4
```

3.57.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2}\sin(bx)\,dx$$

$$\Big\downarrow \text{ 4975}$$

$$\int \left(\frac{1}{2}ie^{x^2-ibx}-\frac{1}{2}ie^{x^2+ibx}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}}\text{erfi}\left(\frac{1}{2}(2x-ib)\right)-\frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}}\text{erfi}\left(\frac{1}{2}(2x+ib)\right)$$

input
```
Int[E^x^2*Sin[b*x],x]
```

output
```
(I/4)*E^(b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2] - (I/4)*E^(b^2/4)*Sqrt[Pi]
*Erfi[(I*b + 2*x)/2]
```

3.57.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.57.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{erf}\left(-ix+\frac{b}{2}\right)}{4}+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{erf}\left(ix+\frac{b}{2}\right)}{4}$	42

input
```
int(exp(x^2)*sin(x*b),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*exp(1/4*b^2)*erf(-I*x+1/2*b)+1/4*Pi^(1/2)*exp(1/4*b^2)*erf(I*
x+1/2*b)
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.43

$$\int e^{x^2}\sin(bx)\,dx=\frac{1}{4}\sqrt{\pi}\left(\mathrm{erf}\left(\frac{1}{2}\,b+i\,x\right)-\mathrm{erf}\left(-\frac{1}{2}\,b+i\,x\right)\right)e^{\left(\frac{1}{4}\,b^2\right)}$$

input
```
integrate(exp(x^2)*sin(b*x),x, algorithm="fricas")
```

output
```
1/4*sqrt(pi)*(erf(1/2*b + I*x) - erf(-1/2*b + I*x))*e^(1/4*b^2)
```

3.57.6 Sympy [F]

$$\int e^{x^2}\sin(bx)\,dx=\int e^{x^2}\sin(bx)\,dx$$

input
```
integrate(exp(x**2)*sin(b*x),x)
```

output
```
Integral(exp(x**2)*sin(b*x), x)
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

$$\int e^{x^2} \sin(bx)\,dx = \frac{1}{4}\sqrt{\pi}\left(\operatorname{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)} - \operatorname{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}\right)$$

input
```
integrate(exp(x^2)*sin(b*x),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*(erf(1/2*b + I*x)*e^(1/4*b^2) - erf(-1/2*b + I*x)*e^(1/4*b^2)
)
```

3.57.8 Giac [F]

$$\int e^{x^2} \sin(bx)\,dx = \int e^{\left(x^2\right)} \sin\left(bx\right)\,dx$$

input
```
integrate(exp(x^2)*sin(b*x),x, algorithm="giac")
```

output
```
integrate(e^(x^2)*sin(b*x), x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int e^{x^2} \sin(bx)\,dx = \int e^{x^2}\,\sin\left(b\,x\right)\,dx$$

input
```
int(exp(x^2)*sin(b*x),x)
```

output
```
int(exp(x^2)*sin(b*x), x)
```

3.58 $\int e^{x^2} \cos(bx)\,dx$

3.58.1 Optimal result

Integrand size = 10, antiderivative size = 65

$$\int e^{x^2} \cos(bx)\,dx = \frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(-ib+2x)\right) + \frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(ib+2x)\right)$$

output
```
-1/4*exp(1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)+1/4*exp(1/4*b^2)*erfi(1/2*I*b+x
)*Pi^(1/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int e^{x^2} \cos(bx)\,dx = \frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\left(\text{erfi}\left(\frac{1}{2}(-ib+2x)\right) + \text{erfi}\left(\frac{1}{2}(ib+2x)\right)\right)$$

input
```
Integrate[E^x^2*Cos[b*x],x]
```

output
```
(E^(b^2/4)*Sqrt[Pi]*(Erfi[((-I)*b + 2*x)/2] + Erfi[(I*b + 2*x)/2]))/4
```

3.58.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2}\cos(bx)\,dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{1}{2}e^{x^2-ibx}+\frac{1}{2}e^{x^2+ibx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}}\operatorname{erfi}\left(\frac{1}{2}(2x-ib)\right)+\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}}\operatorname{erfi}\left(\frac{1}{2}(2x+ib)\right)$$

input
```
Int[E^x^2*Cos[b*x],x]
```

output
```
(E^(b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2])/4 + (E^(b^2/4)*Sqrt[Pi]*Erfi[(
I*b + 2*x)/2])/4
```

3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.58.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{erf}\left(ix+\frac{b}{2}\right)}{4}+\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{erf}\left(-ix+\frac{b}{2}\right)}{4}$	44

input
```
int(exp(x^2)*cos(x*b),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*Pi^(1/2)*exp(1/4*b^2)*erf(I*x+1/2*b)+1/4*I*Pi^(1/2)*exp(1/4*b^2)*er
f(-I*x+1/2*b)
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.49

$$\int e^{x^2}\cos(bx)\,dx = \frac{1}{4}\sqrt{\pi}\left(-i\ \mathrm{erf}\left(\frac{1}{2}\,b+i\,x\right)-i\ \mathrm{erf}\left(-\frac{1}{2}\,b+i\,x\right)\right)e^{\left(\frac{1}{4}\,b^2\right)}$$

input
```
integrate(exp(x^2)*cos(b*x),x, algorithm="fricas")
```

output
```
1/4*sqrt(pi)*(-I*erf(1/2*b + I*x) - I*erf(-1/2*b + I*x))*e^(1/4*b^2)
```

3.58.6 Sympy [F]

$$\int e^{x^2}\cos(bx)\,dx = \int e^{x^2}\cos{(bx)}\,dx$$

input
```
integrate(exp(x**2)*cos(b*x),x)
```

output
```
Integral(exp(x**2)*cos(b*x), x)
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int e^{x^2}\cos(bx)\,dx = -\frac{1}{4}\sqrt{\pi}\left(i\,\text{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}+i\,\text{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}\right)$$

input
```
integrate(exp(x^2)*cos(b*x),x, algorithm="maxima")
```

output
```
-1/4*sqrt(pi)*(I*erf(1/2*b + I*x)*e^(1/4*b^2) + I*erf(-1/2*b + I*x)*e^(1/4
*b^2))
```

3.58.8 Giac [F]

$$\int e^{x^2}\cos(bx)\,dx = \int \cos(bx)\,e^{(x^2)}\,dx$$

input
```
integrate(exp(x^2)*cos(b*x),x, algorithm="giac")
```

output
```
integrate(cos(b*x)*e^(x^2), x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\cos(bx)\,dx = \int e^{x^2}\cos(b\,x)\,dx$$

input
```
int(exp(x^2)*cos(b*x),x)
```

output
```
int(exp(x^2)*cos(b*x), x)
```

3.59 $\int e^{x^2} \sin(a+bx)\,dx$

3.59.1 Optimal result

Integrand size = 12, antiderivative size = 81

$$\int e^{x^2} \sin(a+bx)\,dx = \frac{1}{4} i e^{-ia+\frac{b^2}{4}} \sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-ib+2x)\right) - \frac{1}{4} i e^{ia+\frac{b^2}{4}} \sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(ib+2x)\right)$$

output
```
-1/4*I*exp(-I*a+1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)-1/4*I*exp(I*a+1/4*b^2)*e
rfi(1/2*I*b+x)*Pi^(1/2)
```

3.59.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int e^{x^2} \sin(a+bx)\,dx = \frac{1}{4} e^{\frac{b^2}{4}} \sqrt{\pi}\left(\cos(a)\operatorname{erf}\left(\frac{b}{2}-ix\right) + \cos(a)\operatorname{erf}\left(\frac{b}{2}+ix\right) + \left(\operatorname{erfi}\left(\frac{1}{2}(-ib+2x)\right) + \operatorname{erfi}\left(\frac{1}{2}(ib+2x)\right)\right)\sin(a)\right)$$

input
```
Integrate[E^x^2*Sin[a + b*x],x]
```

output
```
(E^(b^2/4)*Sqrt[Pi]*(Cos[a]*Erf[b/2 - I*x] + Cos[a]*Erf[b/2 + I*x] + (Erfi
[((-I)*b + 2*x)/2] + Erfi[(I*b + 2*x)/2])*Sin[a]))/4
```

3.59.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} \sin(a+bx)\,dx$$

$$\Big\downarrow \text{ 4975}$$

$$\int \left(\frac{1}{2}ie^{-ia-ibx+x^2}-\frac{1}{2}ie^{ia+ibx+x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}-ia}\text{erfi}\left(\frac{1}{2}(2x-ib)\right)-\frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}+ia}\text{erfi}\left(\frac{1}{2}(2x+ib)\right)$$

input
```
Int[E^x^2*Sin[a + b*x],x]
```

output
```
(I/4)*E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2] - (I/4)*E^(I*a +
b^2/4)*Sqrt[Pi]*Erfi[(I*b + 2*x)/2]
```

3.59.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.59.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{e}^{ia}\,\mathrm{erf}\left(-ix+\frac{b}{2}\right)}{4}+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{e}^{-ia}\,\mathrm{erf}\left(ix+\frac{b}{2}\right)}{4}$	52

input
```
int(exp(x^2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*exp(1/4*b^2)*exp(I*a)*erf(-I*x+1/2*b)+1/4*Pi^(1/2)*exp(1/4*b^
2)*exp(-I*a)*erf(I*x+1/2*b)
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.56

$$\int e^{x^2}\sin(a+bx)\,dx=-\frac{1}{4}\sqrt{\pi}\left(\operatorname{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2+i\,a\right)}-\operatorname{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2-i\,a\right)}\right)$$

input
```
integrate(exp(x^2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*sqrt(pi)*(erf(-1/2*b + I*x)*e^(1/4*b^2 + I*a) - erf(1/2*b + I*x)*e^(1
/4*b^2 - I*a))
```

3.59.6 Sympy [F]

$$\int e^{x^2}\sin(a+bx)\,dx=\int e^{x^2}\sin\left(a+bx\right)dx$$

input
```
integrate(exp(x**2)*sin(b*x+a),x)
```

output
```
Integral(exp(x**2)*sin(a + b*x), x)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.63

$$\int e^{x^2}\sin(a+bx)\,dx$$
$$=\frac{1}{4}\sqrt{\pi}\left((\cos(a)-i\sin(a))\operatorname{erf}\left(\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}-(\cos(a)+i\sin(a))\operatorname{erf}\left(-\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}\right)$$

input
```
integrate(exp(x^2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*((cos(a) - I*sin(a))*erf(1/2*b + I*x)*e^(1/4*b^2) - (cos(a) +
 I*sin(a))*erf(-1/2*b + I*x)*e^(1/4*b^2))
```

3.59.8 Giac [F]

$$\int e^{x^2}\sin(a+bx)\,dx=\int e^{(x^2)}\sin(bx+a)\,dx$$

input
```
integrate(exp(x^2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(e^(x^2)*sin(b*x + a), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\sin(a+bx)\,dx=\int e^{x^2}\sin(a+bx)\,dx$$

input
```
int(exp(x^2)*sin(a + b*x),x)
```

output
```
int(exp(x^2)*sin(a + b*x), x)
```

3.60 $\int e^{x^2} \cos(a+bx)\,dx$

3.60.1 Optimal result

Integrand size = 12, antiderivative size = 77

$$\int e^{x^2}\cos(a+bx)\,dx=\frac{1}{4}e^{-ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\frac{1}{4}e^{ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)$$

output
```
-1/4*exp(-I*a+1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)+1/4*exp(I*a+1/4*b^2)*erfi(
1/2*I*b+x)*Pi^(1/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int e^{x^2}\cos(a+bx)\,dx=\frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\left(\cos(a)\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\cos(a)\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)-\left(\mathrm{erf}\left(\frac{b}{2}-ix\right)+\mathrm{erf}\left(\frac{b}{2}+ix\right)\right)\sin(a)\right)$$

input
```
Integrate[E^x^2*Cos[a + b*x],x]
```

output
```
(E^(b^2/4)*Sqrt[Pi]*(Cos[a]*Erfi[((-I)*b + 2*x)/2] + Cos[a]*Erfi[(I*b + 2*
x)/2] - (Erf[b/2 - I*x] + Erf[b/2 + I*x])*Sin[a]))/4
```

3.60.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} \cos(a+bx)\,dx$$

$$\Big\downarrow \; 4976$$

$$\int \left(\frac{1}{2}e^{-ia-ibx+x^2}+\frac{1}{2}e^{ia+ibx+x^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}-ia}\text{erfi}\left(\frac{1}{2}(2x-ib)\right)+\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}+ia}\text{erfi}\left(\frac{1}{2}(2x+ib)\right)$$

input
```
Int[E^x^2*Cos[a + b*x],x]
```

output
```
(E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2])/4 + (E^(I*a + b^2/4)*
Sqrt[Pi]*Erfi[(I*b + 2*x)/2])/4
```

3.60.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.60.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{e}^{-ia}\,\mathrm{erf}\left(ix+\frac{b}{2}\right)}{4}+\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{e}^{ia}\,\mathrm{erf}\left(-ix+\frac{b}{2}\right)}{4}$	54

input
```
int(exp(x^2)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*Pi^(1/2)*exp(1/4*b^2)*exp(-I*a)*erf(I*x+1/2*b)+1/4*I*Pi^(1/2)*exp(1
/4*b^2)*exp(I*a)*erf(-I*x+1/2*b)
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.60

$$\int e^{x^2}\cos(a+bx)\,dx = \frac{1}{4}\sqrt{\pi}\left(-i\ \mathrm{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2+i\,a\right)}-i\ \mathrm{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2-i\,a\right)}\right)$$

input
```
integrate(exp(x^2)*cos(b*x+a),x, algorithm="fricas")
```

output
```
1/4*sqrt(pi)*(-I*erf(-1/2*b + I*x)*e^(1/4*b^2 + I*a) - I*erf(1/2*b + I*x)*
e^(1/4*b^2 - I*a))
```

3.60.6 Sympy [F]

$$\int e^{x^2}\cos(a+bx)\,dx = \int e^{x^2}\cos\left(a+bx\right)dx$$

input
```
integrate(exp(x**2)*cos(b*x+a),x)
```

output
```
Integral(exp(x**2)*cos(a + b*x), x)
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int e^{x^2}\cos(a+bx)\,dx =$$
$$-\frac{1}{4}\sqrt{\pi}\left((i\cos(a)+\sin(a))\operatorname{erf}\left(\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}+(i\cos(a)-\sin(a))\operatorname{erf}\left(-\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}\right)$$

input
```
integrate(exp(x^2)*cos(b*x+a),x, algorithm="maxima")
```

output
```
-1/4*sqrt(pi)*((I*cos(a) + sin(a))*erf(1/2*b + I*x)*e^(1/4*b^2) + (I*cos(a
) - sin(a))*erf(-1/2*b + I*x)*e^(1/4*b^2))
```

3.60.8 Giac [F]

$$\int e^{x^2}\cos(a+bx)\,dx = \int \cos(bx+a)\,e^{(x^2)}\,dx$$

input
```
integrate(exp(x^2)*cos(b*x+a),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*e^(x^2), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\cos(a+bx)\,dx = \int \cos(a+b\,x)\;\mathrm{e}^{x^2}\,dx$$

input
```
int(cos(a + b*x)*exp(x^2),x)
```

output
```
int(cos(a + b*x)*exp(x^2), x)
```

3.61 $\int e^{2x^2} x \cos(2x^2)\,dx$

3.61.1 Optimal result

Integrand size = 15, antiderivative size = 35

$$\int e^{2x^2} x \cos(2x^2)\,dx = \frac{1}{8}e^{2x^2}\cos(2x^2) + \frac{1}{8}e^{2x^2}\sin(2x^2)$$

output
```
1/8*exp(2*x^2)*cos(2*x^2)+1/8*exp(2*x^2)*sin(2*x^2)
```

3.61.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int e^{2x^2} x \cos(2x^2)\,dx = \frac{1}{8}e^{2x^2}\left(\cos(2x^2) + \sin(2x^2)\right)$$

input
```
Integrate[E^(2*x^2)*x*Cos[2*x^2],x]
```

output
```
(E^(2*x^2)*(Cos[2*x^2] + Sin[2*x^2]))/8
```

3.61.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7266, 4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2x^2} x \cos\left(2x^2\right) \, dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2} \int e^{2x^2} \cos\left(2x^2\right) dx^2$$

$$\Big\downarrow \text{4933}$$

$$\frac{1}{2} \left(\frac{1}{4} e^{2x^2} \sin\left(2x^2\right) + \frac{1}{4} e^{2x^2} \cos\left(2x^2\right)\right)$$

input
```
Int[E^(2*x^2)*x*Cos[2*x^2],x]
```

output
```
((E^(2*x^2)*Cos[2*x^2])/4 + (E^(2*x^2)*Sin[2*x^2])/4)/2
```

3.61.3.1 Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.61.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{e^{2x^2}(\cos(2x^2)+\sin(2x^2))}{8}$	22
derivativedivides	$\frac{e^{2x^2}\cos(2x^2)}{8}+\frac{e^{2x^2}\sin(2x^2)}{8}$	30
default	$\frac{e^{2x^2}\cos(2x^2)}{8}+\frac{e^{2x^2}\sin(2x^2)}{8}$	30
risch	$\frac{e^{(2+2i)x^2}}{16}-\frac{ie^{(2+2i)x^2}}{16}+\frac{e^{(2-2i)x^2}}{16}+\frac{ie^{(2-2i)x^2}}{16}$	44
norman	$\frac{\frac{e^{2x^2}\tan(x^2)}{4}-\frac{e^{2x^2}\tan(x^2)^2}{8}+\frac{e^{2x^2}}{8}}{1+\tan(x^2)^2}$	47

input
```
int(exp(2*x^2)*x*cos(2*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/8*exp(2*x^2)*(cos(2*x^2)+sin(2*x^2))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int e^{2x^2}x\cos(2x^2)\,dx=\frac{1}{8}\cos(2x^2)e^{(2x^2)}+\frac{1}{8}e^{(2x^2)}\sin(2x^2)$$

input
```
integrate(exp(2*x^2)*x*cos(2*x^2),x, algorithm="fricas")
```

output
```
1/8*cos(2*x^2)*e^(2*x^2) + 1/8*e^(2*x^2)*sin(2*x^2)
```

3.61.6 Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int e^{2x^2}x\cos(2x^2)\,dx=\frac{e^{2x^2}\sin(2x^2)}{8}+\frac{e^{2x^2}\cos(2x^2)}{8}$$

input
```
integrate(exp(2*x**2)*x*cos(2*x**2),x)
```

output
```
exp(2*x**2)*sin(2*x**2)/8 + exp(2*x**2)*cos(2*x**2)/8
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int e^{2x^2} x \cos\left(2x^2\right) \, dx = \frac{1}{8} \cos\left(2\,x^2\right) e^{\left(2\,x^2\right)} + \frac{1}{8} e^{\left(2\,x^2\right)} \sin\left(2\,x^2\right)$$

input
```
integrate(exp(2*x^2)*x*cos(2*x^2),x, algorithm="maxima")
```

output
```
1/8*cos(2*x^2)*e^(2*x^2) + 1/8*e^(2*x^2)*sin(2*x^2)
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int e^{2x^2} x \cos\left(2x^2\right) \, dx = \frac{1}{8} \left(\cos\left(2\,x^2\right) + \sin\left(2\,x^2\right)\right) e^{\left(2\,x^2\right)}$$

input
```
integrate(exp(2*x^2)*x*cos(2*x^2),x, algorithm="giac")
```

output
```
1/8*(cos(2*x^2) + sin(2*x^2))*e^(2*x^2)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int e^{2x^2} x \cos\left(2x^2\right) \, dx = \frac{e^{2\,x^2} \left(\cos\left(2\,x^2\right) + \sin\left(2\,x^2\right)\right)}{8}$$

input
```
int(x*exp(2*x^2)*cos(2*x^2),x)
```

output
```
(exp(2*x^2)*(cos(2*x^2) + sin(2*x^2)))/8
```

3.62 $\int e^x \sin(e^x) \, dx$

3.62.1 Optimal result

Integrand size = 8, antiderivative size = 6

$$\int e^x \sin(e^x) \, dx = -\cos(e^x)$$

output
```
-cos(exp(x))
```

3.62.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int e^x \sin(e^x) \, dx = -\cos(e^x)$$

input
```
Integrate[E^x*Sin[E^x],x]
```

output
```
-Cos[E^x]
```

3.62.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2720, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sin(e^x)\,dx$$

$$\Big\downarrow \; 2720$$

$$\int \sin(e^x)\,de^x$$

$$\Big\downarrow \; 3042$$

$$\int \sin(e^x)\,de^x$$

$$\Big\downarrow \; 3118$$

$$-\cos(e^x)$$

input
```
Int[E^x*Sin[E^x],x]
```

output
```
-Cos[E^x]
```

3.62.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

3.62.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\cos(\mathrm{e}^{x})$	6
default	$-\cos(\mathrm{e}^{x})$	6
risch	$-\cos(\mathrm{e}^{x})$	6
parallelrisch	$-\cos(\mathrm{e}^{x})-1$	8
norman	$-\frac{2}{1+\tan\left(\frac{\mathrm{e}^{x}}{2}\right)^{2}}$	14

input
```
int(exp(x)*sin(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-cos(exp(x))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^{x}\sin(e^{x})\,dx=-\cos(e^{x})$$

input
```
integrate(exp(x)*sin(exp(x)),x, algorithm="fricas")
```

output
```
-cos(e^x)
```

3.62.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x \sin\left(e^x\right)\,dx = -\cos\left(e^x\right)$$

input
```
integrate(exp(x)*sin(exp(x)),x)
```

output
```
-cos(exp(x))
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x \sin\left(e^x\right)\,dx = -\cos\left(e^x\right)$$

input
```
integrate(exp(x)*sin(exp(x)),x, algorithm="maxima")
```

output
```
-cos(e^x)
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x \sin\left(e^x\right)\,dx = -\cos\left(e^x\right)$$

input
```
integrate(exp(x)*sin(exp(x)),x, algorithm="giac")
```

output
```
-cos(e^x)
```

3.62.9 Mupad [B] (verification not implemented)

Time = 27.65 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x \sin \left(e^x\right) \, dx = -\cos \left(\mathrm{e}^x\right)$$

input
```
int(sin(exp(x))*exp(x),x)
```

output
```
-cos(exp(x))
```

3.63 $\int e^x \csc(e^x) \sec(e^x)\, dx$

3.63.1 Optimal result

Integrand size = 12, antiderivative size = 5

$$\int e^x \csc(e^x) \sec(e^x)\, dx = \log(\tan(e^x))$$

output
```
ln(tan(exp(x)))
```

3.63.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(5) = 10.

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 4.20

$$\int e^x \csc(e^x) \sec(e^x)\, dx = 2\left(-\frac{1}{2}\log(\cos(e^x)) + \frac{1}{2}\log(\sin(e^x))\right)$$

input
```
Integrate[E^x*Csc[E^x]*Sec[E^x],x]
```

output
```
2*(-1/2*Log[Cos[E^x]] + Log[Sin[E^x]]/2)
```

3.63.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2720, 3042, 3100, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \csc(e^x) \sec(e^x) \, dx$$
$$\Big\downarrow \text{ 2720}$$
$$\int \csc(e^x) \sec(e^x) de^x$$
$$\Big\downarrow \text{ 3042}$$
$$\int \csc(e^x) \sec(e^x) de^x$$
$$\Big\downarrow \text{ 3100}$$
$$\int e^{-x} d\tan(e^x)$$
$$\Big\downarrow \text{ 14}$$
$$\log(\tan(e^x))$$

input
```
Int[E^x*Csc[E^x]*Sec[E^x],x]
```

output
```
Log[Tan[E^x]]
```

3.63.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.63.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\ln\left(\tan\left(\mathrm{e}^{x}\right)\right)$	5
default	$\ln\left(\tan\left(\mathrm{e}^{x}\right)\right)$	5
risch	$-\ln\left(\mathrm{e}^{2i\mathrm{e}^{x}}+1\right)+\ln\left(\mathrm{e}^{2i\mathrm{e}^{x}}-1\right)$	22
norman	$-\ln\left(\tan\left(\frac{\mathrm{e}^{x}}{2}\right)-1\right)-\ln\left(\tan\left(\frac{\mathrm{e}^{x}}{2}\right)+1\right)+\ln\left(\tan\left(\frac{\mathrm{e}^{x}}{2}\right)\right)$	28
parallelrisch	$-\ln\left(\tan\left(\frac{\mathrm{e}^{x}}{2}\right)-1\right)-\ln\left(\tan\left(\frac{\mathrm{e}^{x}}{2}\right)+1\right)+\ln\left(\tan\left(\frac{\mathrm{e}^{x}}{2}\right)\right)$	28

input
```
int(exp(x)*csc(exp(x))*sec(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(tan(exp(x)))
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(4) = 8.

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 4.20

$$\int e^{x}\csc\left(e^{x}\right)\sec\left(e^{x}\right)\,dx=-\frac{1}{2}\log\left(\cos\left(e^{x}\right)^{2}\right)+\frac{1}{2}\log\left(-\frac{1}{4}\cos\left(e^{x}\right)^{2}+\frac{1}{4}\right)$$

input
```
integrate(exp(x)*csc(exp(x))*sec(exp(x)),x, algorithm="fricas")
```

output
```
-1/2*log(cos(e^x)^2) + 1/2*log(-1/4*cos(e^x)^2 + 1/4)
```

3.63.6 Sympy [F]

$$\int e^x \csc(e^x)\sec(e^x)\,dx = \int e^x \csc(e^x)\sec(e^x)\,dx$$

input
```
integrate(exp(x)*csc(exp(x))*sec(exp(x)),x)
```

output
```
Integral(exp(x)*csc(exp(x))*sec(exp(x)), x)
```

3.63.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(4) = 8$.

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 3.80

$$\int e^x \csc(e^x)\sec(e^x)\,dx = -\frac{1}{2}\log\left(\sin(e^x)^2 - 1\right) + \frac{1}{2}\log\left(\sin(e^x)^2\right)$$

input
```
integrate(exp(x)*csc(exp(x))*sec(exp(x)),x, algorithm="maxima")
```

output
```
-1/2*log(sin(e^x)^2 - 1) + 1/2*log(sin(e^x)^2)
```

3.63.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(4) = 8$.

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 3.40

$$\int e^x \csc(e^x)\sec(e^x)\,dx = -\frac{1}{2}\log\left(\left|\sin(e^x)^2 - 1\right|\right) + \log\left(\left|\sin(e^x)\right|\right)$$

input
```
integrate(exp(x)*csc(exp(x))*sec(exp(x)),x, algorithm="giac")
```

output
```
-1/2*log(abs(sin(e^x)^2 - 1)) + log(abs(sin(e^x)))
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 43, normalized size of antiderivative = 8.60

$$\int e^x \csc(e^x) \sec(e^x)\, dx = -\ln\left(-16\,e^{2\,x} - 16\,e^{2\,x}\,e^{e^x\,2i}\right) + \ln\left(16\,e^{2\,x} - 16\,e^{2\,x}\,e^{e^x\,2i}\right)$$

input
```
int(exp(x)/(cos(exp(x))*sin(exp(x))),x)
```

output
```
log(16*exp(2*x) - 16*exp(2*x)*exp(exp(x)*2i)) - log(- 16*exp(2*x) - 16*exp
(2*x)*exp(exp(x)*2i))
```

3.64 $\int e^x \cos(e^x) \, dx$

3.64.1 Optimal result

Integrand size = 8, antiderivative size = 4

$$\int e^x \cos(e^x) \, dx = \sin(e^x)$$

output
```
sin(exp(x))
```

3.64.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int e^x \cos(e^x) \, dx = \sin(e^x)$$

input
```
Integrate[E^x*Cos[E^x],x]
```

output
```
Sin[E^x]
```

3.64.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2720, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cos\left(e^x\right) dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int \cos\left(e^x\right) de^x$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(e^x+\frac{\pi}{2}\right) de^x$$

$$\Big\downarrow \text{ 3117}$$

$$\sin\left(e^x\right)$$

input
```
Int[E^x*Cos[E^x],x]
```

output
```
Sin[E^x]
```

3.64.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.64.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\sin(e^x)$	4
default	$\sin(e^x)$	4
risch	$\sin(e^x)$	4
parallelrisch	$\sin(e^x)$	4
norman	$\frac{2\tan\left(\frac{e^x}{2}\right)}{1+\tan\left(\frac{e^x}{2}\right)^2}$	19

input
```
int(exp(x)*cos(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
sin(exp(x))
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^x \cos(e^x) \, dx = \sin(e^x)$$

input
```
integrate(exp(x)*cos(exp(x)),x, algorithm="fricas")
```

output
```
sin(e^x)
```

3.64.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^x \cos(e^x) \, dx = \sin(e^x)$$

input
```
integrate(exp(x)*cos(exp(x)),x)
```

output
```
sin(exp(x))
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^x \cos(e^x) \, dx = \sin(e^x)$$

input
```
integrate(exp(x)*cos(exp(x)),x, algorithm="maxima")
```

output
```
sin(e^x)
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^x \cos(e^x) \, dx = \sin(e^x)$$

input
```
integrate(exp(x)*cos(exp(x)),x, algorithm="giac")
```

output
```
sin(e^x)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 27.70 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^x \cos(e^x)\, dx = \sin(e^x)$$

input
```
int(cos(exp(x))*exp(x),x)
```

output
```
sin(exp(x))
```

3.65 $\int e^{2x} \cos(e^{2x}) \, dx$

3.65.1 Optimal result

Integrand size = 12, antiderivative size = 10

$$\int e^{2x} \cos(e^{2x}) \, dx = \frac{1}{2} \sin(e^{2x})$$

output
```
1/2*sin(exp(2*x))
```

3.65.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int e^{2x} \cos(e^{2x}) \, dx = \frac{1}{2} \sin(e^{2x})$$

input
```
Integrate[E^(2*x)*Cos[E^(2*x)],x]
```

output
```
Sin[E^(2*x)]/2
```

3.65.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2x} \cos \left(e^{2x}\right) \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2} \int \cos \left(e^{2x}\right) de^{2x}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2} \int \sin \left(e^{2x} + \frac{\pi}{2}\right) de^{2x}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{1}{2} \sin \left(e^{2x}\right)$$

input
```
Int[E^(2*x)*Cos[E^(2*x)],x]
```

output
```
Sin[E^(2*x)]/2
```

3.65.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.65.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\sin\left(e^{2x}\right)}{2}$	8
default	$\frac{\sin\left(e^{2x}\right)}{2}$	8
risch	$\frac{\sin\left(e^{2x}\right)}{2}$	8
parallelrisch	$\frac{\sin\left(e^{2x}\right)}{2}$	8
norman	$\frac{\tan\left(\frac{e^{2x}}{2}\right)}{1+\tan\left(\frac{e^{2x}}{2}\right)^{2}}$	22

input
```
int(exp(2*x)*cos(exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(exp(2*x))
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2x}\cos\left(e^{2x}\right)\,dx = \frac{1}{2}\,\sin\left(e^{(2\,x)}\right)$$

input
```
integrate(exp(2*x)*cos(exp(2*x)),x, algorithm="fricas")
```

output
```
1/2*sin(e^(2*x))
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2x}\cos\left(e^{2x}\right)\,dx=\frac{\sin\left(e^{2x}\right)}{2}$$

input
```
integrate(exp(2*x)*cos(exp(2*x)),x)
```

output
```
sin(exp(2*x))/2
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2x}\cos\left(e^{2x}\right)\,dx=\frac{1}{2}\,\sin\left(e^{(2\,x)}\right)$$

input
```
integrate(exp(2*x)*cos(exp(2*x)),x, algorithm="maxima")
```

output
```
1/2*sin(e^(2*x))
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2x}\cos\left(e^{2x}\right)\,dx=\frac{1}{2}\,\sin\left(e^{(2\,x)}\right)$$

input
```
integrate(exp(2*x)*cos(exp(2*x)),x, algorithm="giac")
```

output
```
1/2*sin(e^(2*x))
```

3.65.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2x}\cos\left(e^{2x}\right)\,dx=\frac{\sin\left(\mathrm{e}^{2\,x}\right)}{2}$$

input
```
int(exp(2*x)*cos(exp(2*x)),x)
```

output
```
sin(exp(2*x))/2
```

3.66 $\int e^{-2x} \cos(e^{-2x})\,dx$

3.66.1 Optimal result

Integrand size = 12, antiderivative size = 10

$$\int e^{-2x} \cos\left(e^{-2x}\right)\,dx = -\frac{1}{2}\sin\left(e^{-2x}\right)$$

output
```
-1/2*sin(exp(-2*x))
```

3.66.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int e^{-2x} \cos\left(e^{-2x}\right)\,dx = -\frac{1}{2}\sin\left(e^{-2x}\right)$$

input
```
Integrate[Cos[E^(-2*x)]/E^(2*x),x]
```

output
```
-1/2*Sin[E^(-2*x)]
```

3.66.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2x}\cos\left(e^{-2x}\right)\,dx$$

$$\Big\downarrow \text{2720}$$

$$-\frac{1}{2}\int \cos\left(e^{-2x}\right)de^{-2x}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \sin\left(e^{-2x}+\frac{\pi}{2}\right)de^{-2x}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{1}{2}\sin\left(e^{-2x}\right)$$

input
```
Int[Cos[E^(-2*x)]/E^(2*x),x]
```

output
```
-1/2*Sin[E^(-2*x)]
```

3.66.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.66.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\sin(e^{-2x})}{2}$	8
risch	$-\frac{\sin(e^{-2x})}{2}$	8
norman	$-\frac{\tan\left(\frac{e^{-2x}}{2}\right)}{1+\tan\left(\frac{e^{-2x}}{2}\right)^2}$	23

input
```
int(cos(exp(-2*x))/exp(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*sin(exp(-2*x))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{-2x}\cos\left(e^{-2x}\right)\,dx = -\frac{1}{2}\,\sin\left(e^{(-2\,x)}\right)$$

input
```
integrate(cos(exp(-2*x))/exp(2*x),x, algorithm="fricas")
```

output
```
-1/2*sin(e^(-2*x))
```

3.66.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int e^{-2x}\cos\left(e^{-2x}\right)\,dx=-\frac{\sin\left(e^{-2x}\right)}{2}$$

input
```
integrate(cos(exp(-2*x))/exp(2*x),x)
```

output
```
-sin(exp(-2*x))/2
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{-2x}\cos\left(e^{-2x}\right)\,dx=-\frac{1}{2}\,\sin\left(e^{(-2\,x)}\right)$$

input
```
integrate(cos(exp(-2*x))/exp(2*x),x, algorithm="maxima")
```

output
```
-1/2*sin(e^(-2*x))
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{-2x}\cos\left(e^{-2x}\right)\,dx=-\frac{1}{2}\,\sin\left(e^{(-2\,x)}\right)$$

input
```
integrate(cos(exp(-2*x))/exp(2*x),x, algorithm="giac")
```

output
```
-1/2*sin(e^(-2*x))
```

3.66.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{-2x} \cos \left(e^{-2x}\right) \, dx = -\frac{\sin \left(\mathrm{e}^{-2\,x}\right)}{2}$$

input
```
int(exp(-2*x)*cos(exp(-2*x)),x)
```

output
```
-sin(exp(-2*x))/2
```

3.67 $\int e^{2x}\cos(e^x)\,dx$

3.67.1 Optimal result

Integrand size = 10, antiderivative size = 13

$$\int e^{2x}\cos(e^x)\,dx=\cos(e^x)+e^x\sin(e^x)$$

output
```
cos(exp(x))+exp(x)*sin(exp(x))
```

3.67.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{2x}\cos(e^x)\,dx=\cos(e^x)+e^x\sin(e^x)$$

input
```
Integrate[E^(2*x)*Cos[E^x],x]
```

output
```
Cos[E^x] + E^x*Sin[E^x]
```

3.67.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2720, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2x}\cos\left(e^x\right)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int e^x\cos\left(e^x\right)de^x$$

$$\Big\downarrow \text{3042}$$

$$\int e^x\sin\left(e^x+\frac{\pi}{2}\right)de^x$$

$$\Big\downarrow \text{3777}$$

$$\int -\sin\left(e^x\right)de^x+e^x\sin\left(e^x\right)$$

$$\Big\downarrow \text{25}$$

$$e^x\sin\left(e^x\right)-\int \sin\left(e^x\right)de^x$$

$$\Big\downarrow \text{3042}$$

$$e^x\sin\left(e^x\right)-\int \sin\left(e^x\right)de^x$$

$$\Big\downarrow \text{3118}$$

$$e^x\sin\left(e^x\right)+\cos\left(e^x\right)$$

input
```
Int[E^(2*x)*Cos[E^x],x]
```

output
```
Cos[E^x] + E^x*Sin[E^x]
```

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.67.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

method	result	size
risch	$\cos\left(e^{x}\right)+e^{x} \sin\left(e^{x}\right)$	11
norman	$\frac{2 e^{x} \tan\left(\frac{e^{x}}{2}\right)+2}{1+\tan\left(\frac{e^{x}}{2}\right)^{2}}$	24

input
```
int(exp(2*x)*cos(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
cos(exp(x))+exp(x)*sin(exp(x))
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{2x} \cos\left(e^x\right) \, dx = e^x \sin\left(e^x\right) + \cos\left(e^x\right)$$

input
```
integrate(exp(2*x)*cos(exp(x)),x, algorithm="fricas")
```

output
```
e^x*sin(e^x) + cos(e^x)
```

3.67.6 Sympy [A] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int e^{2x} \cos\left(e^x\right) \, dx = e^x \sin\left(e^x\right) + \cos\left(e^x\right)$$

input
```
integrate(exp(2*x)*cos(exp(x)),x)
```

output
```
exp(x)*sin(exp(x)) + cos(exp(x))
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{2x} \cos\left(e^x\right) \, dx = e^x \sin\left(e^x\right) + \cos\left(e^x\right)$$

input
```
integrate(exp(2*x)*cos(exp(x)),x, algorithm="maxima")
```

output
```
e^x*sin(e^x) + cos(e^x)
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{2x}\cos\left(e^x\right)\,dx = e^x\sin\left(e^x\right)+\cos\left(e^x\right)$$

input
```
integrate(exp(2*x)*cos(exp(x)),x, algorithm="giac")
```

output
```
e^x*sin(e^x) + cos(e^x)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 27.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{2x}\cos\left(e^x\right)\,dx = \cos\left(\mathrm{e}^x\right)+\sin\left(\mathrm{e}^x\right)\,\mathrm{e}^x$$

input
```
int(cos(exp(x))*exp(2*x),x)
```

output
```
cos(exp(x)) + sin(exp(x))*exp(x)
```

3.68 $\int e^{-1+3x} \cos\left(e^{-1+3x}\right) \sin\left(1+e^{-1+3x}\right) dx$

3.68.1 Optimal result

Integrand size = 26, antiderivative size = 30

$$\int e^{-1+3x} \cos\left(e^{-1+3x}\right) \sin\left(1+e^{-1+3x}\right) dx = -\frac{1}{12} \cos\left(1+2e^{-1+3x}\right) + \frac{1}{6} e^{-1+3x} \sin(1)$$

output
```
-1/12*cos(1+2*exp(-1+3*x))+1/6*exp(-1+3*x)*sin(1)
```

3.68.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int e^{-1+3x} \cos\left(e^{-1+3x}\right) \sin\left(1+e^{-1+3x}\right) dx = -\frac{1}{12} \cos\left(1+2e^{-1+3x}\right) + \frac{1}{6} e^{-1+3x} \sin(1)$$

input
```
Integrate[E^(-1 + 3*x)*Cos[E^(-1 + 3*x)]*Sin[1 + E^(-1 + 3*x)],x]
```

output
```
-1/12*Cos[1 + 2*E^(-1 + 3*x)] + (E^(-1 + 3*x)*Sin[1])/6
```

3.68.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2720, 5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3x-1}\sin\left(e^{3x-1}+1\right)\cos\left(e^{3x-1}\right)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}\int \cos\left(e^{3x-1}\right)\sin\left(1+e^{3x-1}\right)de^{3x-1}$$

$$\Big\downarrow \text{5085}$$

$$\frac{1}{3}\int \left(\frac{1}{2}\sin\left(1+2e^{3x-1}\right)+\frac{\sin(1)}{2}\right)de^{3x-1}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1}{2}e^{3x-1}\sin(1)-\frac{1}{4}\cos\left(2e^{3x-1}+1\right)\right)$$

input
```
Int[E^(-1 + 3*x)*Cos[E^(-1 + 3*x)]*Sin[1 + E^(-1 + 3*x)],x]
```

output
```
(-1/4*Cos[1 + 2*E^(-1 + 3*x)] + (E^(-1 + 3*x)*Sin[1])/2)/3
```

3.68.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

3.68.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{\cos\left(1+2\,{\mathrm e}^{-1+3 x}\right)}{12}+\frac{{\mathrm e}^{-1+3 x} \sin\left(1\right)}{6}$
default	$-\frac{\cos\left(1+2\,{\mathrm e}^{-1+3 x}\right)}{12}+\frac{{\mathrm e}^{-1+3 x} \sin\left(1\right)}{6}$
risch	$-\frac{\cos\left(1+2\,{\mathrm e}^{-1+3 x}\right)}{12}+\frac{{\mathrm e}^{-1+3 x} \sin\left(1\right)}{6}$
parallelrisch	$\frac{{\mathrm e}^{-1+3 x} \sin\left(1\right)}{6}-\frac{\cos\left(1+2\,{\mathrm e}^{-1+3 x}\right)}{12}-\frac{\cos\left(1\right)}{12}+\frac{1}{6}$
norman	$\frac{\frac{2 \tan\left(\frac{{\mathrm e}^{-1+3 x}}{2}\right) \tan\left(\frac{1}{2}+\frac{{\mathrm e}^{-1+3 x}}{2}\right)}{3}-\frac{{\mathrm e}^{-1+3 x} \tan\left(\frac{{\mathrm e}^{-1+3 x}}{2}\right)}{3}+\frac{{\mathrm e}^{-1+3 x} \tan\left(\frac{1}{2}+\frac{{\mathrm e}^{-1+3 x}}{2}\right)}{3}+\frac{{\mathrm e}^{-1+3 x} \tan\left(\frac{{\mathrm e}^{-1+3 x}}{2}\right) \tan\left(\frac{1}{2}+{\mathrm e}^{-}\right.}{3}}{\left(1+\tan\left(\frac{{\mathrm e}^{-1+3 x}}{2}\right)^{2}\right) \left(1+\tan\left(\frac{1}{2}+\frac{{\mathrm e}^{-1+3 x}}{2}\right)^{2}\right)}$

input
```
int(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x,method=_RETURNVERBOS
E)
```

output
```
-1/12*cos(1+2*exp(-1+3*x))+1/6*exp(-1+3*x)*sin(1)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int e^{-1+3x}\cos\left(e^{-1+3x}\right)\sin\left(1+e^{-1+3x}\right)\,dx = -\frac{1}{6}\,\cos\left(1\right)\cos\left(e^{(3\,x-1)}\right)^{2} + \frac{1}{6}\,\cos\left(e^{(3\,x-1)}\right)\sin\left(1\right)\sin\left(e^{(3\,x-1)}\right) + \frac{1}{6}\,e^{(3\,x-1)}\sin\left(1\right)$$

input
```
integrate(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x, algorithm="fr
icas")
```

output
```
-1/6*cos(1)*cos(e^(3*x - 1))^2 + 1/6*cos(e^(3*x - 1))*sin(1)*sin(e^(3*x -
1)) + 1/6*e^(3*x - 1)*sin(1)
```

3.68.6 Sympy [F(-1)]

Timed out.

$$\int e^{-1+3x}\cos\left(e^{-1+3x}\right)\sin\left(1+e^{-1+3x}\right)\,dx=\text{Timed out}$$

input
```
integrate(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x)
```

output
```
Timed out
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int e^{-1+3x}\cos\left(e^{-1+3x}\right)\sin\left(1+e^{-1+3x}\right)\,dx=\frac{1}{6}\,e^{(3\,x-1)}\sin\left(1\right)-\frac{1}{12}\,\cos\left(2\,e^{(3\,x-1)}+1\right)$$

input
```
integrate(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x, algorithm="ma
xima")
```

output
```
1/6*e^(3*x - 1)*sin(1) - 1/12*cos(2*e^(3*x - 1) + 1)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int e^{-1+3x}\cos\left(e^{-1+3x}\right)\sin\left(1+e^{-1+3x}\right)\,dx=\frac{1}{6}\,e^{(3\,x-1)}\sin\left(1\right)-\frac{1}{12}\,\cos\left(2\,e^{(3\,x-1)}+1\right)$$

input
```
integrate(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x, algorithm="gi
ac")
```

output
```
1/6*e^(3*x - 1)*sin(1) - 1/12*cos(2*e^(3*x - 1) + 1)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int e^{-1+3x}\cos\left(e^{-1+3x}\right)\sin\left(1+e^{-1+3x}\right)\,dx=\frac{\mathrm{e}^{3\,x-1}\,\sin\left(1\right)}{6}-\frac{\cos\left(2\,\mathrm{e}^{3\,x-1}+1\right)}{12}$$

input
```
int(exp(3*x - 1)*sin(exp(3*x - 1) + 1)*cos(exp(3*x - 1)),x)
```

output
```
(exp(3*x - 1)*sin(1))/6 - cos(2*exp(3*x - 1) + 1)/12
```

3.69 $\int e^x \tan(e^x) \, dx$

3.69.1 Optimal result

Integrand size = 8, antiderivative size = 7

$$\int e^x \tan(e^x) \, dx = -\log(\cos(e^x))$$

output
```
-ln(cos(exp(x)))
```

3.69.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int e^x \tan(e^x) \, dx = -\log(\cos(e^x))$$

input
```
Integrate[E^x*Tan[E^x],x]
```

output
```
-Log[Cos[E^x]]
```

3.69.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2720, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \tan(e^x)\, dx$$
$$\Big\downarrow \text{ 2720}$$
$$\int \tan(e^x)\, de^x$$
$$\Big\downarrow \text{ 3042}$$
$$\int \tan(e^x)\, de^x$$
$$\Big\downarrow \text{ 3956}$$
$$-\log(\cos(e^x))$$

input
```
Int[E^x*Tan[E^x],x]
```

output
```
-Log[Cos[E^x]]
```

3.69.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.69.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\ln(\cos(e^{x}))$	7
default	$-\ln(\cos(e^{x}))$	7
norman	$\frac{\ln\left(1+\tan(e^{x})^{2}\right)}{2}$	11
parallelrisch	$\frac{\ln\left(1+\tan(e^{x})^{2}\right)}{2}$	11
risch	$i e^{x}-\ln\left(e^{2 i e^{x}}+1\right)$	18

input
```
int(exp(x)*tan(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(cos(exp(x)))
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int e^{x}\tan(e^{x})\,dx=-\frac{1}{2}\log\left(\frac{1}{\tan(e^{x})^{2}+1}\right)$$

input
```
integrate(exp(x)*tan(exp(x)),x, algorithm="fricas")
```

output
```
-1/2*log(1/(tan(e^x)^2 + 1))
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.43

$$\int e^x \tan(e^x) \, dx = \frac{\log(\tan^2(e^x) + 1)}{2}$$

input
```
integrate(exp(x)*tan(exp(x)),x)
```

output
```
log(tan(exp(x))**2 + 1)/2
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.57

$$\int e^x \tan(e^x) \, dx = \log(\sec(e^x))$$

input
```
integrate(exp(x)*tan(exp(x)),x, algorithm="maxima")
```

output
```
log(sec(e^x))
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int e^x \tan(e^x) \, dx = -\log(|\cos(e^x)|)$$

input
```
integrate(exp(x)*tan(exp(x)),x, algorithm="giac")
```

output
```
-log(abs(cos(e^x)))
```

3.69.9 Mupad [B] (verification not implemented)

Time = 27.55 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.43

$$\int e^x \tan(e^x) \, dx = \frac{\ln\left(\tan(e^x)^2 + 1\right)}{2}$$

input
```
int(tan(exp(x))*exp(x),x)
```

output
```
log(tan(exp(x))^2 + 1)/2
```

3.70 $\int e^x \sec(e^x)\, dx$

3.70.1 Optimal result

Integrand size = 8, antiderivative size = 5

$$\int e^x \sec(e^x)\, dx = \text{arctanh}(\sin(e^x))$$

output
```
arctanh(sin(exp(x)))
```

3.70.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int e^x \sec(e^x)\, dx = \text{arctanh}(\sin(e^x))$$

input
```
Integrate[E^x*Sec[E^x],x]
```

output
```
ArcTanh[Sin[E^x]]
```

3.70.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2720, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sec(e^x) \, dx$$

$$\Big\downarrow \; 2720$$

$$\int \sec(e^x) \, de^x$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(e^x + \frac{\pi}{2}\right) de^x$$

$$\Big\downarrow \; 4257$$

$$\text{arctanh}(\sin(e^x))$$

input
```
Int[E^x*Sec[E^x],x]
```

output
```
ArcTanh[Sin[E^x]]
```

3.70.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.70.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

method	result	size
derivativedivides	$\ln(\sec(e^x)+\tan(e^x))$	9
default	$\ln(\sec(e^x)+\tan(e^x))$	9
norman	$-\ln\left(\tan\left(\frac{e^x}{2}\right)-1\right)+\ln\left(\tan\left(\frac{e^x}{2}\right)+1\right)$	20
parallelrisch	$-\ln\left(\tan\left(\frac{e^x}{2}\right)-1\right)+\ln\left(\tan\left(\frac{e^x}{2}\right)+1\right)$	20
risch	$-\ln\left(e^{ie^x}-i\right)+\ln\left(e^{ie^x}+i\right)$	24

input
```
int(exp(x)*sec(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(sec(exp(x))+tan(exp(x)))
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(4) = 8$.

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 3.80

$$\int e^x \sec(e^x)\,dx = \frac{1}{2}\log(\sin(e^x)+1) - \frac{1}{2}\log(-\sin(e^x)+1)$$

input
```
integrate(exp(x)*sec(exp(x)),x, algorithm="fricas")
```

output
```
1/2*log(sin(e^x) + 1) - 1/2*log(-sin(e^x) + 1)
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.00

$$\int e^x \sec(e^x)\, dx = \log(\tan(e^x) + \sec(e^x))$$

input
```
integrate(exp(x)*sec(exp(x)),x)
```

output
```
log(tan(exp(x)) + sec(exp(x)))
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

$$\int e^x \sec(e^x)\, dx = \log(\sec(e^x) + \tan(e^x))$$

input
```
integrate(exp(x)*sec(exp(x)),x, algorithm="maxima")
```

output
```
log(sec(e^x) + tan(e^x))
```

3.70.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(4) = 8.

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 5.80

$$\int e^x \sec(e^x)\, dx = \frac{1}{4}\log\left(\left|\frac{1}{\sin(e^x)} + \sin(e^x) + 2\right|\right) - \frac{1}{4}\log\left(\left|\frac{1}{\sin(e^x)} + \sin(e^x) - 2\right|\right)$$

input
```
integrate(exp(x)*sec(exp(x)),x, algorithm="giac")
```

output
```
1/4*log(abs(1/sin(e^x) + sin(e^x) + 2)) - 1/4*log(abs(1/sin(e^x) + sin(e^x
) - 2))
```

3.70.9 Mupad [B] (verification not implemented)

Time = 27.97 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.00

$$\int e^x \sec(e^x)\, dx = -\mathrm{atan}\left(\mathrm{e}^{\mathrm{e}^x\,1\mathrm{i}}\right)\, 2\mathrm{i}$$

input
```
int(exp(x)/cos(exp(x)),x)
```

output
```
-atan(exp(exp(x)*1i))*2i
```

3.71 $\int e^x \sec(e^x) \tan(e^x) \, dx$

3.71.1 Optimal result

Integrand size = 12, antiderivative size = 4

$$\int e^x \sec(e^x) \tan(e^x) \, dx = \sec(e^x)$$

output
```
sec(exp(x))
```

3.71.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int e^x \sec(e^x) \tan(e^x) \, dx = \sec(e^x)$$

input
```
Integrate[E^x*Sec[E^x]*Tan[E^x],x]
```

output
```
Sec[E^x]
```

3.71.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2720, 3042, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \tan(e^x) \sec(e^x) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\int \tan(e^x) \sec(e^x) de^x$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e^x) \sec(e^x) de^x$$

$$\Big\downarrow \text{3086}$$

$$\int 1 d \sec(e^x)$$

$$\Big\downarrow \text{24}$$

$$\sec(e^x)$$

input
```
Int[E^x*Sec[E^x]*Tan[E^x],x]
```

output
```
Sec[E^x]
```

3.71.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.71.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\sec\left(e^{x}\right)$	4
default	$\sec\left(e^{x}\right)$	4
risch	$\frac{2\,e^{i e^{x}}}{e^{2 i e^{x}}+1}$	19

input
```
int(exp(x)*sec(exp(x))*tan(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
sec(exp(x))
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int e^{x} \sec\left(e^{x}\right) \tan\left(e^{x}\right) \, dx = \frac{1}{\cos\left(e^{x}\right)}$$

input
```
integrate(exp(x)*sec(exp(x))*tan(exp(x)),x, algorithm="fricas")
```

output
```
1/cos(e^x)
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^x \sec(e^x)\tan(e^x)\,dx = \sec(e^x)$$

input
```
integrate(exp(x)*sec(exp(x))*tan(exp(x)),x)
```

output
```
sec(exp(x))
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int e^x \sec(e^x)\tan(e^x)\,dx = \frac{1}{\cos(e^x)}$$

input
```
integrate(exp(x)*sec(exp(x))*tan(exp(x)),x, algorithm="maxima")
```

output
```
1/cos(e^x)
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int e^x \sec(e^x)\tan(e^x)\,dx = \frac{1}{\cos(e^x)}$$

input
```
integrate(exp(x)*sec(exp(x))*tan(exp(x)),x, algorithm="giac")
```

output
```
1/cos(e^x)
```

3.71.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int e^x \sec(e^x) \tan(e^x) \, dx = \frac{1}{\cos(e^x)}$$

input
```
int((tan(exp(x))*exp(x))/cos(exp(x)),x)
```

output
```
1/cos(exp(x))
```

3.72 $\int e^x \csc^2(e^x)\,dx$

3.72.1 Optimal result

Integrand size = 10, antiderivative size = 6

$$\int e^x \csc^2(e^x)\,dx = -\cot(e^x)$$

output
```
-cot(exp(x))
```

3.72.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int e^x \csc^2(e^x)\,dx = -\cot(e^x)$$

input
```
Integrate[E^x*Csc[E^x]^2,x]
```

output
```
-Cot[E^x]
```

3.72.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2720, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \csc^2\left(e^x\right) dx$$

$$\Big\downarrow \text{2720}$$

$$\int \csc^2\left(e^x\right) de^x$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e^x\right)^2 de^x$$

$$\Big\downarrow \text{4254}$$

$$-\int 1 d\cot\left(e^x\right)$$

$$\Big\downarrow \text{24}$$

$$-\cot\left(e^x\right)$$

input
```
Int[E^x*Csc[E^x]^2,x]
```

output
```
-Cot[E^x]
```

3.72.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.72.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\cot\left(e^{x}\right)$	6
default	$-\cot\left(e^{x}\right)$	6
risch	$-\frac{2i}{e^{2ie^{x}}-1}$	14
parallelrisch	$-\frac{\cot\left(\frac{e^{x}}{2}\right)}{2}+\frac{\tan\left(\frac{e^{x}}{2}\right)}{2}$	16
norman	$\frac{-\frac{1}{2}+\frac{\tan\left(\frac{e^{x}}{2}\right)^{2}}{2}}{\tan\left(\frac{e^{x}}{2}\right)}$	20

input
```
int(exp(x)*csc(exp(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-cot(exp(x))
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.67

$$\int e^{x}\csc^{2}\left(e^{x}\right)\,dx=-\frac{\cos\left(e^{x}\right)}{\sin\left(e^{x}\right)}$$

input
```
integrate(exp(x)*csc(exp(x))^2,x, algorithm="fricas")
```

output
```
-cos(e^x)/sin(e^x)
```

3.72.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x \csc^2\left(e^x\right) \, dx = -\cot\left(e^x\right)$$

input
```
integrate(exp(x)*csc(exp(x))**2,x)
```

output
```
-cot(exp(x))
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int e^x \csc^2\left(e^x\right) \, dx = -\frac{1}{\tan\left(e^x\right)}$$

input
```
integrate(exp(x)*csc(exp(x))^2,x, algorithm="maxima")
```

output
```
-1/tan(e^x)
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int e^x \csc^2\left(e^x\right) \, dx = -\frac{1}{\tan\left(e^x\right)}$$

input
```
integrate(exp(x)*csc(exp(x))^2,x, algorithm="giac")
```

output
```
-1/tan(e^x)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 28.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 2.17

$$\int e^x \csc^2(e^x)\,dx = -\frac{2\mathrm{i}}{\mathrm{e}^{\mathrm{e}^x\,2\mathrm{i}} - 1}$$

input
```
int(exp(x)/sin(exp(x))^2,x)
```

output
```
-2i/(exp(exp(x)*2i) - 1)
```

3.73 $\int e^x \sin(a+bx)\,dx$

3.73.1 Optimal result

Integrand size = 10, antiderivative size = 37

$$\int e^x \sin(a+bx)\,dx = -\frac{be^x\cos(a+bx)}{1+b^2}+\frac{e^x\sin(a+bx)}{1+b^2}$$

output
```
-b*exp(x)*cos(b*x+a)/(b^2+1)+exp(x)*sin(b*x+a)/(b^2+1)
```

3.73.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int e^x \sin(a+bx)\,dx = \frac{e^x(-b\cos(a+bx)+\sin(a+bx))}{1+b^2}$$

input
```
Integrate[E^x*Sin[a + b*x],x]
```

output
```
(E^x*(-(b*Cos[a + b*x]) + Sin[a + b*x]))/(1 + b^2)
```

3.73.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sin(a+bx)\,dx$$

$$\Big\downarrow \text{4932}$$

$$\frac{e^x \sin(a+bx)}{b^2+1}-\frac{be^x \cos(a+bx)}{b^2+1}$$

input
```
Int[E^x*Sin[a + b*x],x]
```

output
```
-((b*E^x*Cos[a + b*x])/(1 + b^2)) + (E^x*Sin[a + b*x])/(1 + b^2)
```

3.73.3.1 Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.73.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

method	result	size
parallelrisch	$\frac{{\mathrm e}^{x} \left(-b \cos \left(x b +a \right)+\sin \left(x b +a \right)\right)}{b^{2}+1}$	27
default	$-\frac{b \,{\mathrm e}^{x} \cos \left(x b +a \right)}{b^{2}+1}+\frac{{\mathrm e}^{x} \sin \left(x b +a \right)}{b^{2}+1}$	36
risch	$\frac{{\mathrm e}^{x} \left(2 b \cos \left(x b +a \right)-2 \sin \left(x b +a \right)\right)}{2 \left(-b +i\right) \left(i+b \right)}$	37
norman	$\frac{\frac{b \,{\mathrm e}^{x} \tan \left(\frac{a}{2}+\frac{x b}{2}\right)^{2}}{b^{2}+1}-\frac{b \,{\mathrm e}^{x}}{b^{2}+1}+\frac{2 \,{\mathrm e}^{x} \tan \left(\frac{a}{2}+\frac{x b}{2}\right)}{b^{2}+1}}{1+\tan \left(\frac{a}{2}+\frac{x b}{2}\right)^{2}}$	72

input
```
int(exp(x)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
exp(x)/(b^2+1)*(-b*cos(b*x+a)+sin(b*x+a))
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int e^x \sin(a+bx)\,dx = -\frac{b\cos(bx+a)\,e^x - e^x\sin(bx+a)}{b^2+1}$$

input
```
integrate(exp(x)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-(b*cos(b*x + a)*e^x - e^x*sin(b*x + a))/(b^2 + 1)
```

3.73.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.08

$$\int e^x \sin(a+bx)\,dx = \begin{cases} \frac{xe^x\sin(a-ix)}{2} + \frac{ixe^x\cos(a-ix)}{2} + \frac{e^x\sin(a-ix)}{2} & \text{for } b=-i \\ \frac{xe^x\sin(a+ix)}{2} - \frac{ixe^x\cos(a+ix)}{2} + \frac{ie^x\cos(a+ix)}{2} & \text{for } b=i \\ -\frac{be^x\cos(a+bx)}{b^2+1} + \frac{e^x\sin(a+bx)}{b^2+1} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(x)*sin(b*x+a),x)
```

output
```
Piecewise((x*exp(x)*sin(a - I*x)/2 + I*x*exp(x)*cos(a - I*x)/2 + exp(x)*si
n(a - I*x)/2, Eq(b, -I)), (x*exp(x)*sin(a + I*x)/2 - I*x*exp(x)*cos(a + I*
x)/2 + I*exp(x)*cos(a + I*x)/2, Eq(b, I)), (-b*exp(x)*cos(a + b*x)/(b**2 +
 1) + exp(x)*sin(a + b*x)/(b**2 + 1), True))
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int e^x \sin(a + bx)\, dx = -\frac{(b \cos(bx + a) - \sin(bx + a))e^x}{b^2 + 1}$$

input
```
integrate(exp(x)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-(b*cos(b*x + a) - sin(b*x + a))*e^x/(b^2 + 1)
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int e^x \sin(a + bx)\, dx = -\left(\frac{b \cos(bx + a)}{b^2 + 1} - \frac{\sin(bx + a)}{b^2 + 1}\right)e^x$$

input
```
integrate(exp(x)*sin(b*x+a),x, algorithm="giac")
```

output
```
-(b*cos(b*x + a)/(b^2 + 1) - sin(b*x + a)/(b^2 + 1))*e^x
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int e^x \sin(a + bx)\, dx = \frac{e^x (\sin(a + b\,x) - b\,\cos(a + b\,x))}{b^2 + 1}$$

input
```
int(exp(x)*sin(a + b*x),x)
```

output
```
(exp(x)*(sin(a + b*x) - b*cos(a + b*x)))/(b^2 + 1)
```

3.74 $\int e^x \sin(a+cx^2)\,dx$

3.74.1 Optimal result

Integrand size = 12, antiderivative size = 115

$$\int e^x \sin(a+cx^2)\,dx = \frac{(-1)^{3/4}e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{(-1)^{3/4}e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

output
```
1/4*(-1)^(3/4)*exp(1/4*I*(4*a+1/c))*erf(1/2*(-1)^(1/4)*(1+2*I*c*x)/c^(1/2)
)*Pi^(1/2)/c^(1/2)+1/4*(-1)^(3/4)*erfi(1/2*(-1)^(1/4)*(1-2*I*c*x)/c^(1/2))
*Pi^(1/2)/exp(1/4*I*(4*a+1/c))/c^(1/2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

$$\int e^x \sin(a+cx^2)\,dx = -\frac{\sqrt[4]{-1}e^{-\frac{i}{4}/c}\sqrt{\pi}\left(e^{\frac{i}{2}/c}\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(-i+2cx)}{2\sqrt{c}}\right)(\cos(a)+i\sin(a))+\mathrm{erfi}\left(\frac{(-1)^{3/4}(i+2cx)}{2\sqrt{c}}\right)(i\cos(a)+\sin(a))\right)}{4\sqrt{c}}$$

input
```
Integrate[E^x*Sin[a + c*x^2],x]
```

output
```
-1/4*((-1)^(1/4)*Sqrt[Pi]*(E^((I/2)/c)*Erfi[((-1)^(1/4)*(-I + 2*c*x))/(2*S
qrt[c])]*(Cos[a] + I*Sin[a]) + Erfi[((-1)^(3/4)*(I + 2*c*x))/(2*Sqrt[c])]*
(I*Cos[a] + Sin[a])))/(Sqrt[c]*E^((I/4)/c))
```

3.74.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sin\left(a+cx^2\right) dx$$

$$\Big\downarrow \text{ 4975}$$

$$\int \left(\frac{1}{2}ie^{-ia-icx^2+x}-\frac{1}{2}ie^{ia+icx^2+x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(-1)^{3/4}\sqrt{\pi}e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\text{erf}\left(\frac{\sqrt[4]{-1}(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{(-1)^{3/4}\sqrt{\pi}e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\text{erfi}\left(\frac{\sqrt[4]{-1}(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

input
```
Int[E^x*Sin[a + c*x^2],x]
```

output
```
((-1)^(3/4)*E^((I/4)*(4*a + c^(-1)))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 + (2*I)*c
*x))/(2*Sqrt[c])])/(4*Sqrt[c]) + ((-1)^(3/4)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1
- (2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]*E^((I/4)*(4*a + c^(-1))))
```

3.74.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.74.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{i(4ac+1)}{4c}}\operatorname{erf}\left(\sqrt{-ic}\,x-\frac{1}{2\sqrt{-ic}}\right)}{4\sqrt{-ic}}+\frac{i\sqrt{\pi}\,\mathrm{e}^{-\frac{i(4ac+1)}{4c}}\operatorname{erf}\left(\sqrt{ic}\,x-\frac{1}{2\sqrt{ic}}\right)}{4\sqrt{ic}}$	88

input
```
int(exp(x)*sin(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*Pi^(1/2)*exp(1/4*I*(4*a*c+1)/c)/(-I*c)^(1/2)*erf((-I*c)^(1/2)*x-1/2
/(-I*c)^(1/2))+1/4*I*Pi^(1/2)*exp(-1/4*I*(4*a*c+1)/c)/(I*c)^(1/2)*erf((I*c
)^(1/2)*x-1/2/(I*c)^(1/2))
```

3.74.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(73) = 146.

Time = 0.24 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.26

$$\int e^x \sin\left(a+cx^2\right)\,dx$$

$$= \frac{\sqrt{2}\left(i\,\pi\cos\left(\frac{4\,ac+1}{4\,c}\right)+\pi\sin\left(\frac{4\,ac+1}{4\,c}\right)\right)\sqrt{\frac{c}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}(2\,cx+i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)+\sqrt{2}\left(i\,\pi\cos\left(\frac{4\,ac+1}{4\,c}\right)-\pi\sin\left(\frac{4\,ac+1}{4\,c}\right)\right)\sqrt{\frac{c}{\pi}}\,\mathrm{C}\left(-\right.}{}$$

input
```
integrate(exp(x)*sin(c*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*(I*pi*cos(1/4*(4*a*c + 1)/c) + pi*sin(1/4*(4*a*c + 1)/c))*sqr
t(c/pi)*fresnel_cos(1/2*sqrt(2)*(2*c*x + I)*sqrt(c/pi)/c) + sqrt(2)*(I*pi*
cos(1/4*(4*a*c + 1)/c) - pi*sin(1/4*(4*a*c + 1)/c))*sqrt(c/pi)*fresnel_cos
(-1/2*sqrt(2)*(2*c*x - I)*sqrt(c/pi)/c) + sqrt(2)*(pi*cos(1/4*(4*a*c + 1)/
c) - I*pi*sin(1/4*(4*a*c + 1)/c))*sqrt(c/pi)*fresnel_sin(1/2*sqrt(2)*(2*c*
x + I)*sqrt(c/pi)/c) - sqrt(2)*(pi*cos(1/4*(4*a*c + 1)/c) + I*pi*sin(1/4*(
4*a*c + 1)/c))*sqrt(c/pi)*fresnel_sin(-1/2*sqrt(2)*(2*c*x - I)*sqrt(c/pi)/
c))/c
```

3.74.6 Sympy [F]

$$\int e^x \sin\left(a+cx^2\right)\,dx=\int e^x \sin\left(a+cx^2\right)\,dx$$

input
```
integrate(exp(x)*sin(c*x**2+a),x)
```

output
```
Integral(exp(x)*sin(a + c*x**2), x)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

$$\int e^x \sin\left(a+cx^2\right)\,dx=$$
$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\cos\left(\frac{4\,ac+1}{4\,c}\right)+(i-1)\sin\left(\frac{4\,ac+1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{2i\,cx-1}{2\,\sqrt{i\,c}}\right)+\left(-(i-1)\cos\left(\frac{4\,ac+1}{4\,c}\right)+(i+1)\sin\right.}{8\,\sqrt{c}}$$

input
```
integrate(exp(x)*sin(c*x^2+a),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(1/4*(4*a*c + 1)/c) + (I - 1)*sin(1/4*
(4*a*c + 1)/c))*erf(1/2*(2*I*c*x - 1)/sqrt(I*c)) + (-(I - 1)*cos(1/4*(4*a*
c + 1)/c) + (I + 1)*sin(1/4*(4*a*c + 1)/c))*erf(1/2*(2*I*c*x + 1)/sqrt(-I*
c)))/sqrt(c)
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

$$\int e^x \sin\left(a+cx^2\right)\,dx=\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\frac{1}{4}i\,\sqrt{2}\left(2\,x+\frac{i}{c}\right)\left(-\frac{i\,c}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{4i\,ac+i}{4\,c}\right)}}{4\left(-\frac{i\,c}{|c|}+1\right)\sqrt{|c|}}+\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}i\,\sqrt{2}\left(2\,x-\frac{i}{c}\right)\left(\frac{i\,c}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-4i\,ac-i}{4\,c}\right)}}{4\left(\frac{i\,c}{|c|}+1\right)\sqrt{|c|}}$$

input
```
integrate(exp(x)*sin(c*x^2+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*sqrt(pi)*erf(1/4*I*sqrt(2)*(2*x + I/c)*(-I*c/abs(c) + 1)*sqrt(
abs(c)))*e^(-1/4*(4*I*a*c + I)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c))) + 1/4*s
qrt(2)*sqrt(pi)*erf(-1/4*I*sqrt(2)*(2*x - I/c)*(I*c/abs(c) + 1)*sqrt(abs(c
)))*e^(-1/4*(-4*I*a*c - I)/c)/((I*c/abs(c) + 1)*sqrt(abs(c)))
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int e^x \sin\left(a + cx^2\right)\,dx = \int e^x\,\sin\left(c\,x^2 + a\right)\,dx$$

input
```
int(exp(x)*sin(a + c*x^2),x)
```

output
```
int(exp(x)*sin(a + c*x^2), x)
```

3.75 $\int e^x \sin(a+bx+cx^2)\,dx$

3.75.1 Optimal result

Integrand size = 15, antiderivative size = 144

$$\int e^x \sin(a+bx+cx^2)\,dx = \frac{(-1)^{3/4}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt[4]{-1}(1+ib+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{(-1)^{3/4}e^{-ia+\frac{i(i+b)^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt[4]{-1}(1-ib-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

output
```
1/4*(-1)^(3/4)*exp(1/4*I*(4*a+(1+I*b)^2/c))*erf(1/2*(-1)^(1/4)*(1+I*b+2*I*
c*x)/c^(1/2))*Pi^(1/2)/c^(1/2)+1/4*(-1)^(3/4)*exp(-I*a+1/4*I*(I+b)^2/c)*er
fi(1/2*(-1)^(1/4)*(1-I*b-2*I*c*x)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

3.75.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.93

$$\int e^x \sin(a+bx+cx^2)\,dx = -\frac{\sqrt[4]{-1}e^{-\frac{i(1-2ib+b^2)}{4c}}\sqrt{\pi}\left(e^{\frac{i}{2}/c}\text{erfi}\left(\frac{\sqrt[4]{-1}(-i+b+2cx)}{2\sqrt{c}}\right)(\cos(a)+i\sin(a))+e^{\frac{ib^2}{2c}}\text{erfi}\left(\frac{(-1)^{3/4}(i+b+2cx)}{2\sqrt{c}}\right)(i\cos(a)\right.}{4\sqrt{c}}$$

input
```
Integrate[E^x*Sin[a + b*x + c*x^2],x]
```

output
```
-1/4*((-1)^(1/4)*Sqrt[Pi]*(E^((I/2)/c)*Erfi[((-1)^(1/4)*(-I + b + 2*c*x))/
(2*Sqrt[c])]*(Cos[a] + I*Sin[a]) + E^(((I/2)*b^2)/c)*Erfi[((-1)^(3/4)*(I +
 b + 2*c*x))/(2*Sqrt[c])]*(I*Cos[a] + Sin[a])))/(Sqrt[c]*E^(((I/4)*(1 - (2
*I)*b + b^2))/c))
```

3.75.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sin \left(a+bx+cx^2\right) \, dx$$

$$\Big\downarrow \text{ 4975}$$

$$\int \left(\frac{1}{2} i e^{-ia+(1-ib)x-icx^2}-\frac{1}{2} i e^{ia+(1+ib)x+icx^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(-1)^{3/4}\sqrt{\pi}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\text{erf}\left(\frac{\sqrt[4]{-1}(ib+2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{(-1)^{3/4}\sqrt{\pi}e^{\frac{i(b+i)^2}{4c}-ia}\text{erfi}\left(\frac{\sqrt[4]{-1}(-ib-2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

input
```
Int[E^x*Sin[a + b*x + c*x^2],x]
```

output
```
((-1)^(3/4)*E^((I/4)*(4*a + (1 + I*b)^2/c))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 +
I*b + (2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]) + ((-1)^(3/4)*E^((-I)*a + ((I/
4)*(I + b)^2)/c)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 - I*b - (2*I)*c*x))/(2*Sqrt[
c])])/(4*Sqrt[c])
```

3.75.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.75.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{i\left(4ac-b^{2}+2ib+1\right)}{4c}}\operatorname{erf}\left(-\sqrt{-ic}\,x+\frac{ib+1}{2\sqrt{-ic}}\right)}{4\sqrt{-ic}}+\frac{i\sqrt{\pi}\,\mathrm{e}^{-\frac{i\left(4ac-b^{2}-2ib+1\right)}{4c}}\operatorname{erf}\left(\sqrt{ic}\,x-\frac{-ib+1}{2\sqrt{ic}}\right)}{4\sqrt{ic}}$	119

input
```
int(exp(x)*sin(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*Pi^(1/2)*exp(1/4*I*(-b^2+2*I*b+4*a*c+1)/c)/(-I*c)^(1/2)*erf(-(-I*c)^
(1/2)*x+1/2*(1+I*b)/(-I*c)^(1/2))+1/4*I*Pi^(1/2)*exp(-1/4*I*(-b^2-2*I*b+4*
a*c+1)/c)/(I*c)^(1/2)*erf((I*c)^(1/2)*x-1/2*(-I*b+1)/(I*c)^(1/2))
```

3.75.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(91) = 182.

Time = 0.25 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.59

$$\int e^{x}\sin\left(a+bx+cx^{2}\right)\,dx = \frac{i\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{ib^{2}-4i\,ac-2\,b-i}{4\,c}\right)}\operatorname{C}\left(\frac{\sqrt{2}(2\,cx+b+i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)+i\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{-i\,b^{2}+4i\,ac-2\,b+i}{4\,c}\right)}\operatorname{C}\left(-\frac{\sqrt{2}(2\,cx+b-i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)+\sqrt{2}\pi\sqrt{}}{4\,c}$$

input
```
integrate(exp(x)*sin(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/4*(I*sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(I*b^2 - 4*I*a*c - 2*b - I)/c)*fresnel
_cos(1/2*sqrt(2)*(2*c*x + b + I)*sqrt(c/pi)/c) + I*sqrt(2)*pi*sqrt(c/pi)*e
^(1/4*(-I*b^2 + 4*I*a*c - 2*b + I)/c)*fresnel_cos(-1/2*sqrt(2)*(2*c*x + b
- I)*sqrt(c/pi)/c) + sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(I*b^2 - 4*I*a*c - 2*b -
 I)/c)*fresnel_sin(1/2*sqrt(2)*(2*c*x + b + I)*sqrt(c/pi)/c) - sqrt(2)*pi*
sqrt(c/pi)*e^(1/4*(-I*b^2 + 4*I*a*c - 2*b + I)/c)*fresnel_sin(-1/2*sqrt(2)
*(2*c*x + b - I)*sqrt(c/pi)/c))/c
```

3.75.6 Sympy [F]

$$\int e^x \sin\left(a + bx + cx^2\right)\, dx = \int e^x \sin\left(a + bx + cx^2\right) dx$$

input
```
integrate(exp(x)*sin(c*x**2+b*x+a),x)
```

output
```
Integral(exp(x)*sin(a + b*x + c*x**2), x)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91

$$\int e^x \sin\left(a + bx + cx^2\right)\, dx =$$

$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left((i+1)\cos\left(-\frac{b^2-4\,ac-1}{4\,c}\right) - (i-1)\sin\left(-\frac{b^2-4\,ac-1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{i\,(2i\,cx+i\,b-1)\sqrt{i\,c}}{2\,c}\right) + \left(-(i-1)\cos\left(-\right.\right.}{8\,\sqrt{c}}$$

input
```
integrate(exp(x)*sin(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*sqrt(pi)*(((I + 1)*cos(-1/4*(b^2 - 4*a*c - 1)/c) - (I - 1)*si
n(-1/4*(b^2 - 4*a*c - 1)/c))*erf(1/2*I*(2*I*c*x + I*b - 1)*sqrt(I*c)/c) +
(-(I - 1)*cos(-1/4*(b^2 - 4*a*c - 1)/c) + (I + 1)*sin(-1/4*(b^2 - 4*a*c -
1)/c))*erf(1/2*I*(2*I*c*x + I*b + 1)*sqrt(-I*c)/c))*e^(-1/2*b/c)/sqrt(c)
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

$$\int e^x \sin\left(a+bx+cx^2\right)\,dx = \frac{\sqrt{2}\sqrt{\pi}\,\text{erf}\left(-\frac{1}{4}i\sqrt{2}\left(2x+\frac{b-i}{c}\right)\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{ib^2-4iac+2b-i}{4c}\right)}}{4\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}} + \frac{\sqrt{2}\sqrt{\pi}\,\text{erf}\left(\frac{1}{4}i\sqrt{2}\left(2x+\frac{b+i}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-ib^2+4iac+2b+i}{4c}\right)}}{4\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}}$$

input
```
integrate(exp(x)*sin(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*sqrt(pi)*erf(-1/4*I*sqrt(2)*(2*x + (b - I)/c)*(I*c/abs(c) + 1)
*sqrt(abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c + 2*b - I)/c)/((I*c/abs(c) + 1)*sq
rt(abs(c))) + 1/4*sqrt(2)*sqrt(pi)*erf(1/4*I*sqrt(2)*(2*x + (b + I)/c)*(-I
*c/abs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(-I*b^2 + 4*I*a*c + 2*b + I)/c)/((-I*
c/abs(c) + 1)*sqrt(abs(c)))
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int e^x \sin\left(a+bx+cx^2\right)\,dx = \int \sin\left(c\,x^2+b\,x+a\right)\,e^x\,dx$$

input
```
int(sin(a + b*x + c*x^2)*exp(x),x)
```

output
```
int(sin(a + b*x + c*x^2)*exp(x), x)
```

3.76 $\int e^{x^2} \sin(a+bx)\,dx$

3.76.1 Optimal result

Integrand size = 12, antiderivative size = 81

$$\int e^{x^2} \sin(a+bx)\,dx = \frac{1}{4} i e^{-ia+\frac{b^2}{4}} \sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right) - \frac{1}{4} i e^{ia+\frac{b^2}{4}} \sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)$$

output
```
-1/4*I*exp(-I*a+1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)-1/4*I*exp(I*a+1/4*b^2)*e
rfi(1/2*I*b+x)*Pi^(1/2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int e^{x^2} \sin(a+bx)\,dx = \frac{1}{4} e^{\frac{b^2}{4}} \sqrt{\pi} \left(\cos(a)\mathrm{erf}\left(\frac{b}{2}-ix\right) + \cos(a)\mathrm{erf}\left(\frac{b}{2}+ix\right) + \left(\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right) + \mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)\right)\sin(a)\right)$$

input
```
Integrate[E^x^2*Sin[a + b*x],x]
```

output
```
(E^(b^2/4)*Sqrt[Pi]*(Cos[a]*Erf[b/2 - I*x] + Cos[a]*Erf[b/2 + I*x] + (Erfi
[((-I)*b + 2*x)/2] + Erfi[(I*b + 2*x)/2])*Sin[a]))/4
```

3.76.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2}\sin(a+bx)\,dx$$

$$\Big\downarrow \text{4975}$$

$$\int \left(\frac{1}{2}ie^{-ia-ibx+x^2}-\frac{1}{2}ie^{ia+ibx+x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}-ia}\text{erfi}\left(\frac{1}{2}(2x-ib)\right)-\frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}+ia}\text{erfi}\left(\frac{1}{2}(2x+ib)\right)$$

input
```
Int[E^x^2*Sin[a + b*x],x]
```

output
```
(I/4)*E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2] - (I/4)*E^(I*a +
b^2/4)*Sqrt[Pi]*Erfi[(I*b + 2*x)/2]
```

3.76.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.76.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{e}^{ia}\,\mathrm{erf}\left(-ix+\frac{b}{2}\right)}{4}+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{e}^{-ia}\,\mathrm{erf}\left(ix+\frac{b}{2}\right)}{4}$	52

input
```
int(exp(x^2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*exp(1/4*b^2)*exp(I*a)*erf(-I*x+1/2*b)+1/4*Pi^(1/2)*exp(1/4*b^
2)*exp(-I*a)*erf(I*x+1/2*b)
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.56

$$\int e^{x^2}\sin(a+bx)\,dx = -\frac{1}{4}\sqrt{\pi}\left(\mathrm{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2+i\,a\right)}-\mathrm{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2-i\,a\right)}\right)$$

input
```
integrate(exp(x^2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*sqrt(pi)*(erf(-1/2*b + I*x)*e^(1/4*b^2 + I*a) - erf(1/2*b + I*x)*e^(1
/4*b^2 - I*a))
```

3.76.6 Sympy [F]

$$\int e^{x^2}\sin(a+bx)\,dx = \int e^{x^2}\sin\left(a+bx\right)dx$$

input
```
integrate(exp(x**2)*sin(b*x+a),x)
```

output
```
Integral(exp(x**2)*sin(a + b*x), x)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.63

$$\int e^{x^2}\sin(a+bx)\,dx$$
$$=\frac{1}{4}\sqrt{\pi}\left((\cos(a)-i\sin(a))\operatorname{erf}\left(\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}-(\cos(a)+i\sin(a))\operatorname{erf}\left(-\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}\right)$$

input
```
integrate(exp(x^2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*((cos(a) - I*sin(a))*erf(1/2*b + I*x)*e^(1/4*b^2) - (cos(a) +
 I*sin(a))*erf(-1/2*b + I*x)*e^(1/4*b^2))
```

3.76.8 Giac [F]

$$\int e^{x^2}\sin(a+bx)\,dx=\int e^{(x^2)}\sin(bx+a)\,dx$$

input
```
integrate(exp(x^2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(e^(x^2)*sin(b*x + a), x)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\sin(a+bx)\,dx=\int e^{x^2}\sin(a+b\,x)\,dx$$

input
```
int(exp(x^2)*sin(a + b*x),x)
```

output
```
int(exp(x^2)*sin(a + b*x), x)
```

3.77 $\int e^{x^2} \sin(a + cx^2)\, dx$

3.77.1 Optimal result

Integrand size = 14, antiderivative size = 87

$$\int e^{x^2} \sin(a + cx^2)\, dx = \frac{ie^{-ia}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{1-ic}x\right)}{4\sqrt{1-ic}} - \frac{ie^{ia}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{1+ic}x\right)}{4\sqrt{1+ic}}$$

output
```
1/4*I*erfi(x*(1-I*c)^(1/2))*Pi^(1/2)/exp(I*a)/(1-I*c)^(1/2)-1/4*I*exp(I*a)
*erfi(x*(1+I*c)^(1/2))*Pi^(1/2)/(1+I*c)^(1/2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.48

$$\int e^{x^2} \sin(a + cx^2)\, dx = -\frac{\sqrt[4]{-1}\sqrt{\pi}\left(\sqrt{-i+c}(i+c)\operatorname{erfi}\left(\sqrt[4]{-1}\sqrt{-i+c}x\right)(\cos(a)+i\sin(a)) + \sqrt{i+c}\left(\operatorname{erf}\left(\frac{(1+i)\sqrt{i+c}x}{\sqrt{2}}\right)\sin(a) + \operatorname{erf}\right.\right.}{4(1+c^2)}$$

input
```
Integrate[E^x^2*Sin[a + c*x^2],x]
```

output
```
-1/4*((-1)^(1/4)*Sqrt[Pi]*(Sqrt[-I + c]*(I + c)*Erfi[(-1)^(1/4)*Sqrt[-I +
c]*x]*(Cos[a] + I*Sin[a]) + Sqrt[I + c]*(Erf[((1 + I)*Sqrt[I + c]*x)/Sqrt[
2]]*Sin[a] + Erfi[(-1)^(3/4)*Sqrt[I + c]*x]*(Cos[a] + I*c*Cos[a] + c*Sin[a
]))))/(1 + c^2)
```

3.77.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} \sin \left(a+c x^2\right) \, dx$$

$$\Big\downarrow \text{4975}$$

$$\int \left(\frac{1}{2} i e^{(1-i c) x^2-i a}-\frac{1}{2} i e^{i a+(1+i c) x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{i \sqrt{\pi } e^{-i a} \text{erfi}\left(\sqrt{1-i c} x\right)}{4 \sqrt{1-i c}}-\frac{i \sqrt{\pi } e^{i a} \text{erfi}\left(\sqrt{1+i c} x\right)}{4 \sqrt{1+i c}}$$

input
```
Int[E^x^2*Sin[a + c*x^2],x]
```

output
```
((I/4)*Sqrt[Pi]*Erfi[Sqrt[1 - I*c]*x])/(Sqrt[1 - I*c]*E^(I*a)) - ((I/4)*E^
(I*a)*Sqrt[Pi]*Erfi[Sqrt[1 + I*c]*x])/Sqrt[1 + I*c]
```

3.77.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.77.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

method	result	size
risch	$-\frac{i\sqrt{\pi}\,\mathrm{e}^{ia}\,\mathrm{erf}\left(\sqrt{-ic-1}\,x\right)}{4\sqrt{-ic-1}}+\frac{i\sqrt{\pi}\,\mathrm{e}^{-ia}\,\mathrm{erf}\left(\sqrt{ic-1}\,x\right)}{4\sqrt{ic-1}}$	62

input
```
int(exp(x^2)*sin(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*Pi^(1/2)*exp(I*a)/(-I*c-1)^(1/2)*erf((-I*c-1)^(1/2)*x)+1/4*I*Pi^(1/
2)*exp(-I*a)/(I*c-1)^(1/2)*erf((I*c-1)^(1/2)*x)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int e^{x^2}\sin\left(a+cx^2\right)\,dx = \frac{\sqrt{\pi}((c-i)\cos(a)+(-i\,c-1)\sin(a))\sqrt{i\,c-1}\,\mathrm{erf}\left(\sqrt{i\,c-1}x\right)+\sqrt{\pi}((c+i)\cos(a)+(i\,c-1)\sin(a))\sqrt{-}}{4\left(c^2+1\right)}$$

input
```
integrate(exp(x^2)*sin(c*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*((c - I)*cos(a) + (-I*c - 1)*sin(a))*sqrt(I*c - 1)*erf(sqrt(
I*c - 1)*x) + sqrt(pi)*((c + I)*cos(a) + (I*c - 1)*sin(a))*sqrt(-I*c - 1)*
erf(sqrt(-I*c - 1)*x))/(c^2 + 1)
```

3.77.6 Sympy [F]

$$\int e^{x^2}\sin\left(a+cx^2\right)\,dx = \int e^{x^2}\sin\left(a+cx^2\right)dx$$

input
```
integrate(exp(x**2)*sin(c*x**2+a),x)
```

output
```
Integral(exp(x**2)*sin(a + c*x**2), x)
```

3.77.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(53) = 106$.

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

$$\int e^{x^2} \sin\left(a + cx^2\right)\,dx$$
$$= \frac{\sqrt{\pi}\sqrt{2\,c^2+2}\left((\cos(a) - i\,\sin(a))\operatorname{erf}\left(\sqrt{i\,c-1}x\right) + (\cos(a) + i\,\sin(a))\operatorname{erf}\left(\sqrt{-i\,c-1}x\right)\right)\sqrt{\sqrt{c^2+1}+1}}{8\,(}$$

input
```
integrate(exp(x^2)*sin(c*x^2+a),x, algorithm="maxima")
```

output
```
1/8*(sqrt(pi)*sqrt(2*c^2 + 2)*((cos(a) - I*sin(a))*erf(sqrt(I*c - 1)*x) +
(cos(a) + I*sin(a))*erf(sqrt(-I*c - 1)*x))*sqrt(sqrt(c^2 + 1) + 1) - sqrt(
pi)*sqrt(2*c^2 + 2)*((-I*cos(a) - sin(a))*erf(sqrt(I*c - 1)*x) + (I*cos(a)
 - sin(a))*erf(sqrt(-I*c - 1)*x))*sqrt(sqrt(c^2 + 1) - 1))/(c^2 + 1)
```

3.77.8 Giac [F]

$$\int e^{x^2} \sin\left(a + cx^2\right)\,dx = \int e^{\left(x^2\right)} \sin\left(cx^2 + a\right)\,dx$$

input
```
integrate(exp(x^2)*sin(c*x^2+a),x, algorithm="giac")
```

output
```
integrate(e^(x^2)*sin(c*x^2 + a), x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int e^{x^2} \sin\left(a + cx^2\right)\,dx = \int \mathrm{e}^{x^2} \sin\left(c\,x^2 + a\right)\,dx$$

input
```
int(exp(x^2)*sin(a + c*x^2),x)
```

output
```
int(exp(x^2)*sin(a + c*x^2), x)
```

3.78 $\int e^{x^2} \sin(a+bx+cx^2)\,dx$

3.78.1 Optimal result

Integrand size = 17, antiderivative size = 155

$$\int e^{x^2} \sin(a+bx+cx^2)\,dx = -\frac{ie^{-i\left(a-\frac{b^2}{4i+4c}\right)}\sqrt{\pi}\operatorname{erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}} - \frac{ie^{ia+\frac{b^2}{4(1+ic)}}\sqrt{\pi}\operatorname{erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}$$

output
```
-1/4*I*erfi(1/2*(I*b-2*(1-I*c)*x)/(1-I*c)^(1/2))*Pi^(1/2)/exp(I*(a-b^2/(4*
I+4*c)))/(1-I*c)^(1/2)-1/4*I*exp(I*a+1/4*b^2/(1+I*c))*erfi(1/2*(I*b+2*(1+I
*c)*x)/(1+I*c)^(1/2))*Pi^(1/2)/(1+I*c)^(1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.06

$$\int e^{x^2} \sin(a+bx+cx^2)\,dx = -\frac{(-1)^{3/4}e^{\frac{ib^2}{4i-4c}}\sqrt{\pi}\left((-i+c)\sqrt{i+c}e^{\frac{ib^2c}{2+2c^2}}\operatorname{erfi}\left(\frac{(-1)^{3/4}(b+2(i+c)x)}{2\sqrt{i+c}}\right)(\cos(a)-i\sin(a))+\sqrt{-i+c}(i+c)\operatorname{erfi}\left(\right.\right.}{4(1+c^2)}$$

input
```
Integrate[E^x^2*Sin[a + b*x + c*x^2],x]
```

output
```
-1/4*((-1)^(3/4)*E^((I*b^2)/(4*I - 4*c))*Sqrt[Pi]*((-I + c)*Sqrt[I + c]*E^
((I*b^2*c)/(2 + 2*c^2))*Erfi[((-1)^(3/4)*(b + 2*(I + c)*x))/(2*Sqrt[I + c]
)]*(Cos[a] - I*Sin[a]) + Sqrt[-I + c]*(I + c)*Erfi[((-1)^(1/4)*(b + 2*(-I
+ c)*x))/(2*Sqrt[-I + c])]*((-I)*Cos[a] + Sin[a])))/(1 + c^2)
```

3.78.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2}\sin\left(a+bx+cx^2\right)dx$$

$$\Big\downarrow \text{4975}$$

$$\int \left(\frac{1}{2}ie^{-ia-ibx+(1-ic)x^2}-\frac{1}{2}ie^{ia+ibx+(1+ic)x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\sqrt{\pi}e^{-i\left(a-\frac{b^2}{4c+4i}\right)}\text{erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}-\frac{i\sqrt{\pi}e^{ia+\frac{b^2}{4(1+ic)}}\text{erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}$$

input
```
Int[E^x^2*Sin[a + b*x + c*x^2],x]
```

output
```
((-1/4*I)*Sqrt[Pi]*Erfi[(I*b - 2*(1 - I*c)*x)/(2*Sqrt[1 - I*c])])/(Sqrt[1
- I*c]*E^(I*(a - b^2/(4*I + 4*c)))) - ((I/4)*E^(I*a + b^2/(4*(1 + I*c)))*S
qrt[Pi]*Erfi[(I*b + 2*(1 + I*c)*x)/(2*Sqrt[1 + I*c])])/Sqrt[1 + I*c]
```

3.78.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.78.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{i\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-4ia-b^{2}}{4(ic+1)}}\operatorname{erf}\left(-\sqrt{-ic-1}\,x+\frac{ib}{2\sqrt{-ic-1}}\right)}{4\sqrt{-ic-1}}+\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{4ac+4ia-b^{2}}{4ic-4}}\operatorname{erf}\left(\sqrt{ic-1}\,x+\frac{ib}{2\sqrt{ic-1}}\right)}{4\sqrt{ic-1}}$	129

input
```
int(exp(x^2)*sin(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*Pi^(1/2)*exp(-1/4*(4*a*c-4*I*a-b^2)/(1+I*c))/(-I*c-1)^(1/2)*erf(-(-I
*c-1)^(1/2)*x+1/2*I*b/(-I*c-1)^(1/2))+1/4*I*Pi^(1/2)*exp(1/4*(4*a*c+4*I*a-
b^2)/(I*c-1))/(I*c-1)^(1/2)*erf((I*c-1)^(1/2)*x+1/2*I*b/(I*c-1)^(1/2))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.04

$$\int e^{x^2}\sin\left(a+bx+cx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}(c-i)\sqrt{i\,c-1}\operatorname{erf}\left(-\frac{\left(bc+2\,\left(c^{2}+1\right)x-i\,b\right)\sqrt{i\,c-1}}{2\,\left(c^{2}+1\right)}\right)e^{\left(\frac{i\,b^{2}c-4i\,ac^{2}+b^{2}-4i\,a}{4\,\left(c^{2}+1\right)}\right)}-\sqrt{\pi}(c+i)\sqrt{-i\,c-1}\operatorname{erf}\left(\frac{(bc+2\,(c^{2}+\cdots}{\cdots}\right.}{4\left(c^{2}+1\right)}$$

input
```
integrate(exp(x^2)*sin(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*(c - I)*sqrt(I*c - 1)*erf(-1/2*(b*c + 2*(c^2 + 1)*x - I*b)*
sqrt(I*c - 1)/(c^2 + 1))*e^(1/4*(I*b^2*c - 4*I*a*c^2 + b^2 - 4*I*a)/(c^2 +
 1)) - sqrt(pi)*(c + I)*sqrt(-I*c - 1)*erf(1/2*(b*c + 2*(c^2 + 1)*x + I*b)
*sqrt(-I*c - 1)/(c^2 + 1))*e^(1/4*(-I*b^2*c + 4*I*a*c^2 + b^2 + 4*I*a)/(c^
2 + 1)))/(c^2 + 1)
```

3.78.6 Sympy [F]

$$\int e^{x^2}\sin\left(a+bx+cx^2\right)\,dx=\int e^{x^2}\sin\left(a+bx+cx^2\right)dx$$

input
```
integrate(exp(x**2)*sin(c*x**2+b*x+a),x)
```

output
```
Integral(exp(x**2)*sin(a + b*x + c*x**2), x)
```

3.78.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(101) = 202.

Time = 0.24 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.06

$$\int e^{x^2}\sin\left(a+bx+cx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}\sqrt{2\,c^2+2}\left(\left(\left(\cos\left(-\frac{b^2c-4\,ac^2-4\,a}{4\,(c^2+1)}\right)e^{\left(\frac{b^2}{4\,(c^2+1)}\right)}-i\,e^{\left(\frac{b^2}{4\,(c^2+1)}\right)}\sin\left(-\frac{b^2c-4\,ac^2-4\,a}{4\,(c^2+1)}\right)\right)\operatorname{erf}\left(-\frac{2\,(-i\,c+1)x-i\,b}{2\,\sqrt{i\,c-1}}\right)\right.\right.}{}$$

input
```
integrate(exp(x^2)*sin(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/8*(sqrt(pi)*sqrt(2*c^2 + 2)*((cos(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)
)*e^(1/4*b^2/(c^2 + 1)) - I*e^(1/4*b^2/(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^
2 - 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-I*c + 1)*x - I*b)/sqrt(I*c - 1)) - (cos
(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1))*e^(1/4*b^2/(c^2 + 1)) + I*e^(1/4*
b^2/(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-
I*c - 1)*x - I*b)/sqrt(-I*c - 1)))*sqrt(sqrt(c^2 + 1) + 1) - sqrt(pi)*sqrt
(2*c^2 + 2)*((-I*cos(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1))*e^(1/4*b^2/(c
^2 + 1)) - e^(1/4*b^2/(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1
)))*erf(-1/2*(2*(-I*c + 1)*x - I*b)/sqrt(I*c - 1)) + (-I*cos(-1/4*(b^2*c -
 4*a*c^2 - 4*a)/(c^2 + 1))*e^(1/4*b^2/(c^2 + 1)) + e^(1/4*b^2/(c^2 + 1))*s
in(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-I*c - 1)*x - I*b
)/sqrt(-I*c - 1)))*sqrt(sqrt(c^2 + 1) - 1))/(c^2 + 1)
```

3.78.8 Giac [F]

$$\int e^{x^2}\sin\left(a+bx+cx^2\right)\,dx=\int e^{\left(x^2\right)}\sin\left(cx^2+bx+a\right)\,dx$$

input
```
integrate(exp(x^2)*sin(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate(e^(x^2)*sin(c*x^2 + b*x + a), x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\sin\left(a+bx+cx^2\right)\,dx=\int \sin\left(c\,x^2+b\,x+a\right)\,\mathrm{e}^{x^2}\,dx$$

input
```
int(sin(a + b*x + c*x^2)*exp(x^2),x)
```

output
```
int(sin(a + b*x + c*x^2)*exp(x^2), x)
```

3.79 $\int f^{a+bx}\sin(d+fx^2)\,dx$

3.79.1 Optimal result

Integrand size = 16, antiderivative size = 142

$$\int f^{a+bx}\sin(d+fx^2)\,dx = \frac{1}{4}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erf}\left(\frac{\sqrt[4]{-1}(2ifx+b\log(f))}{2\sqrt{f}}\right) - \frac{1}{4}(-1)^{3/4}e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt[4]{-1}(2ifx-b\log(f))}{2\sqrt{f}}\right)$$

output
```
1/4*(-1)^(3/4)*exp(1/4*I*(4*d+b^2*ln(f)^2/f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4
)*(2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-1/4*(-1)^(3/4)*f^(-1/2+a)*erfi(1/2*(
-1)^(1/4)*(2*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)/exp(1/4*I*(4*d+b^2*ln(f)^2/f
))
```

3.79.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93

$$\int f^{a+bx}\sin(d+fx^2)\,dx = -\frac{1}{4}\sqrt[4]{-1}e^{-\frac{ib^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(e^{\frac{ib^2\log^2(f)}{2f}}\text{erfi}\left(\frac{\sqrt[4]{-1}(2fx-ib\log(f))}{2\sqrt{f}}\right)(\cos(d)+i\sin(d)) + \text{erfi}\left(\frac{(-1)^{3/4}(2fx+ib\log(f))}{2\sqrt{f}}\right)(i\cos(d)+\sin(d))\right)$$

input
```
Integrate[f^(a + b*x)*Sin[d + f*x^2],x]
```

output
```
-1/4*((-1)^(1/4)*f^(-1/2 + a)*Sqrt[Pi]*(E^(((I/2)*b^2*Log[f]^2)/f)*Erfi[((
-1)^(1/4)*(2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] + I*Sin[d]) + Erfi[((
-1)^(3/4)*(2*f*x + I*b*Log[f]))/(2*Sqrt[f])]*(I*Cos[d] + Sin[d])))/E^(((I/
4)*b^2*Log[f]^2)/f)
```

3.79.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\sin\left(d+fx^2\right)\,dx$$

$$\Big\downarrow \ 4975$$

$$\int \left(\frac{1}{2}ie^{-id-ifx^2}f^{a+bx}-\frac{1}{2}ie^{id+ifx^2}f^{a+bx}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+2ifx)}{2\sqrt{f}}\right)-\frac{1}{4}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+2ifx)}{2\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Sin[d + f*x^2],x]
```

output
```
((-1)^(3/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(
(-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/4 - ((-1)^(3/4)*f^(-1/2 +
 a)*Sqrt[Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(4*E^(
(I/4)*(4*d + (b^2*Log[f]^2)/f)))
```

3.79.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.79.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+4 d f\right)}{4 f}} \operatorname{erf}\left(-\sqrt{-i f}\, x+\frac{\ln(f) b}{2\sqrt{-i f}}\right)}{4\sqrt{-i f}}-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+4 d f\right)}{4 f}} \operatorname{erf}\left(-\sqrt{i f}\, x+\frac{\ln(f) b}{2\sqrt{i f}}\right)}{4\sqrt{i f}}$	116

input
```
int(f^(b*x+a)*sin(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(-I*f)^(1/2)*erf(-(-I*
f)^(1/2)*x+1/2*ln(f)*b/(-I*f)^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^
2*b^2+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*ln(f)*b/(I*f)^(1/2))
```

3.79.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(98) = 196.

Time = 0.25 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.87

$$\int f^{a+bx}\sin\left(d+fx^2\right)\,dx$$
$$= \frac{i\,\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log(f)^2+4\,af\log(f)-4i\,df}{4\,f}\right)}\,\mathrm{C}\left(\frac{\sqrt{2}(2\,fx+i\,b\log(f))\sqrt{\frac{f}{\pi}}}{2\,f}\right)+i\,\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2+4\,af\log(f)+4i\,df}{4\,f}\right)}\,\mathrm{C}\left(-\frac{\sqrt{2}(2\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+d),x, algorithm="fricas")
```

output
```
1/4*(I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*
d*f)/f)*fresnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) + I*sqr
t(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fre
snel_cos(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f) + sqrt(2)*pi*sqrt
(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel_sin(1/
2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) - sqrt(2)*pi*sqrt(f/pi)*e^(1/
4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_sin(-1/2*sqrt(2)*(2
*f*x - I*b*log(f))*sqrt(f/pi)/f))/f
```

3.79.6 Sympy [F]

$$\int f^{a+bx}\sin\left(d+fx^2\right)\,dx=\int f^{a+bx}\sin\left(d+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*sin(f*x**2+d),x)
```

output
```
Integral(f**(a + b*x)*sin(d + f*x**2), x)
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04

$$\int f^{a+bx}\sin\left(d+fx^2\right)\,dx=$$
$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\,f^a\cos\left(\frac{b^2\log(f)^2+4\,df}{4\,f}\right)+(i-1)\,f^a\sin\left(\frac{b^2\log(f)^2+4\,df}{4\,f}\right)\right)\mathrm{erf}\left(\frac{2i\,fx-b\log(f)}{2\,\sqrt{i\,f}}\right)+\left(-(i-1\right.\right.}{8\,\sqrt{f}}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+d),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*f)/f) + (
I - 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1/2*(2*I*f*x - b*log(f))
/sqrt(I*f)) + (-(I - 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*f)/f) + (I + 1)*f^
a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1/2*(2*I*f*x + b*log(f))/sqrt(-I*
f)))/sqrt(f)
```

3.79.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(98) = 196$.

Time = 0.34 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.11

$$\int f^{a+bx}\sin\left(d+fx^2\right)\,dx$$
$$=\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{8}i\sqrt{2}\left(4x-\frac{\pi b\mathrm{sgn}(f)-\pi b+2i\,b\log(|f|)}{f}\right)\left(\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(\frac{i\pi^2b^2\mathrm{sgn}(f)}{8f}+\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4f}-\frac{i\pi^2b^2}{8f}-\frac{\pi b^2\log(|f|)}{4f}+\right.}}{4\left(\frac{if}{|f|}+1\right)\sqrt{|f|}}$$
$$+\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{8}i\sqrt{2}\left(4x+\frac{\pi b\mathrm{sgn}(f)-\pi b+2i\,b\log(|f|)}{f}\right)\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(-\frac{i\pi^2b^2\mathrm{sgn}(f)}{8f}-\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4f}+\frac{i\pi^2b^2}{8f}+\frac{\pi b^2\log(}{4f}\right.}}{4\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+d),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*sqrt(pi)*erf(-1/8*I*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b
*log(abs(f)))/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(f)/f
 + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(abs
(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*lo
g(abs(f)) + I*d)/((I*f/abs(f) + 1)*sqrt(abs(f))) + 1/4*sqrt(2)*sqrt(pi)*er
f(1/8*I*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(-I*f/a
bs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*log(abs(
f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I*b^2*log
(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) - I*d)/((-I*
f/abs(f) + 1)*sqrt(abs(f)))
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sin\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\,\sin\left(f\,x^2+d\right)\,dx$$

input
```
int(f^(a + b*x)*sin(d + f*x^2),x)
```

output
```
int(f^(a + b*x)*sin(d + f*x^2), x)
```

3.80 $\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx$

3.80.1 Optimal result

Integrand size = 18, antiderivative size = 157

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx = \left(\frac{1}{16}+\frac{i}{16}\right)e^{2id+\frac{ib^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx+b\log(f))}{\sqrt{f}}\right) + \left(\frac{1}{16}+\frac{i}{16}\right)e^{-\frac{1}{8}i\left(16d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx-b\log(f))}{\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

output
```
1/2*f^(b*x+a)/b/ln(f)+(1/16+1/16*I)*exp(2*I*d+1/8*I*b^2*ln(f)^2/f)*f^(-1/2
+a)*erf((1/4+1/4*I)*(4*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)+(1/16+1/16*I)*f^(-
1/2+a)*erfi((1/4+1/4*I)*(4*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)/exp(1/8*I*(16*
d+b^2*ln(f)^2/f))
```

3.80.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.99

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx$$
$$=\frac{1}{16}f^a\left(\frac{8f^{bx}}{b\log(f)}-\frac{(1-i)e^{-\frac{ib^2\log^2(f)}{8f}}\sqrt{\pi}\operatorname{erf}\left(\frac{(4+4i)fx-(1-i)b\log(f)}{4\sqrt{f}}\right)(\cos(d)-i\sin(d))^2}{\sqrt{f}}-\frac{(1-i)e^{\frac{ib^2\log^2(f)}{8f}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(4+4i)fx+(1-i)b\log(f)}{4\sqrt{f}}\right)(\cos(d)+i\sin(d))^2}{\sqrt{f}}\right)$$

input
```
Integrate[f^(a + b*x)*Sin[d + f*x^2]^2,x]
```

output
```
(f^a*((8*f^(b*x))/(b*Log[f]) - ((1 - I)*Sqrt[Pi]*Erf[((4 + 4*I)*f*x - (1 -
 I)*b*Log[f])/(4*Sqrt[f])]*(Cos[d] - I*Sin[d])^2)/(E^(((I/8)*b^2*Log[f]^2)
/f)*Sqrt[f]) - ((1 - I)*E^(((I/8)*b^2*Log[f]^2)/f)*Sqrt[Pi]*Erfi[((4 + 4*I
)*f*x + (1 - I)*b*Log[f])/(4*Sqrt[f])]*(Cos[d] + I*Sin[d])^2)/Sqrt[f]))/16
```

3.80.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx$$
$$\Big\downarrow \text{ 4975}$$
$$\int \left(-\frac{1}{4}e^{-2id-2ifx^2}f^{a+bx}-\frac{1}{4}e^{2id+2ifx^2}f^{a+bx}+\frac{1}{2}f^{a+bx}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}\, f^{a-\frac{1}{2}} e^{\frac{ib^2\log^2(f)}{8f}+2id}\operatorname{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+4ifx)}{\sqrt{f}}\right)+\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}\, f^{a-\frac{1}{2}} e^{-\frac{1}{8}i\left(\frac{b^2\log^2(f)}{f}+16d\right)}\operatorname{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-b\log(f)+4ifx)}{\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

input
```
Int[f^(a + b*x)*Sin[d + f*x^2]^2,x]
```

output
```
(1/16 + I/16)*E^((2*I)*d + ((I/8)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*E
rf[((1/4 + I/4)*((4*I)*f*x + b*Log[f]))/Sqrt[f]] + ((1/16 + I/16)*f^(-1/2
+ a)*Sqrt[Pi]*Erfi[((1/4 + I/4)*((4*I)*f*x - b*Log[f]))/Sqrt[f]])/E^((I/8)
*(16*d + (b^2*Log[f]^2)/f)) + f^(a + b*x)/(2*b*Log[f])
```

3.80.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.80.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

method	result	si
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+16 d f\right)}{8 f}} \sqrt{2}\, \operatorname{erf}\left(-\sqrt{2}\, \sqrt{i f}\, x+\frac{b \ln(f) \sqrt{2}}{4 \sqrt{i f}}\right)}{16 \sqrt{i f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+16 d f\right)}{8 f}} \operatorname{erf}\left(-\sqrt{-2 i f}\, x+\frac{b \ln(f)}{2 \sqrt{-2 i f}}\right)}{8 \sqrt{-2 i f}}+\frac{f^{x b+a}}{2 b \ln(f)}$	13

input
```
int(f^(b*x+a)*sin(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*Pi^(1/2)*f^a*exp(-1/8*I*(ln(f)^2*b^2+16*d*f)/f)*2^(1/2)/(I*f)^(1/2)*e
rf(-2^(1/2)*(I*f)^(1/2)*x+1/4*b*ln(f)*2^(1/2)/(I*f)^(1/2))+1/8*Pi^(1/2)*f^
a*exp(1/8*I*(ln(f)^2*b^2+16*d*f)/f)/(-2*I*f)^(1/2)*erf(-(-2*I*f)^(1/2)*x+1
/2*b*ln(f)/(-2*I*f)^(1/2))+1/2*f^(b*x+a)/b/ln(f)
```

3.80.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(103) = 206$.

Time = 0.25 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.72

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx = -\frac{\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{-ib^2\log(f)^2+8\,af\log(f)-16i\,df}{8\,f}\right)}\mathrm{C}\left(\frac{(4\,fx+i\,b\log(f))\sqrt{\frac{f}{\pi}}}{2\,f}\right)\log(f)-\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2+8\,af\log(f)+16i\,df}{8\,f}\right)}\mathrm{C}\left(-\frac{(4}{}\right.}{}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 8*a*f*log(f) - 16*I*d*f)/f
)*fresnel_cos(1/2*(4*f*x + I*b*log(f))*sqrt(f/pi)/f)*log(f) - pi*b*sqrt(f/
pi)*e^(1/8*(I*b^2*log(f)^2 + 8*a*f*log(f) + 16*I*d*f)/f)*fresnel_cos(-1/2*
(4*f*x - I*b*log(f))*sqrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(-I*b
^2*log(f)^2 + 8*a*f*log(f) - 16*I*d*f)/f)*fresnel_sin(1/2*(4*f*x + I*b*log
(f))*sqrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 + 8*a
*f*log(f) + 16*I*d*f)/f)*fresnel_sin(-1/2*(4*f*x - I*b*log(f))*sqrt(f/pi)/
f)*log(f) - 4*f*f^(b*x + a))/(b*f*log(f))
```

3.80.6 Sympy [F]

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx = \int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx$$

input
```
integrate(f**(b*x+a)*sin(f*x**2+d)**2,x)
```

output
```
Integral(f**(a + b*x)*sin(d + f*x**2)**2, x)
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.18

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx$$
$$=\frac{4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left((i-1)\,bf^a\cos\left(\frac{b^2\log(f)^2+16\,df}{8\,f}\right)\log(f)+(i+1)\,bf^a\log(f)\sin\left(\frac{b^2\log(f)^2+16\,df}{8\,f}\right)\right)\text{erf}\left(\frac{4i\,fx-b\log}{2\,\sqrt{2i\,f}}\right.}{}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*b*f^a*cos(1/8*(b^2*log(f)^2 + 16*
d*f)/f)*log(f) + (I + 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 + 16*d*f)/f))*
erf(1/2*(4*I*f*x - b*log(f))/sqrt(2*I*f)) + ((I + 1)*b*f^a*cos(1/8*(b^2*lo
g(f)^2 + 16*d*f)/f)*log(f) + (I - 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 +
16*d*f)/f))*erf(1/2*(4*I*f*x + b*log(f))/sqrt(-2*I*f)))*f^(3/2) + 16*f^(b*
x)*f^(a + 2))/(b*f^2*log(f))
```

3.80.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(103) = 206.

Time = 0.32 (sec) , antiderivative size = 521, normalized size of antiderivative = 3.32

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+d)^2,x, algorithm="giac")
```

output
```
(2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log
(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) - (pi*b*sgn(f) - p
i*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*
b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(f)) + a*log(ab
s(f))) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f) -
1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) - I*e^(-1/2*I*p
i*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*s
gn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(abs(f))) -
 1/8*I*sqrt(pi)*erf(-1/8*I*sqrt(f)*(8*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(
abs(f)))/f)*(I*f/abs(f) + 1))*e^(1/16*I*pi^2*b^2*sgn(f)/f + 1/8*pi*b^2*log
(abs(f))*sgn(f)/f - 1/16*I*pi^2*b^2/f - 1/8*pi*b^2*log(abs(f))/f + 1/8*I*b
^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) + 2*I*
d)/(sqrt(f)*(I*f/abs(f) + 1)) + 1/8*I*sqrt(pi)*erf(1/8*I*sqrt(f)*(8*x + (p
i*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(-I*f/abs(f) + 1))*e^(-1/16*I*pi
^2*b^2*sgn(f)/f - 1/8*pi*b^2*log(abs(f))*sgn(f)/f + 1/16*I*pi^2*b^2/f + 1/
8*pi*b^2*log(abs(f))/f - 1/8*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1
/2*I*pi*a + a*log(abs(f)) - 2*I*d)/(sqrt(f)*(-I*f/abs(f) + 1))
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\,\sin\left(f\,x^2+d\right)^2\,dx$$

input
```
int(f^(a + b*x)*sin(d + f*x^2)^2,x)
```

output
```
int(f^(a + b*x)*sin(d + f*x^2)^2, x)
```

3.81 $\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx$

3.81.1 Optimal result

Integrand size = 18, antiderivative size = 298

$$\begin{aligned}\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx &= \frac{3}{16}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(2ifx+b\log(f))}{2\sqrt{f}}\right)\\ &\quad+\left(\frac{1}{16}-\frac{i}{16}\right)e^{3id+\frac{ib^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(6ifx+b\log(f))}{\sqrt{6}\sqrt{f}}\right)\\ &\quad-\frac{3}{16}(-1)^{3/4}e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(2ifx-b\log(f))}{2\sqrt{f}}\right)\\ &\quad-\left(\frac{1}{16}-\frac{i}{16}\right)e^{-\frac{1}{12}i\left(36d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(6ifx-b\log(f))}{\sqrt{6}\sqrt{f}}\right)\end{aligned}$$

output
```
(1/96-1/96*I)*exp(3*I*d+1/12*I*b^2*ln(f)^2/f)*f^(-1/2+a)*erf((1/12+1/12*I)
*(6*I*f*x+b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)+(-1/96+1/96*I)*f^(-1/
2+a)*erfi((1/12+1/12*I)*(6*I*f*x-b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2
)/exp(1/12*I*(36*d+b^2*ln(f)^2/f))+3/16*(-1)^(3/4)*exp(1/4*I*(4*d+b^2*ln(f
)^2/f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4)*(2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-
3/16*(-1)^(3/4)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(2*I*f*x-b*ln(f))/f^(1/2))*
Pi^(1/2)/exp(1/4*I*(4*d+b^2*ln(f)^2/f))
```

3.81.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.90

$$\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx$$
$$=\frac{1}{48}(-1)^{3/4}e^{-\frac{ib^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(-9\text{erfi}\left(\frac{(-1)^{3/4}(2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d))+9ie^{\frac{ib^2\log^2(f)}{2f}}\text{erfi}\left(\frac{\sqrt[4]{}}{}\right.\right.$$

input
```
Integrate[f^(a + b*x)*Sin[d + f*x^2]^3,x]
```

output
```
((-1)^(3/4)*f^(-1/2 + a)*Sqrt[Pi]*(-9*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f]
))/(2*Sqrt[f])]*(Cos[d] - I*Sin[d]) + (9*I)*E^(((I/2)*b^2*Log[f]^2)/f)*Erf
i[((-1)^(1/4)*(2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] + I*Sin[d]) + Sqr
t[3]*E^(((I/6)*b^2*Log[f]^2)/f)*(Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f]))/(2
*Sqrt[3]*Sqrt[f])]*(Cos[3*d] - I*Sin[3*d]) + E^(((I/6)*b^2*Log[f]^2)/f)*Er
fi[((6 + 6*I)*f*x + (1 - I)*b*Log[f])/(2*Sqrt[6]*Sqrt[f])]*((-I)*Cos[3*d]
+ Sin[3*d]))))/(48*E^(((I/4)*b^2*Log[f]^2)/f))
```

3.81.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx$$
$$\Big\downarrow \text{4975}$$
$$\int \left(\frac{3}{8}ie^{-id-ifx^2}f^{a+bx}-\frac{3}{8}ie^{id+ifx^2}f^{a+bx}-\frac{1}{8}ie^{-3id-3ifx^2}f^{a+bx}+\frac{1}{8}ie^{3id+3ifx^2}f^{a+bx}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{3}{16}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+2ifx)}{2\sqrt{f}}\right)+$$
$$\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{12f}+3id}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\log(f)+6ifx)}{\sqrt{6}\sqrt{f}}\right)-$$
$$\frac{3}{16}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+2ifx)}{2\sqrt{f}}\right)-$$
$$\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{-\frac{1}{12}i\left(\frac{b^2\log^2(f)}{f}+36d\right)}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-b\log(f)+6ifx)}{\sqrt{6}\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Sin[d + f*x^2]^3,x]
```

output
```
(3*(-1)^(3/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Erf
[((-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 + (1/16 - I/16)*E^((
3*I)*d + ((I/12)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6]*Erf[((1/2 + I/2)
*((6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (3*(-1)^(3/4)*f^(-1/2 + a)*S
qrt[Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(16*E^((I/4
)*(4*d + (b^2*Log[f]^2)/f))) - ((1/16 - I/16)*f^(-1/2 + a)*Sqrt[Pi/6]*Erfi
[((1/2 + I/2)*((6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])])/E^((I/12)*(36*d
+ (b^2*Log[f]^2)/f))
```

3.81.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.81.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+36 d f\right)}{12 f}} \operatorname{erf}\left(-\sqrt{-3 i f}\, x+\frac{\ln(f) b}{2\sqrt{-3 i f}}\right)}{16\sqrt{-3 i f}}+\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+36 d f\right)}{12 f}} \sqrt{3}\, \operatorname{erf}\left(-\sqrt{3}\,\sqrt{i f}\, x+\frac{\ln(f) b\sqrt{3}}{6\sqrt{i f}}\right)}{48\sqrt{i f}}-\frac{3 i\sqrt{\pi}\, f^{a} \mathrm{e}}{}$

input
```
int(f^(b*x+a)*sin(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*I*Pi^(1/2)*f^a*exp(1/12*I*(ln(f)^2*b^2+36*d*f)/f)/(-3*I*f)^(1/2)*erf
(-(-3*I*f)^(1/2)*x+1/2*ln(f)*b/(-3*I*f)^(1/2))+1/48*I*Pi^(1/2)*f^a*exp(-1/
12*I*(ln(f)^2*b^2+36*d*f)/f)*3^(1/2)/(I*f)^(1/2)*erf(-3^(1/2)*(I*f)^(1/2)*
x+1/6*ln(f)*b*3^(1/2)/(I*f)^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2
*b^2+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*ln(f)*b/(I*f)^(1/2))+3/1
6*I*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(-I*f)^(1/2)*erf(-(-I*f)
^(1/2)*x+1/2*ln(f)*b/(-I*f)^(1/2))
```

3.81.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(196) = 392.

Time = 0.26 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.76

$$\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx$$
$$=\frac{-i\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log(f)^2+12\,af\log(f)-36i\,df}{12\,f}\right)}\,\mathrm{C}\left(\frac{\sqrt{6}(6\,fx+i\,b\log(f))\sqrt{\frac{f}{\pi}}}{6\,f}\right)-i\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2+12\,af\log(f)+36i\,df}{12\,f}\right)}\,\mathrm{C}\left(-\right.}{}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+d)^3,x, algorithm="fricas")
```

output
```
1/48*(-I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 12*a*f*log(f) -
36*I*d*f)/f)*fresnel_cos(1/6*sqrt(6)*(6*f*x + I*b*log(f))*sqrt(f/pi)/f) -
I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 + 12*a*f*log(f) + 36*I*d*f
)/f)*fresnel_cos(-1/6*sqrt(6)*(6*f*x - I*b*log(f))*sqrt(f/pi)/f) + 9*I*sqr
t(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fr
esnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) + 9*I*sqrt(2)*pi*
sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_cos
(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f) - sqrt(6)*pi*sqrt(f/pi)*e
^(1/12*(-I*b^2*log(f)^2 + 12*a*f*log(f) - 36*I*d*f)/f)*fresnel_sin(1/6*sqr
t(6)*(6*f*x + I*b*log(f))*sqrt(f/pi)/f) + sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I
*b^2*log(f)^2 + 12*a*f*log(f) + 36*I*d*f)/f)*fresnel_sin(-1/6*sqrt(6)*(6*f
*x - I*b*log(f))*sqrt(f/pi)/f) + 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*lo
g(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I*b*l
og(f))*sqrt(f/pi)/f) - 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*
a*f*log(f) + 4*I*d*f)/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqr
t(f/pi)/f))/f
```

3.81.6 Sympy [F]

$$\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx=\int f^{a+bx}\sin^3\left(d+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*sin(f*x**2+d)**3,x)
```

output
```
Integral(f**(a + b*x)*sin(d + f*x**2)**3, x)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.01

$$\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx = \frac{9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\,f^a\cos\left(\frac{b^2\log(f)^2+36\,df}{12\,f}\right)+(i-1)\,f^a\sin\left(\frac{b^2\log(f)^2+36\,df}{12\,f}\right)\right)\operatorname{erf}\left(\frac{6i\,fx-b\log(f)}{2\sqrt{3i\,f}}\right)+\left(-(i-\right.}{}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/96*(9^(1/4)*sqrt(2)*sqrt(pi)*((-(I + 1)*f^a*cos(1/12*(b^2*log(f)^2 + 36*
d*f)/f) + (I - 1)*f^a*sin(1/12*(b^2*log(f)^2 + 36*d*f)/f))*erf(1/2*(6*I*f*
x - b*log(f))/sqrt(3*I*f)) + (-(I - 1)*f^a*cos(1/12*(b^2*log(f)^2 + 36*d*f
)/f) + (I + 1)*f^a*sin(1/12*(b^2*log(f)^2 + 36*d*f)/f))*erf(1/2*(6*I*f*x +
 b*log(f))/sqrt(-3*I*f)))*f^(3/2) - 9*sqrt(2)*sqrt(pi)*((-(I + 1)*f^a*cos(
1/4*(b^2*log(f)^2 + 4*d*f)/f) + (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)
/f))*erf(1/2*(2*I*f*x - b*log(f))/sqrt(I*f)) + (-(I - 1)*f^a*cos(1/4*(b^2*
log(f)^2 + 4*d*f)/f) + (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(
1/2*(2*I*f*x + b*log(f))/sqrt(-I*f)))*f^(3/2))/f^2
```

3.81.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 595 vs. $2(196) = 392$.

Time = 0.39 (sec) , antiderivative size = 595, normalized size of antiderivative = 2.00

$$\int f^{a+bx} \sin^3\left(d + fx^2\right)\, dx = \text{Too large to display}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+d)^3,x, algorithm="giac")
```

output
```
3/16*sqrt(2)*sqrt(pi)*erf(-1/8*I*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*
b*log(abs(f)))/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(f)/
f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(ab
s(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*l
og(abs(f)) + I*d)/((I*f/abs(f) + 1)*sqrt(abs(f))) - 1/48*sqrt(6)*sqrt(pi)*
erf(-1/24*I*sqrt(6)*sqrt(f)*(12*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)
))/f)*(I*f/abs(f) + 1))*e^(1/24*I*pi^2*b^2*sgn(f)/f + 1/12*pi*b^2*log(abs(
f))*sgn(f)/f - 1/24*I*pi^2*b^2/f - 1/12*pi*b^2*log(abs(f))/f + 1/12*I*b^2*
log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) + 3*I*d)/
(sqrt(f)*(I*f/abs(f) + 1)) - 1/48*sqrt(6)*sqrt(pi)*erf(1/24*I*sqrt(6)*sqrt
(f)*(12*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(-I*f/abs(f) + 1))
*e^(-1/24*I*pi^2*b^2*sgn(f)/f - 1/12*pi*b^2*log(abs(f))*sgn(f)/f + 1/24*I*
pi^2*b^2/f + 1/12*pi*b^2*log(abs(f))/f - 1/12*I*b^2*log(abs(f))^2/f - 1/2*
I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) - 3*I*d)/(sqrt(f)*(-I*f/abs(f)
+ 1)) + 3/16*sqrt(2)*sqrt(pi)*erf(1/8*I*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b
 + 2*I*b*log(abs(f)))/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^
2*sgn(f)/f - 1/4*pi*b^2*log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b
^2*log(abs(f))/f - 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*p
i*a + a*log(abs(f)) - I*d)/((-I*f/abs(f) + 1)*sqrt(abs(f)))
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\sin\left(f\,x^2+d\right)^3\,dx$$

input
```
int(f^(a + b*x)*sin(d + f*x^2)^3,x)
```

output
```
int(f^(a + b*x)*sin(d + f*x^2)^3, x)
```

3.82 $\int f^{a+bx} \sin \left(d+ex+fx^2\right) \, dx$

3.82.1 Optimal result

Integrand size = 19, antiderivative size = 162

$$\begin{aligned}\int f^{a+bx} \sin \left(d+ex+fx^2\right) \, dx &= \frac{1}{4}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erf}\left(\frac{\sqrt[4]{-1}(ie+2ifx+b\log(f))}{2\sqrt{f}}\right)\\&\quad-\frac{1}{4}(-1)^{3/4}e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt[4]{-1}(ie+2ifx-b\log(f))}{2\sqrt{f}}\right)\end{aligned}$$

output
```
1/4*(-1)^(3/4)*exp(1/4*I*(4*d+(I*e+b*ln(f))^2/f))*f^(-1/2+a)*erf(1/2*(-1)^
(1/4)*(I*e+2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-1/4*(-1)^(3/4)*exp(-I*d+1/4*
I*(e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(I*e+2*I*f*x-b*ln(f))/
f^(1/2))*Pi^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

$$\begin{aligned}\int f^{a+bx} \sin \left(d+ex+fx^2\right) \, dx &= -\frac{1}{4}\sqrt[4]{-1}e^{-\frac{i\left(e^2+b^2\log^2(f)\right)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(e^{\frac{ib^2\log^2(f)}{2f}}\text{erfi}\left(\frac{\sqrt[4]{-1}(e+2fx-ib\log(f))}{2\sqrt{f}}\right)(\cos(d)\right.\\&\quad\left.+i\sin(d))+e^{\frac{ie^2}{2f}}\text{erfi}\left(\frac{(-1)^{3/4}(e+2fx+ib\log(f))}{2\sqrt{f}}\right)(i\cos(d)+\sin(d))\right)\end{aligned}$$

input
```
Integrate[f^(a + b*x)*Sin[d + e*x + f*x^2],x]
```

output
```
-1/4*((-1)^(1/4)*f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(E^(((I/2)*b^2*Log[f]^2)
/f)*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] + I*Si
n[d]) + E^(((I/2)*e^2)/f)*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f]))/(2*Sq
rt[f])]*(I*Cos[d] + Sin[d])))/E^(((I/4)*(e^2 + b^2*Log[f]^2))/f)
```

3.82.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\sin\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \ 4975$$

$$\int \left(\frac{1}{2}if^{a+bx}e^{-id-iex-ifx^2}-\frac{1}{2}if^{a+bx}e^{id+iex+ifx^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\text{erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)-\frac{1}{4}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(e+ib\log(f))^2}{4f}-id}\text{erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Sin[d + e*x + f*x^2],x]
```

output
```
((-1)^(3/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi]*E
rf[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/4 - ((-1)^(3/4)
*E^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*Erfi[((-1
)^(1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/4
```

3.82.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.82.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.96

method	result	s
risch	$\frac{i\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+4df-e^{2}\right)}{4f}} \operatorname{erf}\left(-\sqrt{-if}\, x+\frac{ie+b\ln(f)}{2\sqrt{-if}}\right)}{4\sqrt{-if}}-\frac{i\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+4df-e^{2}\right)}{4f}} \operatorname{erf}\left(-\sqrt{if}\, x+\frac{b\ln(f)-ie}{2\sqrt{if}}\right)}{4\sqrt{if}}$	1

input
```
int(f^(b*x+a)*sin(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(1/4*I*(ln(f)^2*b^2+4*d*f-e^2)/f)/(-I
*f)^(1/2)*erf(-(-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-I*f)^(1/2))-1/4*I*Pi^(1/
2)*f^a*f^(-1/2/f*b*e)*exp(-1/4*I*(ln(f)^2*b^2+4*d*f-e^2)/f)/(I*f)^(1/2)*er
f(-(I*f)^(1/2)*x+1/2*(b*ln(f)-I*e)/(I*f)^(1/2))
```

3.82.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(109) = 218.

Time = 0.25 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.93

$$\int f^{a+bx}\sin\left(d+ex+fx^2\right)\,dx = \frac{i\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log(f)^2+i\,e^2-4i\,df-2\,(be-2\,af)\log(f)}{4\,f}\right)}\mathrm{C}\left(\frac{\sqrt{2}(2\,fx+i\,b\log(f)+e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)+i\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2-i\,e^2+4i\,df-2\,(be-2\cdots}{4\,f}\cdots\right.}\cdots}{\cdots}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
1/4*(I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + I*e^2 - 4*I*d*f - 2
*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f) + e)
*sqrt(f/pi)/f) + I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 - I*e^2 +
4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/2*sqrt(2)*(2*f*x - I*b
*log(f) + e)*sqrt(f/pi)/f) + sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2
 + I*e^2 - 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*sqrt(2)*(2
*f*x + I*b*log(f) + e)*sqrt(f/pi)/f) - sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2
*log(f)^2 - I*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/2*
sqrt(2)*(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f))/f
```

3.82.6 Sympy [F]

$$\int f^{a+bx}\sin\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\sin\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*sin(f*x**2+e*x+d),x)
```

output
```
Integral(f**(a + b*x)*sin(d + e*x + f*x**2), x)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.17

$$\int f^{a+bx}\sin\left(d+ex+fx^2\right)\,dx=$$
$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(\left(i+1\right)\,f^a\cos\left(\frac{b^2\log(f)^2-e^2+4\,df}{4\,f}\right)-\left(i-1\right)\,f^a\sin\left(\frac{b^2\log(f)^2-e^2+4\,df}{4\,f}\right)\right)\operatorname{erf}\left(\frac{i\,(2i\,fx-b\log(f)+i\,e)\sqrt{i\,f}}{2\,f}\right)\right.}{8\,\sqrt{f}}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*sqrt(pi)*(((I + 1)*f^a*cos(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f
) - (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f))*erf(1/2*I*(2*I*f*
x - b*log(f) + I*e)*sqrt(I*f)/f) + (-(I - 1)*f^a*cos(1/4*(b^2*log(f)^2 - e
^2 + 4*d*f)/f) + (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f))*erf(
1/2*I*(2*I*f*x + b*log(f) + I*e)*sqrt(-I*f)/f))/(sqrt(f)*f^(1/2*b*e/f))
```

3.82.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(109) = 218.

Time = 0.34 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.33

$$\int f^{a+bx}\sin\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{8}i\sqrt{2}\left(4x-\frac{\pi b\mathrm{sgn}(f)-\pi b+2ib\log(|f|)-2e}{f}\right)\left(\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(\frac{i\pi^2b^2\mathrm{sgn}(f)}{8f}+\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4f}-\frac{i\pi^2b^2}{8f}-\frac{\pi b^2\log(|\cdots}{4f}\right.}}{4\left(\frac{if}{|f|}+1\right)\sqrt{|f|}}$$
$$+\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{8}i\sqrt{2}\left(4x+\frac{\pi b\mathrm{sgn}(f)-\pi b+2ib\log(|f|)+2e}{f}\right)\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(-\frac{i\pi^2b^2\mathrm{sgn}(f)}{8f}-\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4f}+\frac{i\pi^2b^2}{8f}+\frac{\pi b^{2}\cdots}{}\right.}}{4\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*sqrt(pi)*erf(-1/8*I*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b
*log(abs(f)) - 2*e)/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sg
n(f)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*l
og(abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*
e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs
(f))/f + I*d - 1/4*I*e^2/f)/((I*f/abs(f) + 1)*sqrt(abs(f))) + 1/4*sqrt(2)*
sqrt(pi)*erf(1/8*I*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f))
+ 2*e)/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/
4*pi*b^2*log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/
f - 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f
+ 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f - I*
d + 1/4*I*e^2/f)/((-I*f/abs(f) + 1)*sqrt(abs(f)))
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sin\left(d+ex+fx^2\right)\,dx=\int f^{a+b\,x}\,\sin\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int(f^(a + b*x)*sin(d + e*x + f*x^2),x)
```

output
```
int(f^(a + b*x)*sin(d + e*x + f*x^2), x)
```

3.82. $\int f^{a+bx} \sin\left(d + ex + fx^2\right) \, dx$

3.83 $\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)\,dx$

3.83.1 Optimal result

Integrand size = 21, antiderivative size = 179

$$\begin{aligned}
\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)\,dx
&= \left(\frac{1}{16}+\frac{i}{16}\right)e^{2id+\frac{i(2ie+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2ie+4ifx+b\log(f))}{\sqrt{f}}\right)\\
&\quad+\left(\frac{1}{16}+\frac{i}{16}\right)e^{-2id+\frac{i(2e+ib\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2ie+4ifx-b\log(f))}{\sqrt{f}}\right)\\
&\quad+\frac{f^{a+bx}}{2b\log(f)}
\end{aligned}$$

output
```
1/2*f^(b*x+a)/b/ln(f)+(1/16+1/16*I)*exp(2*I*d+1/8*I*(2*I*e+b*ln(f))^2/f)*f
^(-1/2+a)*erf((1/4+1/4*I)*(2*I*e+4*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)+(1/16+
1/16*I)*exp(-2*I*d+1/8*I*(2*e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi((1/4+1/4*I)*
(2*I*e+4*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.36

$$\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)\,dx$$
$$=\frac{e^{-\frac{i\left(4e^2+b^2\log^2(f)\right)}{8f}}f^{a-\frac{be+f}{2f}}\left(8e^{\frac{i\left(4e^2+b^2\log^2(f)\right)}{8f}}f^{\frac{1}{2}+b\left(\frac{e}{2f}+x\right)}+\sqrt[4]{-1}be^{\frac{ib^2\log^2(f)}{4f}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2i(e+2fx)+b\log(f))}{\sqrt{f}}\right)\log(\ldots}{16b\log(f)\ldots}$$

input
```
Integrate[f^(a + b*x)*Sin[d + e*x + f*x^2]^2,x]
```

output
```
(f^(a - (b*e + f)/(2*f))*(8*E^(((I/8)*(4*e^2 + b^2*Log[f]^2))/f)*f^(1/2 +
b*(e/(2*f) + x)) + (-1)^(1/4)*b*E^(((I/4)*b^2*Log[f]^2)/f)*Sqrt[2*Pi]*Erf[
((1/4 + I/4)*((2*I)*(e + 2*f*x) + b*Log[f]))/Sqrt[f]]*Log[f]*(Cos[2*d] + I
*Sin[2*d]) + (-1)^(1/4)*b*E^((I*e^2)/f)*Sqrt[2*Pi]*Erf[((1/4 + I/4)*(2*e +
 4*f*x + I*b*Log[f]))/Sqrt[f]]*Log[f]*(I*Cos[2*d] + Sin[2*d])))/(16*b*E^((
(I/8)*(4*e^2 + b^2*Log[f]^2))/f)*Log[f])
```

3.83.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)\,dx$$
$$\Big\downarrow \text{4975}$$
$$\int\left(-\frac{1}{4}f^{a+bx}e^{-2id-2iex-2ifx^2}-\frac{1}{4}f^{a+bx}e^{2id+2iex+2ifx^2}+\frac{1}{2}f^{a+bx}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+2ie)^2}{8f}+2id}\operatorname{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+2ie+4ifx)}{\sqrt{f}}\right)+\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(2e+ib\log(f))^2}{8f}-2id}\operatorname{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-b\log(f)+2ie+4ifx)}{\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

input
```
Int[f^(a + b*x)*Sin[d + e*x + f*x^2]^2,x]
```

output
```
(1/16 + I/16)*E^((2*I)*d + ((I/8)*((2*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*
Sqrt[Pi]*Erf[((1/4 + I/4)*((2*I)*e + (4*I)*f*x + b*Log[f]))/Sqrt[f]] + (1/
16 + I/16)*E^((-2*I)*d + ((I/8)*(2*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt
[Pi]*Erfi[((1/4 + I/4)*((2*I)*e + (4*I)*f*x - b*Log[f]))/Sqrt[f]] + f^(a +
 b*x)/(2*b*Log[f])
```

3.83.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.83.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+16df-4e^{2}\right)}{8f}} \sqrt{2}\, \mathrm{erf}\left(-\sqrt{2}\,\sqrt{if}\, x+\frac{(b\ln(f)-2ie)\sqrt{2}}{4\sqrt{if}}\right)}{16\sqrt{if}}+\frac{\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+16df-4e^{2}\right)}{8f}} \mathrm{erf}\left(-\sqrt{-2if}\, x+\right.}{8\sqrt{-2if}}$

input
```
int(f^(b*x+a)*sin(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(-1/8*I*(ln(f)^2*b^2+16*d*f-4*e^2)/f)*
2^(1/2)/(I*f)^(1/2)*erf(-2^(1/2)*(I*f)^(1/2)*x+1/4*(b*ln(f)-2*I*e)*2^(1/2)
/(I*f)^(1/2))+1/8*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(1/8*I*(ln(f)^2*b^2+16*d*
f-4*e^2)/f)/(-2*I*f)^(1/2)*erf(-(-2*I*f)^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-2*I
*f)^(1/2))+1/2*f^(b*x+a)/b/ln(f)
```

3.83.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(116) = 232$.

Time = 0.26 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.82

$$\int f^{a+bx} \sin^2 \left(d+ex+fx^2\right) \, dx =$$

$$-\frac{\pi b\sqrt{\frac{f}{\pi}} e^{\left(\frac{-i b^2 \log(f)^2 + 4i e^2 - 16i df - 4(be - 2af)\log(f)}{8f}\right)} C\left(\frac{(4fx + ib\log(f) + 2e)\sqrt{\frac{f}{\pi}}}{2f}\right)\log(f) - \pi b\sqrt{\frac{f}{\pi}} e^{\left(\frac{i b^2 \log(f)^2 - 4i e^2 + 16i df - \cdots}{8f}\right)} \cdots}{\cdots}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 4*I*e^2 - 16*I*d*f - 4*(b*
e - 2*a*f)*log(f))/f)*fresnel_cos(1/2*(4*f*x + I*b*log(f) + 2*e)*sqrt(f/pi
)/f)*log(f) - pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 - 4*I*e^2 + 16*I*d*f
- 4*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/2*(4*f*x - I*b*log(f) + 2*e)*s
qrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 4*I*e^2
- 16*I*d*f - 4*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*(4*f*x + I*b*log(f
) + 2*e)*sqrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 -
 4*I*e^2 + 16*I*d*f - 4*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/2*(4*f*x -
 I*b*log(f) + 2*e)*sqrt(f/pi)/f)*log(f) - 4*f*f^(b*x + a))/(b*f*log(f))
```

3.83.6 Sympy [F]

$$\int f^{a+bx} \sin^2 \left(d+ex+fx^2\right) \, dx = \int f^{a+bx} \sin^2 \left(d+ex+fx^2\right) dx$$

input
```
integrate(f**(b*x+a)*sin(f*x**2+e*x+d)**2,x)
```

output
```
Integral(f**(a + b*x)*sin(d + e*x + f*x**2)**2, x)
```

3.83.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(116) = 232.

Time = 0.33 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.34

$$\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)\,dx$$
$$= \frac{4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(-(i-1)\,bf^a\cos\left(\frac{b^2\log(f)^2-4\,e^2+16\,df}{8\,f}\right)\log(f)-(i+1)\,bf^a\log(f)\sin\left(\frac{b^2\log(f)^2-4\,e^2+16\,df}{8\,f}\right)\right)\mathrm{erf}}{}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*((-(I - 1)*b*f^a*cos(1/8*(b^2*log(f)^2 - 4*
e^2 + 16*d*f)/f)*log(f) - (I + 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 - 4*e
^2 + 16*d*f)/f))*erf(1/4*I*(4*I*f*x - b*log(f) + 2*I*e)*sqrt(2*I*f)/f) + (
(I + 1)*b*f^a*cos(1/8*(b^2*log(f)^2 - 4*e^2 + 16*d*f)/f)*log(f) + (I - 1)*
b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 - 4*e^2 + 16*d*f)/f))*erf(1/4*I*(4*I*f*
x + b*log(f) + 2*I*e)*sqrt(-2*I*f)/f))*f^(3/2) + 16*f^(a + 2)*e^(b*x*log(f
) + 1/2*b*e*log(f)/f))/(b*f^2*f^(1/2*b*e/f)*log(f))
```

3.83.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(116) = 232.

Time = 0.37 (sec) , antiderivative size = 599, normalized size of antiderivative = 3.35

$$\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
(2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log
(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) - (pi*b*sgn(f) - p
i*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*
b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(f)) + a*log(ab
s(f))) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f) -
1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) - I*e^(-1/2*I*p
i*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*s
gn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(abs(f))) -
 1/8*I*sqrt(pi)*erf(-1/8*I*sqrt(f)*(8*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(
abs(f)) - 4*e)/f)*(I*f/abs(f) + 1))*e^(1/16*I*pi^2*b^2*sgn(f)/f + 1/8*pi*b
^2*log(abs(f))*sgn(f)/f - 1/16*I*pi^2*b^2/f - 1/8*pi*b^2*log(abs(f))/f + 1
/8*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2
*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f + 2*I*d -
 1/2*I*e^2/f)/(sqrt(f)*(I*f/abs(f) + 1)) + 1/8*I*sqrt(pi)*erf(1/8*I*sqrt(f
)*(8*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 4*e)/f)*(-I*f/abs(f) +
1))*e^(-1/16*I*pi^2*b^2*sgn(f)/f - 1/8*pi*b^2*log(abs(f))*sgn(f)/f + 1/16*
I*pi^2*b^2/f + 1/8*pi*b^2*log(abs(f))/f - 1/8*I*b^2*log(abs(f))^2/f - 1/2*
I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*lo
g(abs(f)) - 1/2*b*e*log(abs(f))/f - 2*I*d + 1/2*I*e^2/f)/(sqrt(f)*(-I*f/ab
s(f) + 1))
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx} \sin^2\left(d+ex+fx^2\right) \, dx = \int f^{a+b\,x} \sin\left(f\,x^2+e\,x+d\right)^2 dx$$

input
```
int(f^(a + b*x)*sin(d + e*x + f*x^2)^2,x)
```

output
```
int(f^(a + b*x)*sin(d + e*x + f*x^2)^2, x)
```

3.84 $\int f^{a+bx}\sin^3(d+ex+fx^2)\,dx$

3.84.1 Optimal result

Integrand size = 21, antiderivative size = 340

$$\begin{aligned}\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx &= \frac{3}{16}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}(ie+2ifx+b\log(f))}{2\sqrt{f}}\right)\\ &\quad+\left(\frac{1}{16}-\frac{i}{16}\right)e^{3id+\frac{i(3ie+b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\,\mathrm{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(3ie+6ifx+b\log(f))}{\sqrt{6}\sqrt{f}}\right)\\ &\quad-\frac{3}{16}(-1)^{3/4}e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(ie+2ifx-b\log(f))}{2\sqrt{f}}\right)\\ &\quad-\left(\frac{1}{16}-\frac{i}{16}\right)e^{-3id+\frac{i(3e+ib\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\,\mathrm{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(3ie+6ifx-b\log(f))}{\sqrt{6}\sqrt{f}}\right)\end{aligned}$$

output
```
(1/96-1/96*I)*exp(3*I*d+1/12*I*(3*I*e+b*ln(f))^2/f)*f^(-1/2+a)*erf((1/12+1
/12*I)*(3*I*e+6*I*f*x+b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)+(-1/96+1/
96*I)*exp(-3*I*d+1/12*I*(3*e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi((1/12+1/12*I)
*(3*I*e+6*I*f*x-b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)+3/16*(-1)^(3/4)
*exp(1/4*I*(4*d+(I*e+b*ln(f))^2/f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4)*(I*e+2*I
*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-3/16*(-1)^(3/4)*exp(-I*d+1/4*I*(e+I*b*ln(f
))^2/f)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(I*e+2*I*f*x-b*ln(f))/f^(1/2))*Pi^(
1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.95

$$\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{48}(-1)^{3/4}e^{-\frac{i\left(3e^2+b^2\log^2(f)\right)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(9ie^{\frac{i\left(e^2+b^2\log^2(f)\right)}{2f}}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(e+2fx-ib\log(f))}{2\sqrt{f}}\right)(\cos(d)+i\sin(d))\right.$$
$$+e^{\frac{ie^2}{f}}\left(-9\operatorname{erfi}\left(\frac{(-1)^{3/4}(e+2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d))+\sqrt{3}e^{\frac{i\left(3e^2+b^2\log^2(f)\right)}{6f}}\operatorname{erfi}\left(\frac{(-1)^{3/4}(3e+6f}{2\sqrt{3}}\right.\right.$$

input
```
Integrate[f^(a + b*x)*Sin[d + e*x + f*x^2]^3,x]
```

output
```
((-1)^(3/4)*f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*((9*I)*E^(((I/2)*(e^2 + b^2*L
og[f]^2))/f)*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*(Cos[
d] + I*Sin[d]) + E^((I*e^2)/f)*(-9*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f
]))/(2*Sqrt[f])]*(Cos[d] - I*Sin[d]) + Sqrt[3]*E^(((I/6)*(3*e^2 + b^2*Log[
f]^2))/f)*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f]))/(2*Sqrt[3]*Sqrt[f])
]*(Cos[3*d] - I*Sin[3*d])) + Sqrt[3]*E^(((I/3)*b^2*Log[f]^2)/f)*Erfi[((1/2
 + I/2)*(3*e + 6*f*x - I*b*Log[f]))/(Sqrt[6]*Sqrt[f])]*((-I)*Cos[3*d] + Si
n[3*d])))/(48*E^(((I/4)*(3*e^2 + b^2*Log[f]^2))/f))
```

3.84.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx$$
$$\Big\downarrow \text{4975}$$
$$\int \left(\frac{3}{8}if^{a+bx}\exp\left(-3i\left(d+ex+fx^2\right)+2id+2iex+2ifx^2\right)-\frac{3}{8}if^{a+bx}\exp\left(-3i\left(d+ex+fx^2\right)+4id+4iex+4\right.\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{3}{16}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)+\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+3ie)^2}{12f}+3id}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\log(f)+3ie+6ifx)}{\sqrt{6}\sqrt{f}}\right)-\frac{3}{16}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(e+ib\log(f))^2}{4f}-id}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)-\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{i(3e+ib\log(f))^2}{12f}-3id}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-b\log(f)+3ie+6ifx)}{\sqrt{6}\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Sin[d + e*x + f*x^2]^3,x]
```

output
```
(3*(-1)^(3/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi]
*Erf[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 + (1/16 -
I/16)*E^((3*I)*d + ((I/12)*((3*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi
/6]*Erf[((1/2 + I/2)*((3*I)*e + (6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])]
- (3*(-1)^(3/4)*E^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqr
t[Pi]*Erfi[((-1)^(1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/16 - (1
/16 - I/16)*E^((-3*I)*d + ((I/12)*(3*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sq
rt[Pi/6]*Erfi[((1/2 + I/2)*((3*I)*e + (6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt
[f])]
```

3.84.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.84.4 Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+36df-9e^{2}\right)}{12f}} \operatorname{erf}\left(-\sqrt{-3if}\, x+\frac{3ie+b\ln(f)}{2\sqrt{-3if}}\right)}{16\sqrt{-3if}}+\frac{i\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+36df-9e^{2}\right)}{12f}} \sqrt{3}\, \operatorname{erf}\left(-\sqrt{3}\, \sqrt{if}\, x+\right.}{48\sqrt{if}}$

input
```
int(f^(b*x+a)*sin(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*I*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(1/12*I*(ln(f)^2*b^2+36*d*f-9*e^2)/
f)/(-3*I*f)^(1/2)*erf(-(-3*I*f)^(1/2)*x+1/2*(3*I*e+b*ln(f))/(-3*I*f)^(1/2)
)+1/48*I*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(-1/12*I*(ln(f)^2*b^2+36*d*f-9*e^2
)/f)*3^(1/2)/(I*f)^(1/2)*erf(-3^(1/2)*(I*f)^(1/2)*x+1/6*(b*ln(f)-3*I*e)*3^
(1/2)/(I*f)^(1/2))-3/16*I*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(-1/4*I*(ln(f)^2*
b^2+4*d*f-e^2)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*(b*ln(f)-I*e)/(I*f)^(
1/2))+3/16*I*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(1/4*I*(ln(f)^2*b^2+4*d*f-e^2)
/f)/(-I*f)^(1/2)*erf(-(-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-I*f)^(1/2))
```

3.84.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(220) = 440.

Time = 0.26 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.85

$$\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx$$

$$=\frac{-i\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log(f)^2+9i\,e^2-36i\,df-6\,(be-2\,af)\log(f)}{12\,f}\right)}\,\mathrm{C}\left(\frac{\sqrt{6}(6\,fx+i\,b\log(f)+3\,e)\sqrt{\frac{f}{\pi}}}{6\,f}\right)-i\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2-9i\,e^2+36i\,df}{12\,f}\right.}}{}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
1/48*(-I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 9*I*e^2 - 36*I*d
*f - 6*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(1/6*sqrt(6)*(6*f*x + I*b*log(f
) + 3*e)*sqrt(f/pi)/f) - I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 -
 9*I*e^2 + 36*I*d*f - 6*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/6*sqrt(6)*
(6*f*x - I*b*log(f) + 3*e)*sqrt(f/pi)/f) + 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/
4*(-I*b^2*log(f)^2 + I*e^2 - 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_
cos(1/2*sqrt(2)*(2*f*x + I*b*log(f) + e)*sqrt(f/pi)/f) + 9*I*sqrt(2)*pi*sq
rt(f/pi)*e^(1/4*(I*b^2*log(f)^2 - I*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f)
)/f)*fresnel_cos(-1/2*sqrt(2)*(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f) - sqr
t(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 9*I*e^2 - 36*I*d*f - 6*(b*e
- 2*a*f)*log(f))/f)*fresnel_sin(1/6*sqrt(6)*(6*f*x + I*b*log(f) + 3*e)*sqr
t(f/pi)/f) + sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 - 9*I*e^2 + 36*
I*d*f - 6*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/6*sqrt(6)*(6*f*x - I*b*l
og(f) + 3*e)*sqrt(f/pi)/f) + 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)
^2 + I*e^2 - 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*sqrt(2)*
(2*f*x + I*b*log(f) + e)*sqrt(f/pi)/f) - 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I
*b^2*log(f)^2 - I*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-
1/2*sqrt(2)*(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f))/f
```

3.84.6 Sympy [F]

$$\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*sin(f*x**2+e*x+d)**3,x)
```

output
```
Integral(f**(a + b*x)*sin(d + e*x + f*x**2)**3, x)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.11

$$\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx$$

$$=\frac{9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left((i+1)\,f^a\cos\left(\frac{b^2\log(f)^2-9\,e^2+36\,df}{12\,f}\right)-(i-1)\,f^a\sin\left(\frac{b^2\log(f)^2-9\,e^2+36\,df}{12\,f}\right)\right)\operatorname{erf}\left(\frac{i\,(6i\,fx-b\log(f)+3i\,e)}{6\,f}\right.\cdots}{\cdots}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
1/96*(9^(1/4)*sqrt(2)*sqrt(pi)*(((I + 1)*f^a*cos(1/12*(b^2*log(f)^2 - 9*e^
2 + 36*d*f)/f) - (I - 1)*f^a*sin(1/12*(b^2*log(f)^2 - 9*e^2 + 36*d*f)/f))*
erf(1/6*I*(6*I*f*x - b*log(f) + 3*I*e)*sqrt(3*I*f)/f) + (-(I - 1)*f^a*cos(
1/12*(b^2*log(f)^2 - 9*e^2 + 36*d*f)/f) + (I + 1)*f^a*sin(1/12*(b^2*log(f)
^2 - 9*e^2 + 36*d*f)/f))*erf(1/6*I*(6*I*f*x + b*log(f) + 3*I*e)*sqrt(-3*I*
f)/f))*f^(3/2) - 9*sqrt(2)*sqrt(pi)*(((I + 1)*f^a*cos(1/4*(b^2*log(f)^2 -
e^2 + 4*d*f)/f) - (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f))*erf
(1/2*I*(2*I*f*x - b*log(f) + I*e)*sqrt(I*f)/f) + (-(I - 1)*f^a*cos(1/4*(b^
2*log(f)^2 - e^2 + 4*d*f)/f) + (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 - e^2 + 4
*d*f)/f))*erf(1/2*I*(2*I*f*x + b*log(f) + I*e)*sqrt(-I*f)/f))*f^(3/2))/(f^
2*f^(1/2*b*e/f))
```

3.84.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(220) = 440$.

Time = 0.46 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.21

$$\int f^{a+bx} \sin^3\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
3/16*sqrt(2)*sqrt(pi)*erf(-1/8*I*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*
b*log(abs(f)) - 2*e)/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*s
gn(f)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*
log(abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b
*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(ab
s(f))/f + I*d - 1/4*I*e^2/f)/((I*f/abs(f) + 1)*sqrt(abs(f))) - 1/48*sqrt(6
)*sqrt(pi)*erf(-1/24*I*sqrt(6)*sqrt(f)*(12*x - (pi*b*sgn(f) - pi*b + 2*I*b
*log(abs(f)) - 6*e)/f)*(I*f/abs(f) + 1))*e^(1/24*I*pi^2*b^2*sgn(f)/f + 1/1
2*pi*b^2*log(abs(f))*sgn(f)/f - 1/24*I*pi^2*b^2/f - 1/12*pi*b^2*log(abs(f)
)/f + 1/12*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)
/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f +
 3*I*d - 3/4*I*e^2/f)/(sqrt(f)*(I*f/abs(f) + 1)) - 1/48*sqrt(6)*sqrt(pi)*e
rf(1/24*I*sqrt(6)*sqrt(f)*(12*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f))
+ 6*e)/f)*(-I*f/abs(f) + 1))*e^(-1/24*I*pi^2*b^2*sgn(f)/f - 1/12*pi*b^2*lo
g(abs(f))*sgn(f)/f + 1/24*I*pi^2*b^2/f + 1/12*pi*b^2*log(abs(f))/f - 1/12*
I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*
pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f - 3*I*d + 3/
4*I*e^2/f)/(sqrt(f)*(-I*f/abs(f) + 1)) + 3/16*sqrt(2)*sqrt(pi)*erf(1/8*I*s
qrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 2*e)/f)*(-I*f/abs(
f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*log(abs(...
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx=\int f^{a+b\,x}\,\sin\left(f\,x^2+e\,x+d\right)^3\,dx$$

input
```
int(f^(a + b*x)*sin(d + e*x + f*x^2)^3,x)
```

output
```
int(f^(a + b*x)*sin(d + e*x + f*x^2)^3, x)
```

3.85 $\int f^{a+cx^2}\sin(d+ex)\,dx$

3.85.1 Optimal result

Integrand size = 16, antiderivative size = 151

$$\int f^{a+cx^2}\sin(d+ex)\,dx = -\frac{ie^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{ie^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/4*I*exp(-I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-I*e+2*c*x*ln(f))/c^(1/2)/ln
(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/4*I*exp(I*d+1/4*e^2/c/ln(f))*f^a
*erfi(1/2*(I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1
/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.79

$$\int f^{a+cx^2}\sin(d+ex)\,dx = \frac{e^{\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(i\text{erfi}\left(\frac{-ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))+\text{erfi}\left(\frac{-ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(i\cos(d)+\sin(d))\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + c*x^2)*Sin[d + e*x],x]
```

output
```
(E^(e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*(I*Erfi[((-I)*e - 2*c*x*Log[f])/(2*Sqrt
[c]*Sqrt[Log[f]])]*(Cos[d] + I*Sin[d]) + Erfi[((-I)*e + 2*c*x*Log[f])/(2*S
qrt[c]*Sqrt[Log[f]])]*(I*Cos[d] + Sin[d])))/(4*Sqrt[c]*Sqrt[Log[f]])
```

3.85.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\sin(d+ex)\,dx$$

$$\Big\downarrow \text{4975}$$

$$\int \left(\frac{1}{2}ie^{-id-iex}f^{a+cx^2}-\frac{1}{2}ie^{id+iex}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}-id}\text{erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}+id}\text{erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Sin[d + e*x],x]
```

output
```
((-1/4*I)*E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*Log
[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) - ((I/4)*E^(I*d + e
^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Lo
g[f]])])/(Sqrt[c]*Sqrt[Log[f]])
```

3.85.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.85.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{4 i d \ln(f) c+e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{i e}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}+\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{4 i d \ln(f) c-e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(\sqrt{-c\ln(f)}\, x+\frac{i e}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}$	123

input
```
int(f^(c*x^2+a)*sin(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*e
rf(-(-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2))+1/4*I*Pi^(1/2)*f^a*exp(-1
/4*(4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/
2*I*e/(-c*ln(f))^(1/2))
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.95

$$\int f^{a+cx^2} \sin(d+ex)\, dx$$
$$= \frac{i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2\,cx\log(f)+i\,e)\sqrt{-c\log(f)}}{2\,c\log(f)}\right)e^{\left(\frac{4\,ac\log(f)^2+4i\,cd\log(f)+e^2}{4\,c\log(f)}\right)} - i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2\,cx\log(f)-i\,e)}{2\,c\log(}\right.}{4\,c\log(f)}$$

input
```
integrate(f^(c*x^2+a)*sin(e*x+d),x, algorithm="fricas")
```

output
```
1/4*(I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + I*e)*sqrt(-c*log(f
))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 4*I*c*d*log(f) + e^2)/(c*log(f)))
- I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - I*e)*sqrt(-c*log(f))/
(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 - 4*I*c*d*log(f) + e^2)/(c*log(f))))/(c
*log(f))
```

3.85.6 Sympy [F]

$$\int f^{a+cx^2}\sin(d+ex)\,dx = \int f^{a+cx^2}\sin{(d+ex)}\,dx$$

input
```
integrate(f**(c*x**2+a)*sin(e*x+d),x)
```

output
```
Integral(f**(a + c*x**2)*sin(d + e*x), x)
```

3.85.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.36

$$\int f^{a+cx^2}\sin(d+ex)\,dx =$$
$$-\frac{\sqrt{\pi}\left(f^a(i\cos{(d)}+\sin{(d)})\operatorname{erf}\left(x\overline{\sqrt{-c\log{(f)}}}+\frac{1}{2}ie\overline{\frac{1}{\sqrt{-c\log{(f)}}}}\right)e^{\left(\frac{e^2}{4c\log{(f)}}\right)}+f^a(-i\cos{(d)}+\sin{(d)})\operatorname{erf}\right.}{}$$

input
```
integrate(f^(c*x^2+a)*sin(e*x+d),x, algorithm="maxima")
```

output
```
-1/8*sqrt(pi)*(f^a*(I*cos(d) + sin(d))*erf(x*conjugate(sqrt(-c*log(f))) +
1/2*I*e*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(-I*cos
(d) + sin(d))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*I*e*conjugate(1/sqrt(
-c*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(I*cos(d) - sin(d))*erf(1/2*(2*c
*x*log(f) + I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f))) + f^a*(-I*cos(d)
- sin(d))*erf(1/2*(2*c*x*log(f) - I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(
f))))*sqrt(-c*log(f))/(c*log(f))
```

3.85.8 Giac [F]

$$\int f^{a+cx^2}\sin(d+ex)\,dx=\int f^{cx^2+a}\sin\left(ex+d\right)\,dx$$

input
```
integrate(f^(c*x^2+a)*sin(e*x+d),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*sin(e*x + d), x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sin(d+ex)\,dx=\int f^{c\,x^2+a}\,\sin\left(d+e\,x\right)\,dx$$

input
```
int(f^(a + c*x^2)*sin(d + e*x),x)
```

output
```
int(f^(a + c*x^2)*sin(d + e*x), x)
```

3.86 $\int f^{a+cx^2}\sin^2(d+ex)\,dx$

3.86.1 Optimal result

Integrand size = 18, antiderivative size = 171

$$\int f^{a+cx^2}\sin^2(d+ex)\,dx=\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{e^{2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

output
```
-1/8*exp(-2*I*d+e^2/c/ln(f))*f^a*erfi((-I*e+c*x*ln(f))/c^(1/2)/ln(f)^(1/2)
)*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/8*exp(2*I*d+e^2/c/ln(f))*f^a*erfi((I*e+c*
x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^a*erfi(x*
c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.77

$$\int f^{a+cx^2}\sin^2(d+ex)\,dx$$
$$=\frac{f^a\sqrt{\pi}\left(2\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)-e^{\frac{e^2}{c\log(f)}}\left(\text{erfi}\left(\frac{-ie+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)(\cos(2d)-i\sin(2d))+\text{erfi}\left(\frac{ie+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)(\cos(2d)\right.\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + c*x^2)*Sin[d + e*x]^2,x]
```

output
```
(f^a*Sqrt[Pi]*(2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] - E^(e^2/(c*Log[f]))*(Erfi[(
(-I)*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] - I*Sin[2*d]) + Erf
i[(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] + I*Sin[2*d]))))/(8
*Sqrt[c]*Sqrt[Log[f]])
```

3.86.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\sin^2(d+ex)\,dx$$

$$\Big\downarrow \text{4975}$$

$$\int \left(-\frac{1}{4}e^{-2id-2iex}f^{a+cx^2}-\frac{1}{4}e^{2id+2iex}f^{a+cx^2}+\frac{1}{2}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^a e^{\frac{e^2}{c\log(f)}-2id}\text{erfi}\left(\frac{-cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{e^2}{c\log(f)}+2id}\text{erfi}\left(\frac{cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Sin[d + e*x]^2,x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^
((-2*I)*d + e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - c*x*Log[f])/(Sqrt[c]*
Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) - (E^((2*I)*d + e^2/(c*Log[f]))*f
^a*Sqrt[Pi]*Erfi[(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sq
rt[Log[f]])
```

3.86.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.86.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{2 i d \ln(f) c-e^{2}}{\ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x+\frac{i e}{\sqrt{-c \ln(f)}}\right)}{8 \sqrt{-c \ln(f)}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{2 i d \ln(f) c+e^{2}}{\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{i e}{\sqrt{-c \ln(f)}}\right)}{8 \sqrt{-c \ln(f)}}+\frac{f^{a} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{\right.}}{4 \sqrt{-c}}$

input
```
int(f^(c*x^2+a)*sin(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*Pi^(1/2)*f^a*exp(-(2*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((
-c*ln(f))^(1/2)*x+I*e/(-c*ln(f))^(1/2))+1/8*Pi^(1/2)*f^a*exp((2*I*d*ln(f)*
c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+I*e/(-c*ln(f))^(1
/2))+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.94

$$\int f^{a+cx^2} \sin^2(d+ex)\, dx =$$

$$-\frac{2\sqrt{\pi}\sqrt{-c\log(f)} f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)}x\right)-\sqrt{\pi}\sqrt{-c\log(f)} \operatorname{erf}\left(\frac{(cx\log(f)+i\,e)\sqrt{-c\log(f)}}{c\log(f)}\right) e^{\left(\frac{ac\log(f)^{2}+2i\,cd\log(f}{c\log(f)}\right.}}{8\,c\log(f)}$$

input
```
integrate(f^(c*x^2+a)*sin(e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(pi)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log(f))*x) - sqrt(pi)*sqr
t(-c*log(f))*erf((c*x*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^((a*c*lo
g(f)^2 + 2*I*c*d*log(f) + e^2)/(c*log(f))) - sqrt(pi)*sqrt(-c*log(f))*erf(
(c*x*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^((a*c*log(f)^2 - 2*I*c*d*
log(f) + e^2)/(c*log(f))))/(c*log(f))
```

3.86.6 Sympy [F]

$$\int f^{a+cx^2}\sin^2(d+ex)\,dx=\int f^{a+cx^2}\sin^2{(d+ex)}\,dx$$

input
```
integrate(f**(c*x**2+a)*sin(e*x+d)**2,x)
```

output
```
Integral(f**(a + c*x**2)*sin(d + e*x)**2, x)
```

3.86.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.38

$$\int f^{a+cx^2}\sin^2(d+ex)\,dx=$$
$$-\frac{\sqrt{\pi}\left(f^a(\cos(2d)-i\sin(2d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}+ie\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^2}{c\log(f)}\right)}+f^a(\cos(2d)+i\sin(2d))\,e\right.}{}$$

input
```
integrate(f^(c*x^2+a)*sin(e*x+d)^2,x, algorithm="maxima")
```

output
```
-1/16*sqrt(pi)*(f^a*(cos(2*d) - I*sin(2*d))*erf(x*conjugate(sqrt(-c*log(f)
)) + I*e*conjugate(1/sqrt(-c*log(f))))*e^(e^2/(c*log(f))) + f^a*(cos(2*d)
+ I*sin(2*d))*erf(x*conjugate(sqrt(-c*log(f))) - I*e*conjugate(1/sqrt(-c*l
og(f))))*e^(e^2/(c*log(f))) - f^a*(cos(2*d) + I*sin(2*d))*erf((c*x*log(f)
+ I*e)/sqrt(-c*log(f)))*e^(e^2/(c*log(f))) - f^a*(cos(2*d) - I*sin(2*d))*e
rf((c*x*log(f) - I*e)/sqrt(-c*log(f)))*e^(e^2/(c*log(f))) - 2*f^a*erf(x*co
njugate(sqrt(-c*log(f)))) - 2*f^a*erf(sqrt(-c*log(f))*x))/sqrt(-c*log(f))
```

3.86.8 Giac [F]

$$\int f^{a+cx^2} \sin^2(d+ex)\,dx = \int f^{cx^2+a} \sin{(ex+d)}^2\,dx$$

input
```
integrate(f^(c*x^2+a)*sin(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*sin(e*x + d)^2, x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2} \sin^2(d+ex)\,dx = \int f^{c\,x^2+a} \sin{(d+e\,x)}^2\,dx$$

input
```
int(f^(a + c*x^2)*sin(d + e*x)^2,x)
```

output
```
int(f^(a + c*x^2)*sin(d + e*x)^2, x)
```

3.87 $\int f^{a+cx^2}\sin^3(d+ex)\,dx$

3.87.1 Optimal result

Integrand size = 18, antiderivative size = 301

$$\int f^{a+cx^2}\sin^3(d+ex)\,dx = -\frac{3ie^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{ie^{-3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{3ie^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{ie^{3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

output
```
3/16*I*exp(-I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-I*e+2*c*x*ln(f))/c^(1/2)/l
n(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/16*I*exp(-3*I*d+9/4*e^2/c/ln(f)
)*f^a*erfi(1/2*(-3*I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/
ln(f)^(1/2)-3/16*I*exp(I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(I*e+2*c*x*ln(f))
/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*I*exp(3*I*d+9/4*e^
2/c/ln(f))*f^a*erfi(1/2*(3*I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/
c^(1/2)/ln(f)^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.74

$$\int f^{a+cx^2}\sin^3(d+ex)\,dx$$
$$=\frac{e^{\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(3i\text{erfi}\left(\frac{-ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))+3\text{erfi}\left(\frac{-ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(i\cos(d)+\sin(d))-ie^{\frac{2e^2}{c\log(f)}}(\ldots}{16\sqrt{c}\sqrt{\log(f)}}$$

input

```
Integrate[f^(a + c*x^2)*Sin[d + e*x]^3,x]
```

output

```
(E^(e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*((3*I)*Erfi[((-I)*e - 2*c*x*Log[f])/(2*
Sqrt[c]*Sqrt[Log[f]])]*(Cos[d] + I*Sin[d]) + 3*Erfi[((-I)*e + 2*c*x*Log[f]
)/(2*Sqrt[c]*Sqrt[Log[f]])]*(I*Cos[d] + Sin[d]) - I*E^((2*e^2)/(c*Log[f]))
*(Erfi[((-3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[3*d] - I*S
in[3*d]) - Erfi[((3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[3*
d] + I*Sin[3*d]))))/(16*Sqrt[c]*Sqrt[Log[f]])
```

3.87.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\sin^3(d+ex)\,dx$$
$$\Big\downarrow \text{4975}$$
$$\int \left(\frac{3}{8}ie^{-id-iex}f^{a+cx^2}-\frac{3}{8}ie^{id+iex}f^{a+cx^2}-\frac{1}{8}ie^{-3id-3iex}f^{a+cx^2}+\frac{1}{8}ie^{3id+3iex}f^{a+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{3i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}-id}\text{erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{i\sqrt{\pi}f^a e^{\frac{9e^2}{4c\log(f)}-3id}\text{erfi}\left(\frac{-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}+id}\text{erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{i\sqrt{\pi}f^a e^{\frac{9e^2}{4c\log(f)}+3id}\text{erfi}\left(\frac{2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Sin[d + e*x]^3,x]
```

output
```
(((-3*I)/16)*E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*
Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) + ((I/16)*E^((-3
*I)*d + (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e - 2*c*x*Log[f])/(
2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) - (((3*I)/16)*E^(I*d + e^
2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log
[f]])])/(Sqrt[c]*Sqrt[Log[f]]) + ((I/16)*E^((3*I)*d + (9*e^2)/(4*c*Log[f])
)*f^a*Sqrt[Pi]*Erfi[((3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(S
qrt[c]*Sqrt[Log[f]])
```

3.87.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.87.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{3 i d \ln(f) c+\frac{9 e^{2}}{4}}{c \ln(f)}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{3 i e}{2\sqrt{-c\ln(f)}}\right)}{16\sqrt{-c\ln(f)}}-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4 i d \ln(f) c-3 e^{2}\right)}{4 \ln(f) c}} \operatorname{erf}\left(\sqrt{-c\ln(f)}\, x+\frac{3 i e}{2\sqrt{-c\ln(f)}}\right)}{16\sqrt{-c\ln(f)}}+\frac{3i\sqrt{}}{}$

input
```
int(f^(c*x^2+a)*sin(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*I*Pi^(1/2)*f^a*exp(3/4*(4*I*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/
2)*erf(-(-c*ln(f))^(1/2)*x+3/2*I*e/(-c*ln(f))^(1/2))-1/16*I*Pi^(1/2)*f^a*e
xp(-3/4*(4*I*d*ln(f)*c-3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/
2)*x+3/2*I*e/(-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c
-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^
(1/2))+3/16*I*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))
^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2))
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.94

$$\int f^{a+cx^2}\sin^3(d+ex)\,dx$$
$$= \frac{-i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2\,cx\log(f)+3i\,e)\sqrt{-c\log(f)}}{2\,c\log(f)}\right)e^{\left(\frac{4\,ac\log(f)^2+12i\,cd\log(f)+9\,e^2}{4\,c\log(f)}\right)}+3i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2\,cx\log(\ldots}{\ldots}\right.}{\ldots}$$

input
```
integrate(f^(c*x^2+a)*sin(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/16*(-I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + 3*I*e)*sqrt(-c*l
og(f))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 12*I*c*d*log(f) + 9*e^2)/(c*lo
g(f))) + 3*I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + I*e)*sqrt(-c
*log(f))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 4*I*c*d*log(f) + e^2)/(c*log
(f))) - 3*I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - I*e)*sqrt(-c*
log(f))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 - 4*I*c*d*log(f) + e^2)/(c*log(
f))) + I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - 3*I*e)*sqrt(-c*l
og(f))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 - 12*I*c*d*log(f) + 9*e^2)/(c*lo
g(f))))/(c*log(f))
```

3.87.6 Sympy [F]

$$\int f^{a+cx^2}\sin^3(d+ex)\,dx = \int f^{a+cx^2}\sin^3{(d+ex)}\,dx$$

input
```
integrate(f**(c*x**2+a)*sin(e*x+d)**3,x)
```

output
```
Integral(f**(a + c*x**2)*sin(d + e*x)**3, x)
```

3.87.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.37

$$\int f^{a+cx^2}\sin^3(d+ex)\,dx$$
$$= \frac{\sqrt{\pi}\left(f^a(i\,\cos(3\,d)+\sin(3\,d))\,\mathrm{erf}\left(x\overline{\sqrt{-c\log(f)}}+\frac{3}{2}i\,e\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{9\,e^2}{4\,c\log(f)}\right)}+f^a(-i\,\cos(3\,d)+\sin(3\,d)\right.}{}$$

input
```
integrate(f^(c*x^2+a)*sin(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/32*sqrt(pi)*(f^a*(I*cos(3*d) + sin(3*d))*erf(x*conjugate(sqrt(-c*log(f))
) + 3/2*I*e*conjugate(1/sqrt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*(-I
*cos(3*d) + sin(3*d))*erf(x*conjugate(sqrt(-c*log(f))) - 3/2*I*e*conjugate
(1/sqrt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*(I*cos(3*d) - sin(3*d))*
erf(1/2*(2*c*x*log(f) + 3*I*e)/sqrt(-c*log(f)))*e^(9/4*e^2/(c*log(f))) + f
^a*(-I*cos(3*d) - sin(3*d))*erf(1/2*(2*c*x*log(f) - 3*I*e)/sqrt(-c*log(f))
)*e^(9/4*e^2/(c*log(f))) - 3*f^a*(I*cos(d) + sin(d))*erf(x*conjugate(sqrt(
-c*log(f))) + 1/2*I*e*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f)))
 - 3*f^a*(-I*cos(d) + sin(d))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*I*e*c
onjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) - 3*f^a*(I*cos(d) - si
n(d))*erf(1/2*(2*c*x*log(f) + I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f)))
 - 3*f^a*(-I*cos(d) - sin(d))*erf(1/2*(2*c*x*log(f) - I*e)/sqrt(-c*log(f))
)*e^(1/4*e^2/(c*log(f))))*sqrt(-c*log(f))/(c*log(f))
```

3.87.8 Giac [F]

$$\int f^{a+cx^2}\sin^3(d+ex)\,dx = \int f^{cx^2+a}\sin(ex+d)^3\,dx$$

input
```
integrate(f^(c*x^2+a)*sin(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*sin(e*x + d)^3, x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2} \sin^3(d+ex)\,dx = \int f^{c\,x^2+a} \sin\left(d+e\,x\right)^3 dx$$

input
```
int(f^(a + c*x^2)*sin(d + e*x)^3,x)
```

output
```
int(f^(a + c*x^2)*sin(d + e*x)^3, x)
```

3.88 $\int f^{a+cx^2} \sin(d+fx^2)\,dx$

3.88.1 Optimal result

Integrand size = 18, antiderivative size = 107

$$\int f^{a+cx^2}\sin\left(d+fx^2\right)\,dx=\frac{ie^{-id}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{if-c\log(f)}\right)}{4\sqrt{if-c\log(f)}}-\frac{ie^{id}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{if+c\log(f)}\right)}{4\sqrt{if+c\log(f)}}$$

output
```
1/4*I*f^a*erf(x*(I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(I*d)/(I*f-c*ln(f))^(1/2)
-1/4*I*exp(I*d)*f^a*erfi(x*(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/
2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.59

$$\int f^{a+cx^2}\sin\left(d+fx^2\right)\,dx=$$

$$-\frac{\sqrt[4]{-1}f^a\sqrt{\pi}\left(\operatorname{erfi}\left(\sqrt[4]{-1}x\sqrt{f-ic\log(f)}\right)\sqrt{f-ic\log(f)}(f+ic\log(f))(\cos(d)+i\sin(d))+\sqrt{f+ic\log\cdots}\right.}{4\left(f^2\cdots\right.}$$

input
```
Integrate[f^(a + c*x^2)*Sin[d + f*x^2],x]
```

output
```
-1/4*((-1)^(1/4)*f^a*Sqrt[Pi]*(Erfi[(-1)^(1/4)*x*Sqrt[f - I*c*Log[f]]]*Sqr
t[f - I*c*Log[f]]*(f + I*c*Log[f])*(Cos[d] + I*Sin[d]) + Sqrt[f + I*c*Log[
f]]*(c*Erf[((1 + I)*x*Sqrt[f + I*c*Log[f]])/Sqrt[2]]*Log[f]*Sin[d] + Erfi[
(-1)^(3/4)*x*Sqrt[f + I*c*Log[f]]]*(Cos[d]*(I*f + c*Log[f]) + f*Sin[d]))))
/(f^2 + c^2*Log[f]^2)
```

3.88.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2} \sin\left(d+fx^2\right) dx$$

$$\Big\downarrow \text{ 4975}$$

$$\int \left(\frac{1}{2}ie^{-id-ifx^2}f^{a+cx^2}-\frac{1}{2}ie^{id+ifx^2}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i\sqrt{\pi}e^{-id}f^a\text{erf}\left(x\sqrt{-c\log(f)+if}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}e^{id}f^a\text{erfi}\left(x\sqrt{c\log(f)+if}\right)}{4\sqrt{c\log(f)+if}}$$

input
```
Int[f^(a + c*x^2)*Sin[d + f*x^2],x]
```

output
```
((I/4)*f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(E^(I*d)*Sqrt[I*f - c*Log
[f]]) - ((I/4)*E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[I*f + c*Log[f]]])/Sqrt[I*f
 + c*Log[f]]
```

3.88.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.88.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{i d} \operatorname{erf}\left(\sqrt{-c\ln\left(f\right)-i f}\, x\right)}{4\sqrt{-c\ln\left(f\right)-i f}}+\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-i d} \operatorname{erf}\left(x\sqrt{i f-c\ln\left(f\right)}\right)}{4\sqrt{i f-c\ln\left(f\right)}}$	84

input
```
int(f^(c*x^2+a)*sin(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*Pi^(1/2)*f^a*exp(I*d)/(-c*ln(f)-I*f)^(1/2)*erf((-c*ln(f)-I*f)^(1/2)
*x)+1/4*I*Pi^(1/2)*f^a*exp(-I*d)/(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(
1/2))
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int f^{a+cx^2}\sin\left(d+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}(i\,c\log\left(f\right)+f)\sqrt{-c\log\left(f\right)-i\,f}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-i\,f}x\right)e^{\left(a\log\left(f\right)+i\,d\right)}+\sqrt{\pi}(-i\,c\log\left(f\right)+f)\sqrt{-c\log\left(f\right)+i\,f}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+i\,f}x\right)e^{\left(a\log\left(f\right)-i\,d\right)}}{4\left(c^2\log\left(f\right)^2+f^2\right)}$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+d),x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*(I*c*log(f) + f)*sqrt(-c*log(f) - I*f)*erf(sqrt(-c*log(f) -
I*f)*x)*e^(a*log(f) + I*d) + sqrt(pi)*(-I*c*log(f) + f)*sqrt(-c*log(f) + I
*f)*erf(sqrt(-c*log(f) + I*f)*x)*e^(a*log(f) - I*d))/(c^2*log(f)^2 + f^2)
```

3.88.6 Sympy [F]

$$\int f^{a+cx^2} \sin\left(d+fx^2\right) \, dx = \int f^{a+cx^2} \sin\left(d+fx^2\right) dx$$

input
```
integrate(f**(c*x**2+a)*sin(f*x**2+d),x)
```

output
```
Integral(f**(a + c*x**2)*sin(d + f*x**2), x)
```

3.88.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(73) = 146.

Time = 0.24 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.95

$$\int f^{a+cx^2} \sin\left(d+fx^2\right) \, dx$$
$$= \frac{\sqrt{\pi}\sqrt{2\,c^2\log(f)^2+2\,f^2}\left(f^a(\cos(d)-i\,\sin(d))\,\mathrm{erf}\left(\sqrt{-c\log(f)+i\,f}x\right)+f^a(\cos(d)+i\,\sin(d))\,\mathrm{erf}\left(\sqrt{\right.\right.}{}$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+d),x, algorithm="maxima")
```

output
```
1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(f^a*(cos(d) - I*sin(d))*erf(sq
rt(-c*log(f) + I*f)*x) + f^a*(cos(d) + I*sin(d))*erf(sqrt(-c*log(f) - I*f)
*x))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) - sqrt(pi)*sqrt(2*c^2*log(f
)^2 + 2*f^2)*(f^a*(-I*cos(d) - sin(d))*erf(sqrt(-c*log(f) + I*f)*x) + f^a*
(I*cos(d) - sin(d))*erf(sqrt(-c*log(f) - I*f)*x))*sqrt(-c*log(f) + sqrt(c^
2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

3.88.8 Giac [F]

$$\int f^{a+cx^2}\sin\left(d+fx^2\right)\,dx=\int f^{cx^2+a}\sin\left(fx^2+d\right)\,dx$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+d),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*sin(f*x^2 + d), x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sin\left(d+fx^2\right)\,dx=\int f^{c\,x^2+a}\,\sin\left(f\,x^2+d\right)\,dx$$

input
```
int(f^(a + c*x^2)*sin(d + f*x^2),x)
```

output
```
int(f^(a + c*x^2)*sin(d + f*x^2), x)
```

3.89 $\int f^{a+cx^2} \sin^2(d+fx^2)\, dx$

3.89.1 Optimal result

Integrand size = 20, antiderivative size = 140

$$\int f^{a+cx^2}\sin^2(d+fx^2)\,dx=\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{2if-c\log(f)}\right)}{8\sqrt{2if-c\log(f)}}-\frac{e^{2id}f^a\sqrt{\pi}\,\mathrm{erfi}\left(x\sqrt{2if+c\log(f)}\right)}{8\sqrt{2if+c\log(f)}}$$

output
```
1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/8*f^a*e
rf(x*(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(2*I*d)/(2*I*f-c*ln(f))^(1/2)-1/8*
exp(2*I*d)*f^a*erfi(x*(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*ln(f))^(1/2
)
```

3.89.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.34

$$\int f^{a+cx^2}\sin^2(d+fx^2)\,dx=\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt[4]{-1}\left(\mathrm{erf}\left(\sqrt[4]{-1}x\sqrt{2f+ic\log(f)}\right)\sqrt{2f+ic\log(f)}(2if+c\log(f))(\cos(2d)-i\sin(2d))+\mathrm{erf}\left((-1)^{3/4}\right.\right.}{4f^2+c^2\log^2(f)}\right.$$

input
```
Integrate[f^(a + c*x^2)*Sin[d + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*((2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + (
(-1)^(1/4)*(Erf[(-1)^(1/4)*x*Sqrt[2*f + I*c*Log[f]]]*Sqrt[2*f + I*c*Log[f]
]*((2*I)*f + c*Log[f])*(Cos[2*d] - I*Sin[2*d]) + Erf[(-1)^(3/4)*x*Sqrt[2*f
 - I*c*Log[f]]]*Sqrt[2*f - I*c*Log[f]]*(2*f + I*c*Log[f])*(Cos[2*d] + I*Si
n[2*d])))/(4*f^2 + c^2*Log[f]^2)))/8
```

3.89.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2} \sin^2\left(d+fx^2\right) dx$$

$$\Big\downarrow \text{4975}$$

$$\int \left(-\frac{1}{4}e^{-2id-2ifx^2}f^{a+cx^2}-\frac{1}{4}e^{2id+2ifx^2}f^{a+cx^2}+\frac{1}{2}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}e^{-2id}f^a\text{erf}\left(x\sqrt{-c\log(f)+2if}\right)}{8\sqrt{-c\log(f)+2if}}-\frac{\sqrt{\pi}e^{2id}f^a\text{erfi}\left(x\sqrt{c\log(f)+2if}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}f^a\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Sin[d + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (f^
a*Sqrt[Pi]*Erf[x*Sqrt[(2*I)*f - c*Log[f]]])/(8*E^((2*I)*d)*Sqrt[(2*I)*f -
c*Log[f]]) - (E^((2*I)*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[(2*I)*f + c*Log[f]]])/(
8*Sqrt[(2*I)*f + c*Log[f]])
```

3.89.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.89.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

method	result	size
risch	$-\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{-2 i d} \operatorname{erf}\left(x \sqrt{2 i f -c \ln \left(f \right)}\right)}{8 \sqrt{2 i f -c \ln \left(f \right)}}-\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{2 i d} \operatorname{erf}\left(\sqrt{-c \ln \left(f \right)-2 i f}\, x \right)}{8 \sqrt{-c \ln \left(f \right)-2 i f}}+\frac{f^{a} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-c \ln \left(f \right)}\, x \right)}{4 \sqrt{-c \ln \left(f \right)}}$	107

input
```
int(f^(c*x^2+a)*sin(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*Pi^(1/2)*f^a*exp(-2*I*d)/(2*I*f-c*ln(f))^(1/2)*erf(x*(2*I*f-c*ln(f))^
(1/2))-1/8*Pi^(1/2)*f^a*exp(2*I*d)/(-c*ln(f)-2*I*f)^(1/2)*erf((-c*ln(f)-2*
I*f)^(1/2)*x)+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.21

$$\int f^{a+cx^2} \sin^2\left(d+fx^2\right)\,dx =$$

$$-\frac{2\,\sqrt{\pi}\left(c^2\log\left(f\right)^2+4\,f^2\right)\sqrt{-c\log\left(f\right)}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x\right)-\sqrt{\pi}\left(c^2\log\left(f\right)^2-2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(\right.}}{}$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*lo
g(f))*x) - sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f
)*erf(sqrt(-c*log(f) - 2*I*f)*x)*e^(a*log(f) + 2*I*d) - sqrt(pi)*(c^2*log(
f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf(sqrt(-c*log(f) + 2*I*f)
*x)*e^(a*log(f) - 2*I*d))/(c^3*log(f)^3 + 4*c*f^2*log(f))
```

3.89.6 Sympy [F]

$$\int f^{a+cx^2}\sin^2\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\sin^2\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*sin(f*x**2+d)**2,x)
```

output
```
Integral(f**(a + c*x**2)*sin(d + f*x**2)**2, x)
```

3.89.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.25

$$\int f^{a+cx^2}\sin^2\left(d+fx^2\right)\,dx$$

$$=\frac{\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+8\,f^2}\left(f^a(i\,\cos\left(2\,d\right)+\sin\left(2\,d\right))\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)+2i\,f}x\right)+f^a(-i\,\cos\left(2\,d\right)+\sin\left(2\,d\right)\right.}{}$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*(f^a*(I*cos(2*d) + sin(2*d))*e
rf(sqrt(-c*log(f) + 2*I*f)*x) + f^a*(-I*cos(2*d) + sin(2*d))*erf(sqrt(-c*l
og(f) - 2*I*f)*x))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log
(f)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*(f^a*(cos(2*d) - I*sin(2*d))*
erf(sqrt(-c*log(f) + 2*I*f)*x) + f^a*(cos(2*d) + I*sin(2*d))*erf(sqrt(-c*l
og(f) - 2*I*f)*x))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*lo
g(f)) + 2*sqrt(pi)*((c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(x*conjugate(sqrt(
-c*log(f)))) + (c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(sqrt(-c*log(f))*x)))/(
(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f)))
```

3.89.8 Giac [F]

$$\int f^{a+cx^2} \sin^2\left(d+fx^2\right) dx = \int f^{cx^2+a} \sin\left(fx^2+d\right)^2 dx$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*sin(f*x^2 + d)^2, x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2} \sin^2\left(d+fx^2\right) dx = \int f^{c\,x^2+a} \sin\left(f\,x^2+d\right)^2 dx$$

input
```
int(f^(a + c*x^2)*sin(d + f*x^2)^2,x)
```

output
```
int(f^(a + c*x^2)*sin(d + f*x^2)^2, x)
```

3.90 $\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx$

3.90.1 Optimal result

Integrand size = 20, antiderivative size = 213

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx=\frac{3ie^{-id}f^a\sqrt{\pi}\mathrm{erf}\left(x\sqrt{if-c\log(f)}\right)}{16\sqrt{if-c\log(f)}}-\frac{ie^{-3id}f^a\sqrt{\pi}\mathrm{erf}\left(x\sqrt{3if-c\log(f)}\right)}{16\sqrt{3if-c\log(f)}}-\frac{3ie^{id}f^a\sqrt{\pi}\mathrm{erfi}\left(x\sqrt{if+c\log(f)}\right)}{16\sqrt{if+c\log(f)}}+\frac{ie^{3id}f^a\sqrt{\pi}\mathrm{erfi}\left(x\sqrt{3if+c\log(f)}\right)}{16\sqrt{3if+c\log(f)}}$$

output
```
3/16*I*f^a*erf(x*(I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(I*d)/(I*f-c*ln(f))^(1/2
)-1/16*I*f^a*erf(x*(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(3*I*d)/(3*I*f-c*ln(
f))^(1/2)-3/16*I*exp(I*d)*f^a*erfi(x*(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*
ln(f))^(1/2)+1/16*I*exp(3*I*d)*f^a*erfi(x*(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/
(3*I*f+c*ln(f))^(1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.81

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx$$
$$=\frac{\sqrt[4]{-1}f^a\sqrt{\pi}\left(-3\mathrm{erfi}\left(\sqrt[4]{-1}x\sqrt{f-ic\log(f)}\right)\sqrt{f-ic\log(f)}\left(9f^3+9icf^2\log(f)+c^2f\log^2(f)+ic^3\log^3(f\right.\right.}{}$$

input
```
Integrate[f^(a + c*x^2)*Sin[d + f*x^2]^3,x]
```

output
```
((-1)^(1/4)*f^a*Sqrt[Pi]*(-3*Erfi[(-1)^(1/4)*x*Sqrt[f - I*c*Log[f]]]*Sqrt[
f - I*c*Log[f]]*(9*f^3 + (9*I)*c*f^2*Log[f] + c^2*f*Log[f]^2 + I*c^3*Log[f
]^3)*(Cos[d] + I*Sin[d]) + (f - I*c*Log[f])*(Erfi[(-1)^(1/4)*x*Sqrt[3*f -
I*c*Log[f]]]*Sqrt[3*f - I*c*Log[f]]*(3*f^2 + (4*I)*c*f*Log[f] - c^2*Log[f]
^2)*(Cos[3*d] + I*Sin[3*d]) + (3*f - I*c*Log[f])*(3*Erfi[(-1)^(3/4)*x*Sqrt
[f + I*c*Log[f]]]*Sqrt[f + I*c*Log[f]]*(c*Cos[d]*Log[f] - 3*f*Sin[d]) + 3*
Erf[((1 + I)*x*Sqrt[f + I*c*Log[f]])/Sqrt[2]]*Sqrt[f + I*c*Log[f]]*(3*f*Co
s[d] + c*Log[f]*Sin[d]) + Erfi[(-1)^(3/4)*x*Sqrt[3*f + I*c*Log[f]]]*(f + I
*c*Log[f])*Sqrt[3*f + I*c*Log[f]]*(I*Cos[3*d] + Sin[3*d])))))/(16*(9*f^4 +
 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

3.90.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx$$
$$\Big\downarrow \text{4975}$$
$$\int\left(\frac{3}{8}ie^{-id-ifx^2}f^{a+cx^2}-\frac{3}{8}ie^{id+ifx^2}f^{a+cx^2}-\frac{1}{8}ie^{-3id-3ifx^2}f^{a+cx^2}+\frac{1}{8}ie^{3id+3ifx^2}f^{a+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{3i\sqrt{\pi}e^{-id}f^a\text{erf}\left(x\sqrt{-c\log(f)+if}\right)}{16\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}e^{-3id}f^a\text{erf}\left(x\sqrt{-c\log(f)+3if}\right)}{16\sqrt{-c\log(f)+3if}}-\frac{3i\sqrt{\pi}e^{id}f^a\text{erfi}\left(x\sqrt{c\log(f)+if}\right)}{16\sqrt{c\log(f)+if}}+\frac{i\sqrt{\pi}e^{3id}f^a\text{erfi}\left(x\sqrt{c\log(f)+3if}\right)}{16\sqrt{c\log(f)+3if}}$$

input
```
Int[f^(a + c*x^2)*Sin[d + f*x^2]^3,x]
```

output
```
(((3*I)/16)*f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(E^(I*d)*Sqrt[I*f -
c*Log[f]]) - ((I/16)*f^a*Sqrt[Pi]*Erf[x*Sqrt[(3*I)*f - c*Log[f]]])/(E^((3*
I)*d)*Sqrt[(3*I)*f - c*Log[f]]) - (((3*I)/16)*E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*
Sqrt[I*f + c*Log[f]]])/Sqrt[I*f + c*Log[f]] + ((I/16)*E^((3*I)*d)*f^a*Sqrt
[Pi]*Erfi[x*Sqrt[(3*I)*f + c*Log[f]]])/Sqrt[(3*I)*f + c*Log[f]]
```

3.90.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.90.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.78

method	result
risch	$\frac{i\sqrt{\pi}\,f^a\mathrm{e}^{3id}\,\mathrm{erf}\left(\sqrt{-c\ln(f)-3if}\,x\right)}{16\sqrt{-c\ln(f)-3if}}-\frac{i\sqrt{\pi}\,f^a\mathrm{e}^{-3id}\,\mathrm{erf}\left(x\sqrt{3if-c\ln(f)}\right)}{16\sqrt{3if-c\ln(f)}}+\frac{3i\sqrt{\pi}\,f^a\mathrm{e}^{-id}\,\mathrm{erf}\left(x\sqrt{if-c\ln(f)}\right)}{16\sqrt{if-c\ln(f)}}-\frac{3i\sqrt{\pi}\,f^a\mathrm{e}^{id}\,\mathrm{erf}}{16\sqrt{-}}$

input
```
int(f^(c*x^2+a)*sin(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*I*Pi^(1/2)*f^a*exp(3*I*d)/(-c*ln(f)-3*I*f)^(1/2)*erf((-c*ln(f)-3*I*f)
^(1/2)*x)-1/16*I*Pi^(1/2)*f^a*exp(-3*I*d)/(3*I*f-c*ln(f))^(1/2)*erf(x*(3*I
*f-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-I*d)/(I*f-c*ln(f))^(1/2)*erf(x
*(I*f-c*ln(f))^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(I*d)/(-c*ln(f)-I*f)^(1/2)*er
f((-c*ln(f)-I*f)^(1/2)*x)
```

3.90.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(145) = 290.

Time = 0.27 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.49

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx$$
$$= \frac{\sqrt{\pi}\left(-i\,c^3\log\left(f\right)^3-3\,c^2f\log\left(f\right)^2-i\,cf^2\log\left(f\right)-3\,f^3\right)\sqrt{-c\log\left(f\right)-3i\,f}\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-3i\,f}x\right)e^{(\cdots}}{\cdots}$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+d)^3,x, algorithm="fricas")
```

output
```
1/16*(sqrt(pi)*(-I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 - I*c*f^2*log(f) - 3*f^
3)*sqrt(-c*log(f) - 3*I*f)*erf(sqrt(-c*log(f) - 3*I*f)*x)*e^(a*log(f) + 3*
I*d) - 3*sqrt(pi)*(-I*c^3*log(f)^3 - c^2*f*log(f)^2 - 9*I*c*f^2*log(f) - 9
*f^3)*sqrt(-c*log(f) - I*f)*erf(sqrt(-c*log(f) - I*f)*x)*e^(a*log(f) + I*d
) - 3*sqrt(pi)*(I*c^3*log(f)^3 - c^2*f*log(f)^2 + 9*I*c*f^2*log(f) - 9*f^3
)*sqrt(-c*log(f) + I*f)*erf(sqrt(-c*log(f) + I*f)*x)*e^(a*log(f) - I*d) +
sqrt(pi)*(I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 + I*c*f^2*log(f) - 3*f^3)*sqrt
(-c*log(f) + 3*I*f)*erf(sqrt(-c*log(f) + 3*I*f)*x)*e^(a*log(f) - 3*I*d))/(
c^4*log(f)^4 + 10*c^2*f^2*log(f)^2 + 9*f^4)
```

3.90.6 Sympy [F]

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\sin^3\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*sin(f*x**2+d)**3,x)
```

output
```
Integral(f**(a + c*x**2)*sin(d + f*x**2)**3, x)
```

3.90.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(145) = 290$.

Time = 0.24 (sec) , antiderivative size = 661, normalized size of antiderivative = 3.10

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{2\,c^2\log(f)^2+18\,f^2}\left(\left((c^2\cos(3\,d)-i\,c^2\sin(3\,d))f^a\log(f)^2+f^{a+2}(\cos(3\,d)-i\,\sin(3\,d))\right)\operatorname{erf}\left(\right.\right.}{}$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*cos(3*d) - I*c^2*sin(
3*d))*f^a*log(f)^2 + f^(a + 2)*(cos(3*d) - I*sin(3*d)))*erf(sqrt(-c*log(f)
 + 3*I*f)*x) + ((c^2*cos(3*d) + I*c^2*sin(3*d))*f^a*log(f)^2 + f^(a + 2)*(
cos(3*d) + I*sin(3*d)))*erf(sqrt(-c*log(f) - 3*I*f)*x))*sqrt(c*log(f) + sq
rt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((c^2
*cos(d) - I*c^2*sin(d))*f^a*log(f)^2 + 9*f^(a + 2)*(cos(d) - I*sin(d)))*er
f(sqrt(-c*log(f) + I*f)*x) + ((c^2*cos(d) + I*c^2*sin(d))*f^a*log(f)^2 + 9
*f^(a + 2)*(cos(d) + I*sin(d)))*erf(sqrt(-c*log(f) - I*f)*x))*sqrt(c*log(f
) + sqrt(c^2*log(f)^2 + f^2)) + sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((
I*c^2*cos(3*d) + c^2*sin(3*d))*f^a*log(f)^2 + f^(a + 2)*(I*cos(3*d) + sin(
3*d)))*erf(sqrt(-c*log(f) + 3*I*f)*x) + ((-I*c^2*cos(3*d) + c^2*sin(3*d))*
f^a*log(f)^2 + f^(a + 2)*(-I*cos(3*d) + sin(3*d)))*erf(sqrt(-c*log(f) - 3*
I*f)*x))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) + 3*sqrt(pi)*sqrt(2*
c^2*log(f)^2 + 2*f^2)*(((-I*c^2*cos(d) - c^2*sin(d))*f^a*log(f)^2 + 9*f^(a
 + 2)*(-I*cos(d) - sin(d)))*erf(sqrt(-c*log(f) + I*f)*x) + ((I*c^2*cos(d)
- c^2*sin(d))*f^a*log(f)^2 + 9*f^(a + 2)*(I*cos(d) - sin(d)))*erf(sqrt(-c*
log(f) - I*f)*x))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^4*log(f)^
4 + 10*c^2*f^2*log(f)^2 + 9*f^4)
```

3.90.8 Giac [F]

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx=\int f^{cx^2+a}\sin\left(fx^2+d\right)^3\,dx$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*sin(f*x^2 + d)^3, x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx=\int f^{c\,x^2+a}\sin\left(f\,x^2+d\right)^3\,dx$$

input
```
int(f^(a + c*x^2)*sin(d + f*x^2)^3,x)
```

output
```
int(f^(a + c*x^2)*sin(d + f*x^2)^3, x)
```

3.91 $\int f^{a+cx^2}\sin\left(d+ex+fx^2\right)\,dx$

3.91.1 Optimal result

Integrand size = 21, antiderivative size = 187

$$\int f^{a+cx^2}\sin\left(d+ex+fx^2\right)\,dx = \frac{ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}} - \frac{ie^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

output
```
1/4*I*exp(-I*d-e^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e+2*x*(I*f-c*ln(f)))/
(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/4*I*exp(I*d+e^2/(4*I*f
+4*c*ln(f)))*f^a*erfi(1/2*(I*e+2*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^
(1/2)/(I*f+c*ln(f))^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.16

$$\int f^{a+cx^2}\sin\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{(-1)^{3/4}e^{\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\left(e^{\frac{ie^2f}{2\left(f^2+c^2\log^2(f)\right)}}\mathrm{erfi}\left(\frac{(-1)^{3/4}(e+2fx+2icx\log(f))}{2\sqrt{f+ic\log(f)}}\right)(f-ic\log(f))\sqrt{f+ic\log(f)}(\cos(\right.}{4\left(f^2+c^2\log\right.}$$

input
```
Integrate[f^(a + c*x^2)*Sin[d + e*x + f*x^2],x]
```

output
```
-1/4*((-1)^(3/4)*E^(e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*(E^(((I/2)*e^
2*f)/(f^2 + c^2*Log[f]^2))*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f])
)/(2*Sqrt[f + I*c*Log[f]])]*(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*(Cos[d]
- I*Sin[d]) + Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f -
 I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*((-I)*f + c*Log[f])*(Cos[d] + I*Sin[d]
)))/(f^2 + c^2*Log[f]^2)
```

3.91.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\sin\left(d+ex+fx^2\right)dx$$

$$\Big\downarrow \text{4975}$$

$$\int \left(\frac{1}{2}if^{a+cx^2}e^{-id-iex-ifx^2}-\frac{1}{2}if^{a+cx^2}e^{id+iex+ifx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\sqrt{\pi}f^a e^{-\frac{e^2}{-4c\log(f)+4if}-id}\operatorname{erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)+4if}+id}\operatorname{erfi}\left(\frac{2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

input
```
Int[f^(a + c*x^2)*Sin[d + e*x + f*x^2],x]
```

output
```
((I/4)*E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + 2*x
*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f - c*Log[f]] - ((I/4
)*E^(I*d + e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*x*(I*f +
 c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f + c*Log[f]]
```

3.91.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.91.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.90

method	result
risch	$\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{4id\ln(f)c-4df+e^{2}}{4if+4c\ln(f)}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-if}\,x+\frac{ie}{2\sqrt{-c\ln(f)-if}}\right)}{4\sqrt{-c\ln(f)-if}}+\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{4id\ln(f)c+4df-e^{2}}{4(c\ln(f)-if)}} \operatorname{erf}\left(x\sqrt{if-c\ln(f)}+\frac{ie}{2\sqrt{if-c\ln(f}}\right.}{4\sqrt{if-c\ln(f)}}$

input
```
int(f^(c*x^2+a)*sin(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c-4*d*f+e^2)/(I*f+c*ln(f)))/(-c*ln
(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*I*e/(-c*ln(f)-I*f)^(1/2))+1
/4*I*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c+4*d*f-e^2)/(c*ln(f)-I*f))/(I*f-c
*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2)+1/2*I*e/(I*f-c*ln(f))^(1/2))
```

3.91.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(135) = 270.

Time = 0.26 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.60

$$\int f^{a+cx^2} \sin\left(d+ex+fx^2\right)\,dx$$

$$= \frac{\sqrt{\pi}(i\,c\log(f)+f)\sqrt{-c\log(f)-i\,f}\,\operatorname{erf}\left(\frac{\left(2\,c^2x\log(f)^2+2\,f^2x+i\,ce\log(f)+ef\right)\sqrt{-c\log(f)-i\,f}}{2\left(c^2\log(f)^2+f^2\right)}\right)e^{\left(\frac{4\,ac^2\log(f)^3+4i\,c^2d\log(f)}{4\,(c}\right.}}{}$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*(I*c*log(f) + f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*c^2*x*log(
f)^2 + 2*f^2*x + I*c*e*log(f) + e*f)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 +
 f^2))*e^(1/4*(4*a*c^2*log(f)^3 + 4*I*c^2*d*log(f)^2 - I*e^2*f + 4*I*d*f^2
 + (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(-I*c*log(f)
 + f)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x - I*c*e*lo
g(f) + e*f)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*c^2*lo
g(f)^3 - 4*I*c^2*d*log(f)^2 + I*e^2*f - 4*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(
f))/(c^2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

3.91.6 Sympy [F]

$$\int f^{a+cx^2}\sin\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\sin\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*sin(f*x**2+e*x+d),x)
```

output
```
Integral(f**(a + c*x**2)*sin(d + e*x + f*x**2), x)
```

3.91.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 760 vs. 2(135) = 270.

Time = 0.23 (sec) , antiderivative size = 760, normalized size of antiderivative = 4.06

$$\int f^{a+cx^2}\sin\left(d+ex+fx^2\right)\,dx=$$

$$-\frac{\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+2\,f^2}\left(\left(f^{\frac{ce^2}{4\left(c^2\log(f)^2+f^2\right)}}f^a\cos\left(\frac{4\,c^2d\log(f)^2-e^2f+4\,df^2}{4\left(c^2\log(f)^2+f^2\right)}\right)-i\,f^{\frac{ce^2}{4\left(c^2\log(f)^2+f^2\right)}}f^a\sin\left(\frac{4\,c^2d\log}{4\left(c^2\right.}\right.\right.\right.}{}$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
-1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((f^(1/4*c*e^2/(c^2*log(f)^2 +
 f^2))*f^a*cos(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^
2)) - I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(1/4*(4*c^2*d*log(f)^2 -
 e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x - I
*e)/sqrt(-c*log(f) + I*f)) + (f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(1
/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)) + I*f^(1/4*c
*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2
)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + I*e)/sqrt(-c*log(
f) - I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) + sqrt(pi)*sqrt(2*c^
2*log(f)^2 + 2*f^2)*((I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(1/4*(4*
c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)) + f^(1/4*c*e^2/(c^
2*log(f)^2 + f^2))*f^a*sin(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*l
og(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x - I*e)/sqrt(-c*log(f) + I*f
)) + (-I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(1/4*(4*c^2*d*log(f)^2
- e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)) + f^(1/4*c*e^2/(c^2*log(f)^2 + f^
2))*f^a*sin(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2))
)*erf(1/2*(2*(c*log(f) + I*f)*x + I*e)/sqrt(-c*log(f) - I*f)))*sqrt(-c*log
(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

3.91.8 Giac [F]

$$\int f^{a+cx^2} \sin \left(d+ex+fx^2\right) \, dx = \int f^{cx^2+a} \sin \left(fx^2+ex+d\right) \, dx$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*sin(f*x^2 + e*x + d), x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2} \sin \left(d+ex+fx^2\right) \, dx = \int f^{c\,x^2+a} \sin \left(f\,x^2+e\,x+d\right) \, dx$$

input
```
int(f^(a + c*x^2)*sin(d + e*x + f*x^2),x)
```

output
```
int(f^(a + c*x^2)*sin(d + e*x + f*x^2), x)
```

3.92 $\int f^{a+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx$

3.92.1 Optimal result

Integrand size = 23, antiderivative size = 211

$$\int f^{a+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx=\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{ie+x(2if-c\log(f))}{\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}-\frac{e^{2id+\frac{e^2}{2if+c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+x(2if+c\log(f))}{\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}$$

output
```
1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/8*exp(-
2*I*d-e^2/(2*I*f-c*ln(f)))*f^a*erf((I*e+x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))
^(1/2))*Pi^(1/2)/(2*I*f-c*ln(f))^(1/2)-1/8*exp(2*I*d+e^2/(2*I*f+c*ln(f)))*
f^a*erfi((I*e+x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*
ln(f))^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.19

$$\int f^{a+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx=\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{\sqrt{c}\sqrt{\log(f)}}\right.$$

$$+\frac{\sqrt[4]{-1}\left(e^{\frac{e^2}{2if+c\log(f)}}\text{erf}\left(\frac{(-1)^{3/4}(e+2fx-icx\log(f))}{\sqrt{2f-ic\log(f)}}\right)\sqrt{2f-ic\log(f)}(2f+ic\log(f))(\cos(2d)+i\sin(2d))+e^{-2}\right.}{4f^2+c^2\log^2(j}$$

input
```
Integrate[f^(a + c*x^2)*Sin[d + e*x + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*((2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + (
(-1)^(1/4)*(E^(e^2/((2*I)*f + c*Log[f]))*Erf[((-1)^(3/4)*(e + 2*f*x - I*c*
x*Log[f]))/Sqrt[2*f - I*c*Log[f]]]*Sqrt[2*f - I*c*Log[f]]*(2*f + I*c*Log[f
])*(Cos[2*d] + I*Sin[2*d]) + E^(e^2/((-2*I)*f + c*Log[f]))*Erf[((-1)^(1/4)
*(e + 2*f*x + I*c*x*Log[f]))/Sqrt[2*f + I*c*Log[f]]]*(2*f - I*c*Log[f])*Sq
rt[2*f + I*c*Log[f]]*(I*Cos[2*d] + Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/8
```

3.92.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{4975}$$

$$\int \left(-\frac{1}{4}f^{a+cx^2}e^{-2id-2iex-2ifx^2}-\frac{1}{4}f^{a+cx^2}e^{2id+2iex+2ifx^2}+\frac{1}{2}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{-c\log(f)+2if}-2id}\operatorname{erf}\left(\frac{x(-c\log(f)+2if)+ie}{\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}-\frac{\sqrt{\pi}f^a e^{\frac{e^2}{c\log(f)+2if}+2id}\operatorname{erfi}\left(\frac{x(c\log(f)+2if)+ie}{\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}f^a\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Sin[d + e*x + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^
((-2*I)*d - e^2/((2*I)*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + x*((2*I)*f -
 c*Log[f]))/Sqrt[(2*I)*f - c*Log[f]]])/(8*Sqrt[(2*I)*f - c*Log[f]]) - (E^(
(2*I)*d + e^2/((2*I)*f + c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + x*((2*I)*f +
c*Log[f]))/Sqrt[(2*I)*f + c*Log[f]]])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

3.92.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.92.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{2 i d \ln(f) c+4 d f-e^{2}}{c \ln(f)-2 i f}} \operatorname{erf}\left(x \sqrt{2 i f-c \ln(f)}+\frac{i e}{\sqrt{2 i f-c \ln(f)}}\right)}{8 \sqrt{2 i f-c \ln(f)}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{2 i d \ln(f) c-4 d f+e^{2}}{2 i f+c \ln(f)}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-2 i f}\, x+\frac{i e}{\sqrt{-c \ln(f)}}\right.}{8 \sqrt{-c \ln(f)-2 i f}}$

input
```
int(f^(c*x^2+a)*sin(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*Pi^(1/2)*f^a*exp(-(2*I*d*ln(f)*c+4*d*f-e^2)/(c*ln(f)-2*I*f))/(2*I*f-c
*ln(f))^(1/2)*erf(x*(2*I*f-c*ln(f))^(1/2)+I*e/(2*I*f-c*ln(f))^(1/2))+1/8*P
i^(1/2)*f^a*exp((2*I*d*ln(f)*c-4*d*f+e^2)/(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f
)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+I*e/(-c*ln(f)-2*I*f)^(1/2))+1/4*f^a*
Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

3.92.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(155) = 310$.

Time = 0.26 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.72

$$\int f^{a+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{2\sqrt{\pi}\left(c^2\log\left(f\right)^2+4\,f^2\right)\sqrt{-c\log\left(f\right)}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x\right)-\sqrt{\pi}\left(c^2\log\left(f\right)^2-2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(\right.}}{}$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*lo
g(f))*x) - sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f
)*erf((c^2*x*log(f)^2 + 4*f^2*x + I*c*e*log(f) + 2*e*f)*sqrt(-c*log(f) - 2
*I*f)/(c^2*log(f)^2 + 4*f^2))*e^((a*c^2*log(f)^3 + 2*I*c^2*d*log(f)^2 - 2*
I*e^2*f + 8*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) -
sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf((c^2*
x*log(f)^2 + 4*f^2*x - I*c*e*log(f) + 2*e*f)*sqrt(-c*log(f) + 2*I*f)/(c^2*
log(f)^2 + 4*f^2))*e^((a*c^2*log(f)^3 - 2*I*c^2*d*log(f)^2 + 2*I*e^2*f - 8
*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)))/(c^3*log(f)^
3 + 4*c*f^2*log(f))
```

3.92.6 Sympy [F]

$$\int f^{a+cx^2} \sin^2\left(d+ex+fx^2\right) dx = \int f^{a+cx^2} \sin^2\left(d+ex+fx^2\right) dx$$

input
```
integrate(f**(c*x**2+a)*sin(f*x**2+e*x+d)**2,x)
```

output
```
Integral(f**(a + c*x**2)*sin(d + e*x + f*x**2)**2, x)
```

3.92.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 863, normalized size of antiderivative = 4.09

$$\int f^{a+cx^2} \sin^2\left(d+ex+fx^2\right) dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
-1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^(c*e^2/(c^2*log(f)^2 +
4*f^2))*f^a*cos(2*(c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2
)) + f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*sin(2*(c^2*d*log(f)^2 - e^2*f +
4*d*f^2)/(c^2*log(f)^2 + 4*f^2)))*erf(((c*log(f) - 2*I*f)*x - I*e)/sqrt(-c
*log(f) + 2*I*f)) + (-I*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*cos(2*(c^2*d*
log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2)) + f^(c*e^2/(c^2*log(f)
^2 + 4*f^2))*f^a*sin(2*(c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 +
4*f^2)))*erf(((c*log(f) + 2*I*f)*x + I*e)/sqrt(-c*log(f) - 2*I*f)))*sqrt(c
*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^
2*log(f)^2 + 8*f^2)*((f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*cos(2*(c^2*d*lo
g(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2)) - I*f^(c*e^2/(c^2*log(f)
^2 + 4*f^2))*f^a*sin(2*(c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 +
4*f^2)))*erf(((c*log(f) - 2*I*f)*x - I*e)/sqrt(-c*log(f) + 2*I*f)) + (f^(c
*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*cos(2*(c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/
(c^2*log(f)^2 + 4*f^2)) + I*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*sin(2*(c^
2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2)))*erf(((c*log(f) +
2*I*f)*x + I*e)/sqrt(-c*log(f) - 2*I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)
^2 + 4*f^2))*sqrt(-c*log(f)) - 2*sqrt(pi)*((c^2*f^a*log(f)^2 + 4*f^(a + 2)
)*erf(x*conjugate(sqrt(-c*log(f)))) + (c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf
(sqrt(-c*log(f))*x)))/((c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f)))
```

3.92.8 Giac [F]

$$\int f^{a+cx^2} \sin^2\left(d+ex+fx^2\right) dx = \int f^{cx^2+a} \sin\left(fx^2+ex+d\right)^2 dx$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*sin(f*x^2 + e*x + d)^2, x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2} \sin^2\left(d+ex+fx^2\right) dx = \int f^{c\,x^2+a} \sin\left(f\,x^2+e\,x+d\right)^2 dx$$

input
```
int(f^(a + c*x^2)*sin(d + e*x + f*x^2)^2,x)
```

output
```
int(f^(a + c*x^2)*sin(d + e*x + f*x^2)^2, x)
```

3.93 $\int f^{a+cx^2} \sin^3\left(d+ex+fx^2\right) dx$

3.93.1 Optimal result

Integrand size = 23, antiderivative size = 377

$$\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)dx=\frac{3ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\text{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}-\frac{ie^{-3id-\frac{9e^2}{4(3if-c\log(f))}}f^a\sqrt{\pi}\text{erf}\left(\frac{3ie+2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}}-\frac{3ie^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{ie+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}}+\frac{ie^{3id+\frac{9e^2}{4(3if+c\log(f))}}f^a\sqrt{\pi}\text{erfi}\left(\frac{3ie+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

output
```
3/16*I*exp(-I*d-e^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e+2*x*(I*f-c*ln(f)))
/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/16*I*exp(-3*I*d-9/4*e
^2/(3*I*f-c*ln(f)))*f^a*erf(1/2*(3*I*e+2*x*(3*I*f-c*ln(f)))/(3*I*f-c*ln(f)
)^(1/2))*Pi^(1/2)/(3*I*f-c*ln(f))^(1/2)-3/16*I*exp(I*d+e^2/(4*I*f+4*c*ln(f
)))*f^a*erfi(1/2*(I*e+2*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*
f+c*ln(f))^(1/2)+1/16*I*exp(3*I*d+9/4*e^2/(3*I*f+c*ln(f)))*f^a*erfi(1/2*(3
*I*e+2*x*(3*I*f+c*ln(f)))/(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(f))^
(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 5.14 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.30

$$\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt[4]{-1}f^a\sqrt{\pi}\left(-3e^{\frac{e^2}{4if+4c\log(f)}}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(e+2fx-2icx\log(f))}{2\sqrt{f-ic\log(f)}}\right)\sqrt{f-ic\log(f)}\left(9f^3+9icf^2\log(f)+c^2f\log^2(f)\right.\right.}{}$$

input
```
Integrate[f^(a + c*x^2)*Sin[d + e*x + f*x^2]^3,x]
```

output
```
((-1)^(1/4)*f^a*Sqrt[Pi]*(-3*E^(e^2/((4*I)*f + 4*c*Log[f]))*Erfi[((-1)^(1/
4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*
Log[f]]*(9*f^3 + (9*I)*c*f^2*Log[f] + c^2*f*Log[f]^2 + I*c^3*Log[f]^3)*(Co
s[d] + I*Sin[d]) + (f - I*c*Log[f])*(E^((9*e^2)/(4*((3*I)*f + c*Log[f])))*
Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f
]])]*Sqrt[3*f - I*c*Log[f]]*(3*f^2 + (4*I)*c*f*Log[f] - c^2*Log[f]^2)*(Cos
[3*d] + I*Sin[3*d]) + (3*f - I*c*Log[f])*(3*E^(e^2/((-4*I)*f + 4*c*Log[f])
)*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]]
)]*Sqrt[f + I*c*Log[f]]*((-3*I)*f + c*Log[f])*(Cos[d] - I*Sin[d]) + E^((9*
e^2)/(4*((-3*I)*f + c*Log[f])))*Erfi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*
Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*(f + I*c*Log[f])*Sqrt[3*f + I*c*Log[f
]]*(I*Cos[3*d] + Sin[3*d])))))/(16*(9*f^4 + 10*c^2*f^2*Log[f]^2 + c^4*Log[
f]^4))
```

3.93.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx$$
$$\Big\downarrow \; 4975$$

$$\int \left(\frac{3}{8} i f^{a+cx^2} \exp\left(-3i(d+ex+fx^2)+2id+2iex+2ifx^2\right)-\frac{3}{8} i f^{a+cx^2} \exp\left(-3i(d+ex+fx^2)+4id+4iex+\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\sqrt{\pi} f^a \exp\left(-\frac{9e^2}{4(-c\log(f)+3if)}-3id\right)\operatorname{erf}\left(\frac{2x(-c\log(f)+3if)+3ie}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+$$
$$\frac{3i\sqrt{\pi} f^a e^{-\frac{e^2}{-4c\log(f)+4if}-id}\operatorname{erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}-\frac{3i\sqrt{\pi} f^a e^{\frac{e^2}{4c\log(f)+4if}+id}\operatorname{erfi}\left(\frac{2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}}+$$
$$\frac{i\sqrt{\pi} f^a e^{\frac{9e^2}{4(c\log(f)+3if)}+3id}\operatorname{erfi}\left(\frac{2x(c\log(f)+3if)+3ie}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}$$

input
```
Int[f^(a + c*x^2)*Sin[d + e*x + f*x^2]^3,x]
```

output
```
(((3*I)/16)*E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e
+ 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f - c*Log[f]] -
((I/16)*E^((-3*I)*d - (9*e^2)/(4*((3*I)*f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(
(3*I)*e + 2*x*((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])])/Sqrt[(3
*I)*f - c*Log[f]] - (((3*I)/16)*E^(I*d + e^2/((4*I)*f + 4*c*Log[f]))*f^a*S
qrt[Pi]*Erfi[(I*e + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[
I*f + c*Log[f]] + ((I/16)*E^((3*I)*d + (9*e^2)/(4*((3*I)*f + c*Log[f])))*f
^a*Sqrt[Pi]*Erfi[((3*I)*e + 2*x*((3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f + c*
Log[f]])])/Sqrt[(3*I)*f + c*Log[f]]
```

3.93.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.93.4 Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{3 i d \ln(f) c-9 d f+\frac{9 e^{2}}{4}}{3 i f+c \ln(f)}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-3 i f}\, x+\frac{3 i e}{2 \sqrt{-c \ln(f)-3 i f}}\right)}{16 \sqrt{-c \ln(f)-3 i f}}-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4 i d \ln(f) c+12 d f-3 e^{2}\right)}{4(c \ln(f)-3 i f)}} \operatorname{erf}\left(x \sqrt{3 i f-c \ln(f)}\right.}{16 \sqrt{3 i f-c \ln(f)}}$

input
```
int(f^(c*x^2+a)*sin(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*I*Pi^(1/2)*f^a*exp(3/4*(4*I*d*ln(f)*c-12*d*f+3*e^2)/(3*I*f+c*ln(f)))
/(-c*ln(f)-3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1/2)*x+3/2*I*e/(-c*ln(f)-3*
I*f)^(1/2))-1/16*I*Pi^(1/2)*f^a*exp(-3/4*(4*I*d*ln(f)*c+12*d*f-3*e^2)/(c*l
n(f)-3*I*f))/(3*I*f-c*ln(f))^(1/2)*erf(x*(3*I*f-c*ln(f))^(1/2)+3/2*I*e/(3*
I*f-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c+4*d*f-e^2)
/(c*ln(f)-I*f))/(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2)+1/2*I*e/(I*f
-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c-4*d*f+e^2)/(I*
f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*I*e/(-c*l
n(f)-I*f)^(1/2))
```

3.93.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(269) = 538$.

Time = 0.29 (sec) , antiderivative size = 713, normalized size of antiderivative = 1.89

$$\int f^{a+cx^2} \sin^3\left(d+ex+fx^2\right) dx$$

$$= \frac{\sqrt{\pi}\left(-i\, c^3 \log\left(f\right)^3 - 3\, c^2 f \log\left(f\right)^2 - i\, c f^2 \log\left(f\right) - 3\, f^3\right) \sqrt{-c \log\left(f\right) - 3i\, f}\, \operatorname{erf}\left(\frac{\left(2\, c^2 x \log(f)^2 + 18\, f^2 x + 3i\, c e \log(\right.}{2\left(c^2 \log(f)^2\right.}\right.}{}$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
1/16*(sqrt(pi)*(-I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 - I*c*f^2*log(f) - 3*f^
3)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 18*f^2*x + 3*I*c*e*
log(f) + 9*e*f)*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(4*
a*c^2*log(f)^3 + 12*I*c^2*d*log(f)^2 - 27*I*e^2*f + 108*I*d*f^2 + 9*(c*e^2
 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(I*c^3*log(f)^3 - 3
*c^2*f*log(f)^2 + I*c*f^2*log(f) - 3*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*
(2*c^2*x*log(f)^2 + 18*f^2*x - 3*I*c*e*log(f) + 9*e*f)*sqrt(-c*log(f) + 3*
I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(4*a*c^2*log(f)^3 - 12*I*c^2*d*log(f)^
2 + 27*I*e^2*f - 108*I*d*f^2 + 9*(c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 +
 9*f^2)) - 3*sqrt(pi)*(-I*c^3*log(f)^3 - c^2*f*log(f)^2 - 9*I*c*f^2*log(f)
 - 9*f^3)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x + I*c*
e*log(f) + e*f)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*c^
2*log(f)^3 + 4*I*c^2*d*log(f)^2 - I*e^2*f + 4*I*d*f^2 + (c*e^2 + 4*a*f^2)*
log(f))/(c^2*log(f)^2 + f^2)) - 3*sqrt(pi)*(I*c^3*log(f)^3 - c^2*f*log(f)^
2 + 9*I*c*f^2*log(f) - 9*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*c^2*x*log(f
)^2 + 2*f^2*x - I*c*e*log(f) + e*f)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 +
f^2))*e^(1/4*(4*a*c^2*log(f)^3 - 4*I*c^2*d*log(f)^2 + I*e^2*f - 4*I*d*f^2
+ (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*
f^2*log(f)^2 + 9*f^4)
```

3.93.6 Sympy [F]

$$\int f^{a+cx^2} \sin^3\left(d+ex+fx^2\right) \, dx = \int f^{a+cx^2} \sin^3\left(d+ex+fx^2\right) dx$$

input
```
integrate(f**(c*x**2+a)*sin(f*x**2+e*x+d)**3,x)
```

output
```
Integral(f**(a + c*x**2)*sin(d + e*x + f*x**2)**3, x)
```

3.93.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2175 vs. $2(269) = 538$.

Time = 0.28 (sec) , antiderivative size = 2175, normalized size of antiderivative = 5.77

$$\int f^{a+cx^2} \sin^3\left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*f^(9/4*c*e^2/(c^2*log(
f)^2 + 9*f^2))*f^a*log(f)^2 + f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^(a +
2))*cos(3/4*(4*c^2*d*log(f)^2 - 9*e^2*f + 36*d*f^2)/(c^2*log(f)^2 + 9*f^2)
) + (-I*c^2*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^a*log(f)^2 - I*f^(9/4*c
*e^2/(c^2*log(f)^2 + 9*f^2))*f^(a + 2))*sin(3/4*(4*c^2*d*log(f)^2 - 9*e^2*
f + 36*d*f^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) - 3*I*f)*x - 3
*I*e)/sqrt(-c*log(f) + 3*I*f)) + ((c^2*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2)
)*f^a*log(f)^2 + f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^(a + 2))*cos(3/4*(
4*c^2*d*log(f)^2 - 9*e^2*f + 36*d*f^2)/(c^2*log(f)^2 + 9*f^2)) + (I*c^2*f^
(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^a*log(f)^2 + I*f^(9/4*c*e^2/(c^2*log(
f)^2 + 9*f^2))*f^(a + 2))*sin(3/4*(4*c^2*d*log(f)^2 - 9*e^2*f + 36*d*f^2)/
(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) + 3*I*f)*x + 3*I*e)/sqrt(-c*
log(f) - 3*I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)
*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((c^2*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^
a*log(f)^2 + 9*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^(a + 2))*cos(1/4*(4*c^
2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)) + (-I*c^2*f^(1/4*c*e
^2/(c^2*log(f)^2 + f^2))*f^a*log(f)^2 - 9*I*f^(1/4*c*e^2/(c^2*log(f)^2 + f
^2))*f^(a + 2))*sin(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2
 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x - I*e)/sqrt(-c*log(f) + I*f)) + ((
c^2*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*log(f)^2 + 9*f^(1/4*c*e^2/(c...
```

3.93.8 Giac [F]

$$\int f^{a+cx^2} \sin^3\left(d+ex+fx^2\right)\,dx = \int f^{cx^2+a} \sin\left(fx^2+ex+d\right)^3\,dx$$

input
```
integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*sin(f*x^2 + e*x + d)^3, x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+a}\sin\left(f\,x^2+e\,x+d\right)^3\,dx$$

input
```
int(f^(a + c*x^2)*sin(d + e*x + f*x^2)^3,x)
```

output
```
int(f^(a + c*x^2)*sin(d + e*x + f*x^2)^3, x)
```

3.94 $\int f^{a+bx+cx^2}\sin(d+ex)\,dx$

3.94.1 Optimal result

Integrand size = 19, antiderivative size = 176

$$\int f^{a+bx+cx^2}\sin(d+ex)\,dx = -\frac{ie^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{ie^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/4*I*exp(-I*d+1/4*(e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-I*e+b*ln(f)+2*c
*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/4*I*exp(I*d+
1/4*(e-I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2
)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

$$\int f^{a+bx+cx^2}\sin(d+ex)\,dx = \frac{e^{\frac{e(e-2ib\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(i\,\text{erfi}\left(\frac{-ie-(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))+e^{\frac{ibe}{c}}\text{erfi}\left(\frac{-ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(i\cos(d)+\sin}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x],x]
```

output
```
(E^((e*(e - (2*I)*b*Log[f]))/(4*c*Log[f]))*f^(a - b^2/(4*c))*Sqrt[Pi]*(I*E
rfi[((-I)*e - (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[d] + I*Si
n[d]) + E^((I*b*e)/c)*Erfi[((-I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[L
og[f]])]*(I*Cos[d] + Sin[d])))/(4*Sqrt[c]*Sqrt[Log[f]])
```

3.94.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(d+ex) f^{a+bx+cx^2} \, dx$$

$$\Big\downarrow \ 4975$$

$$\int \left(\frac{1}{2} i e^{-id-iex} f^{a+bx+cx^2} - \frac{1}{2} i e^{id+iex} f^{a+bx+cx^2}\right) dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{i\sqrt{\pi} f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id} \operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{i\sqrt{\pi} f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+id} \operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sin[d + e*x],x]
```

output
```
((-1/4*I)*E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(
I*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Lo
g[f]]) - ((I/4)*E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erf
i[(I*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt
[Log[f]])
```

3.94.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.94.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.98

method	result
risch	$\frac{i\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{-\frac{2i\ln(f)be-4id\ln(f)c-e^{2}}{4\ln(f)c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{ie+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}-\frac{i\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{\frac{2i\ln(f)be-4id\ln(f)c+e^{2}}{4\ln(f)c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\right.}{4\sqrt{-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*sin(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-1/4*(2*I*ln(f)*b*e-4*I*d*ln(f)*c-e^
2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(I*e+b*ln(f))/(-c
*ln(f))^(1/2))-1/4*I*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(1/4*(2*I*ln(f)*b*e-4*
I*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*
ln(f)-I*e)/(-c*ln(f))^(1/2))
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.01

$$\int f^{a+bx+cx^2}\sin(d+ex)\,dx$$
$$=\frac{-i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{((2cx+b)\log(f)-ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{\left(b^2-4ac\right)\log(f)^2-e^2+2(2icd-ibe)\log(f)}{4c\log(f)}\right)}+i\sqrt{\pi}\sqrt{-c\log(f)}\,e}{4c\log(f)}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(e*x+d),x, algorithm="fricas")
```

output
```
1/4*(-I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - I*e)*sqrt(-
c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - e^2 + 2*(2*I*c*d -
 I*b*e)*log(f))/(c*log(f))) + I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x +
 b)*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f
)^2 - e^2 + 2*(-2*I*c*d + I*b*e)*log(f))/(c*log(f))))/(c*log(f))
```

3.94.6 Sympy [F]

$$\int f^{a+bx+cx^2}\sin(d+ex)\,dx = \int f^{a+bx+cx^2}\sin(d+ex)\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sin(e*x+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sin(d + e*x), x)
```

3.94.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.01

$$\int f^{a+bx+cx^2}\sin(d+ex)\,dx$$
$$= \frac{\sqrt{\pi}\left(f^a\left(i\cos\left(-\frac{2cd-be}{2c}\right)+\sin\left(-\frac{2cd-be}{2c}\right)\right)\operatorname{erf}\left(x\sqrt{-c\log(f)}-\frac{1}{2}(b\log(f)+ie)\frac{1}{\sqrt{-c\log(f)}}\right)e^{\left(\frac{e^2}{4c\log(f)}\right)}+f}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(e*x+d),x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*(f^a*(I*cos(-1/2*(2*c*d - b*e)/c) + sin(-1/2*(2*c*d - b*e)/c)
)*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + I*e)*conjugate(1/sqrt
(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(-I*cos(-1/2*(2*c*d - b*e)/c) +
 sin(-1/2*(2*c*d - b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(
f) - I*e)*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(I*co
s(-1/2*(2*c*d - b*e)/c) + sin(-1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f)
 + b*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))) + f^
a*(-I*cos(-1/2*(2*c*d - b*e)/c) + sin(-1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*
x*log(f) + b*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f
))))*sqrt(-c*log(f))/(c*f^(1/4*b^2/c)*log(f))
```

3.94.8 Giac [F]

$$\int f^{a+bx+cx^2}\sin(d+ex)\,dx = \int f^{cx^2+bx+a}\sin(ex+d)\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(e*x+d),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*sin(e*x + d), x)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sin(d+ex)\,dx = \int f^{c\,x^2+b\,x+a}\sin(d+e\,x)\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sin(d + e*x),x)
```

output
```
int(f^(a + b*x + c*x^2)*sin(d + e*x), x)
```

3.95 $\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx$

3.95.1 Optimal result

Integrand size = 21, antiderivative size = 231

$$\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id+\frac{(2e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{e^{2id-\frac{(2ie+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

output
```
-1/8*exp(-2*I*d+1/4*(2*e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-2*I*e+b*ln(f
)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/8*exp(2
*I*d-1/4*(2*I*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(2*I*e+b*ln(f)+2*c*x*ln(f
))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^(a-1/4*b^2/c)*e
rfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.88

$$\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx =$$
$$-\frac{e^{-\frac{ibe}{c}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(-2e^{\frac{ibe}{c}}\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)+e^{\frac{e(e+2ib\log(f))}{c\log(f)}}\text{erfi}\left(\frac{-2ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(2d)-i\sin(2d))+e\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x]^2,x]
```

output
```
-1/8*(f^(a - b^2/(4*c))*Sqrt[Pi]*(-2*E^((I*b*e)/c)*Erfi[((b + 2*c*x)*Sqrt[
Log[f]])/(2*Sqrt[c])] + E^((e*(e + (2*I)*b*Log[f]))/(c*Log[f]))*Erfi[((-2*
I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] - I*Sin[2*d
]) + E^(e^2/(c*Log[f]))*Erfi[((2*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqr
t[Log[f]])]*(Cos[2*d] + I*Sin[2*d])))/(Sqrt[c]*E^((I*b*e)/c)*Sqrt[Log[f]])
```

3.95.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(d+ex)f^{a+bx+cx^2}\,dx$$
$$\Big\downarrow \text{4975}$$
$$\int \left(-\frac{1}{4}e^{-2id-2iex}f^{a+bx+cx^2}-\frac{1}{4}e^{2id+2iex}f^{a+bx+cx^2}+\frac{1}{2}f^{a+bx+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{\frac{(2e+ib\log(f))^2}{4c\log(f)}-2id}\text{erfi}\left(\frac{-b\log(f)-2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{2id-\frac{(b\log(f)+2ie)^2}{4c\log(f)}}\text{erfi}\left(\frac{b\log(f)+2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sin[d + e*x]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) + (E^((-2*I)*d + (2*e + I*b*Log[f])^2/(4*c*Log[f]
))*f^a*Sqrt[Pi]*Erfi[((2*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[L
og[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) - (E^((2*I)*d - ((2*I)*e + b*Log[f])^2/
(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqr
t[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

3.95.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.95.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.94

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{\frac{i\ln(f)be-2id\ln(f)c+e^{2}}{\ln(f)c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{b\ln(f)-2ie}{2\sqrt{-c\ln(f)}}\right)}{8\sqrt{-c\ln(f)}}+\frac{\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{-\frac{i\ln(f)be-2id\ln(f)c-e^{2}}{\ln(f)c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\right.}{8\sqrt{-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*sin(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp((I*ln(f)*b*e-2*I*d*ln(f)*c+e^2)/ln(f)/
c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*I*e)/(-c*ln(f))
^(1/2))+1/8*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-(I*ln(f)*b*e-2*I*d*ln(f)*c-e^
2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*I*e+b*ln(f))/(
-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^(-1/4*b^2/c)*f^a/(-c*ln(f))^(1/2)*erf(-(-c
*ln(f))^(1/2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.97

$$\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx$$

$$=\frac{\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{((2\,cx+b)\log(f)-2i\,e)\sqrt{-c\log(f)}}{2\,c\log(f)}\right)e^{\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-4\,e^2+4\,(2i\,cd-i\,be)\log(f)}{4\,c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}}{8\,c\log(}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/8*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - 2*I*e)*sqrt(-c
*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*e^2 + 4*(2*I*c*d
- I*b*e)*log(f))/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x +
b)*log(f) + 2*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(
f)^2 - 4*e^2 + 4*(-2*I*c*d + I*b*e)*log(f))/(c*log(f))) - 2*sqrt(pi)*sqrt(
-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))
/(c*log(f))
```

3.95.6 Sympy [F]

$$\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx=\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sin(e*x+d)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sin(d + e*x)**2, x)
```

3.95.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.73

$$\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx =$$

$$-\frac{\sqrt{\pi}\left(f^a\left(\cos\left(-\frac{2\,cd-be}{c}\right)-i\,\sin\left(-\frac{2\,cd-be}{c}\right)\right)\operatorname{erf}\left(x\overline{\sqrt{-c\log\left(f\right)}}-\frac{1}{2}\left(b\log\left(f\right)+2i\,e\right)\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^2}{c\log(f)}\right)}+\cdots\right.}{\cdots}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^2,x, algorithm="maxima")
```

output
```
-1/16*sqrt(pi)*(f^a*(cos(-(2*c*d - b*e)/c) - I*sin(-(2*c*d - b*e)/c))*erf(
x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + 2*I*e)*conjugate(1/sqrt(-c*
log(f))))*e^(e^2/(c*log(f))) + f^a*(cos(-(2*c*d - b*e)/c) + I*sin(-(2*c*d
- b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) - 2*I*e)*conju
gate(1/sqrt(-c*log(f))))*e^(e^2/(c*log(f))) + f^a*(cos(-(2*c*d - b*e)/c) -
 I*sin(-(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f) + 2*I*e)*sqrt(-
c*log(f))/(c*log(f)))*e^(e^2/(c*log(f))) + f^a*(cos(-(2*c*d - b*e)/c) + I*
sin(-(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f) - 2*I*e)*sqrt(-c*l
og(f))/(c*log(f)))*e^(e^2/(c*log(f))) - 2*f^a*erf(-1/2*b*conjugate(1/sqrt(
-c*log(f)))*log(f) + x*conjugate(sqrt(-c*log(f)))) + 2*f^a*erf(1/2*(2*c*x*
log(f) + b*log(f))/sqrt(-c*log(f))))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

3.95.8 Giac [F]

$$\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx = \int f^{cx^2+bx+a}\sin\left(ex+d\right)^2\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*sin(e*x + d)^2, x)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx=\int f^{c\,x^2+b\,x+a}\,\sin\left(d+e\,x\right)^2\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sin(d + e*x)^2,x)
```

output
```
int(f^(a + b*x + c*x^2)*sin(d + e*x)^2, x)
```

3.96 $\int f^{a+bx+cx^2}\sin^3(d+ex)\,dx$

3.96.1 Optimal result

Integrand size = 21, antiderivative size = 354

$$\int f^{a+bx+cx^2}\sin^3(d+ex)\,dx=-\frac{3ie^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{ie^{-3id+\frac{(3e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3ie^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{ie^{3id-\frac{(3ie+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

output
```
3/16*I*exp(-I*d+1/4*(e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-I*e+b*ln(f)+2*
c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/16*I*exp(-3
*I*d+1/4*(3*e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-3*I*e+b*ln(f)+2*c*x*ln(
f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-3/16*I*exp(I*d+1/4*(
e-I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(
f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*I*exp(3*I*d-1/4*(3*I*e+b*ln(f)
)^2/c/ln(f))*f^a*erfi(1/2*(3*I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))
*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.10

$$\int f^{a+bx+cx^2}\sin^3(d+ex)\,dx$$
$$= \frac{e^{\frac{e(e-6ib\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(-ie^{\frac{e(2e+3ib\log(f))}{c\log(f)}}\cos(3d)\operatorname{erfi}\left(\frac{-3ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+ie^{\frac{2e^2}{c\log(f)}}\cos(3d)\operatorname{erfi}\left(\frac{3ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right.\right.}{}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x]^3,x]
```

output
```
(E^((e*(e - (6*I)*b*Log[f]))/(4*c*Log[f]))*f^(a - b^2/(4*c))*Sqrt[Pi]*((-I
)*E^((e*(2*e + (3*I)*b*Log[f]))/(c*Log[f]))*Cos[3*d]*Erfi[((-3*I)*e + (b +
 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + I*E^((2*e^2)/(c*Log[f]))*Cos[3
*d]*Erfi[((3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + (3*I)*
E^((I*b*e)/c)*Erfi[((-I)*e - (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]
*(Cos[d] + I*Sin[d]) + 3*E^(((2*I)*b*e)/c)*Erfi[((-I)*e + (b + 2*c*x)*Log[
f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(I*Cos[d] + Sin[d]) - E^((e*(2*e + (3*I)*b*L
og[f]))/(c*Log[f]))*Erfi[((-3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[L
og[f]])]*Sin[3*d] - E^((2*e^2)/(c*Log[f]))*Erfi[((3*I)*e + (b + 2*c*x)*Log
[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*Sin[3*d]))/(16*Sqrt[c]*Sqrt[Log[f]])
```

3.96.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(d+ex)f^{a+bx+cx^2}\,dx$$
$$\Big\downarrow \text{4975}$$
$$\int \left(\frac{3}{8}ie^{-id-iex}f^{a+bx+cx^2}-\frac{3}{8}ie^{id+iex}f^{a+bx+cx^2}-\frac{1}{8}ie^{-3id-3iex}f^{a+bx+cx^2}+\frac{1}{8}ie^{3id+3iex}f^{a+bx+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{3i\sqrt{\pi}f^ae^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{i\sqrt{\pi}f^ae^{\frac{(3e+ib\log(f))^2}{4c\log(f)}-3id}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3i\sqrt{\pi}f^ae^{\frac{(e-ib\log(f))^2}{4c\log(f)}+id}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{i\sqrt{\pi}f^ae^{3id-\frac{(b\log(f)+3ie)^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sin[d + e*x]^3,x]
```

output
```
(((-3*I)/16)*E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erf
i[(I*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt
[Log[f]]) + ((I/16)*E^((-3*I)*d + (3*e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*S
qrt[Pi]*Erfi[((3*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]
)/(Sqrt[c]*Sqrt[Log[f]]) - (((3*I)/16)*E^(I*d + (e - I*b*Log[f])^2/(4*c*Lo
g[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[L
og[f]])])/(Sqrt[c]*Sqrt[Log[f]]) + ((I/16)*E^((3*I)*d - ((3*I)*e + b*Log[f
])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e + b*Log[f] + 2*c*x*Log[f])/(
2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]])
```

3.96.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.96.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{-\frac{3\left(2i\ln(f)be-4id\ln(f)c-3e^{2}\right)}{4\ln(f)c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{3ie+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)}{16\sqrt{-c\ln(f)}}+\frac{i\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{\frac{\frac{3i\ln(f)be}{2}-3id\ln(f)c+\frac{9e^{2}}{4}}{c\ln(f)}} \operatorname{erf}\left(-\sqrt{}\right.}{16\sqrt{-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*sin(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*I*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-3/4*(2*I*ln(f)*b*e-4*I*d*ln(f)*c-
3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(3*I*e+b*ln(f
))/(-c*ln(f))^(1/2))+1/16*I*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(3/4*(2*I*ln(f)
*b*e-4*I*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*
x+1/2*(b*ln(f)-3*I*e)/(-c*ln(f))^(1/2))-3/16*I*Pi^(1/2)*f^a*f^(-1/4*b^2/c)
*exp(1/4*(2*I*ln(f)*b*e-4*I*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-
(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-I*e)/(-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a
*f^(-1/4*b^2/c)*exp(-1/4*(2*I*ln(f)*b*e-4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln
(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(I*e+b*ln(f))/(-c*ln(f))^(1/2))
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.98

$$\int f^{a+bx+cx^2}\sin^3(d+ex)\,dx$$

$$= \frac{-3i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{((2\,cx+b)\log(f)-i\,e)\sqrt{-c\log(f)}}{2\,c\log(f)}\right)e^{\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-e^2+2\,(2i\,cd-i\,be)\log(f)}{4\,c\log(f)}\right)}+3i\sqrt{\pi}\sqrt{-c\log(}}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/16*(-3*I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - I*e)*sqr
t(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - e^2 + 2*(2*I*c*
d - I*b*e)*log(f))/(c*log(f))) + 3*I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*
c*x + b)*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*
log(f)^2 - e^2 + 2*(-2*I*c*d + I*b*e)*log(f))/(c*log(f))) + I*sqrt(pi)*sqr
t(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - 3*I*e)*sqrt(-c*log(f))/(c*log(f
)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - 9*e^2 + 6*(2*I*c*d - I*b*e)*log(f))/
(c*log(f))) - I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) + 3*I
*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - 9*e^2 +
6*(-2*I*c*d + I*b*e)*log(f))/(c*log(f))))/(c*log(f))
```

3.96.6 Sympy [F]

$$\int f^{a+bx+cx^2} \sin^3(d+ex)\,dx = \int f^{a+bx+cx^2} \sin^3{(d+ex)}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sin(e*x+d)**3,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sin(d + e*x)**3, x)
```

3.96.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.92

$$\int f^{a+bx+cx^2} \sin^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^3,x, algorithm="maxima")
```

output
```
-1/32*sqrt(pi)*(f^a*(I*cos(-3/2*(2*c*d - b*e)/c) + sin(-3/2*(2*c*d - b*e)/
c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + 3*I*e)*conjugate(1/
sqrt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*(-I*cos(-3/2*(2*c*d - b*e)/
c) + sin(-3/2*(2*c*d - b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*
log(f) - 3*I*e)*conjugate(1/sqrt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a
*(I*cos(-3/2*(2*c*d - b*e)/c) + sin(-3/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*
log(f) + b*log(f) + 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(9/4*e^2/(c*log(f
))) + f^a*(-I*cos(-3/2*(2*c*d - b*e)/c) + sin(-3/2*(2*c*d - b*e)/c))*erf(1
/2*(2*c*x*log(f) + b*log(f) - 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(9/4*e^
2/(c*log(f))) - 3*f^a*(I*cos(-1/2*(2*c*d - b*e)/c) + sin(-1/2*(2*c*d - b*e
)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + I*e)*conjugate(1/
sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) - 3*f^a*(-I*cos(-1/2*(2*c*d - b*e
)/c) + sin(-1/2*(2*c*d - b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(
b*log(f) - I*e)*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) - 3*f
^a*(I*cos(-1/2*(2*c*d - b*e)/c) + sin(-1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*
x*log(f) + b*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f
))) - 3*f^a*(-I*cos(-1/2*(2*c*d - b*e)/c) + sin(-1/2*(2*c*d - b*e)/c))*erf
(1/2*(2*c*x*log(f) + b*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^
2/(c*log(f))))*sqrt(-c*log(f))/(c*f^(1/4*b^2/c)*log(f))
```

3.96.8 Giac [F]

$$\int f^{a+bx+cx^2}\sin^3(d+ex)\,dx=\int f^{cx^2+bx+a}\sin\left(ex+d\right)^3\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*sin(e*x + d)^3, x)
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sin^3(d+ex)\,dx=\int f^{c\,x^2+b\,x+a}\sin\left(d+e\,x\right)^3\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sin(d + e*x)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)*sin(d + e*x)^3, x)
```

3.97 $\int f^{a+bx+cx^2} \sin(d+fx^2)\,dx$

3.97.1 Optimal result

Integrand size = 21, antiderivative size = 193

$$\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)\,dx=-\frac{ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}-\frac{ie^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

output
```
-1/4*I*exp(-I*d+b^2*ln(f)^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(I
*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/4*I*exp(I
*d-b^2*ln(f)^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(I*f+c*ln(f)))
/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

3.97.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.19

$$\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)\,dx=$$

$$-\frac{\sqrt[4]{-1}e^{\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\left(\mathrm{erfi}\left(\frac{(-1)^{3/4}(2fx+i(b+2cx)\log(f))}{2\sqrt{f+ic\log(f)}}\right)\sqrt{f+ic\log(f)}(if+c\log(f))(\cos(d)-i\sin(d))\right.}{4\left(f^2+c^2\log\right.}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sin[d + f*x^2],x]
```

output
```
-1/4*((-1)^(1/4)*E^((b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*(E
rfi[((-1)^(3/4)*(2*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*
Sqrt[f + I*c*Log[f]]*(I*f + c*Log[f])*(Cos[d] - I*Sin[d]) + E^(((I/2)*b^2*
f*Log[f]^2)/(f^2 + c^2*Log[f]^2))*Erfi[((-1)^(1/4)*(2*f*x - I*(b + 2*c*x)*
Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*(f + I*c*Log[f])*(
Cos[d] + I*Sin[d])))/(f^2 + c^2*Log[f]^2)
```

3.97.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(d+fx^2\right) f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{4975}$$

$$\int \left(\frac{1}{2}ie^{-id-ifx^2}f^{a+bx+cx^2}-\frac{1}{2}ie^{id+ifx^2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\operatorname{erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}f^a e^{id-\frac{b^2\log^2(f)}{4c\log(f)+4if}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sin[d + f*x^2],x]
```

output
```
((-1/4*I)*E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*
Erf[(b*Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f
- c*Log[f]] - ((I/4)*E^(I*d - (b^2*Log[f]^2)/((4*I)*f + 4*c*Log[f]))*f^a*S
qrt[Pi]*Erfi[(b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/
Sqrt[I*f + c*Log[f]]
```

3.97.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.97.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.93

method	result
risch	$\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-4 i d \ln(f) c+4 d f}{4(i f+c \ln(f))}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-i f}\, x+\frac{\ln(f) b}{2\sqrt{-c \ln(f)-i f}}\right)}{4\sqrt{-c \ln(f)-i f}}-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4 i d \ln(f) c+4 d f}{4(c \ln(f)-i f)}} \operatorname{erf}\left(-x\sqrt{i f-c \ln(f}\right.}{4\sqrt{i f-c \ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*sin(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*d*ln(f)*c+4*d*f)/(I*f+c*ln(f)
))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-
I*f)^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*d*ln(f)*c+4*d*f)/
(c*ln(f)-I*f))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/
(I*f-c*ln(f))^(1/2))
```

3.97.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(145) = 290.

Time = 0.27 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.60

$$\int f^{a+bx+cx^2} \sin\left(d+fx^2\right)\,dx$$

$$= \frac{\sqrt{\pi}(i c \log(f)+f)\sqrt{-c\log(f)-i f}\operatorname{erf}\left(\frac{\left(2 f^2 x-i b f\log(f)+\left(2 c^2 x+b c\right)\log(f)^2\right)\sqrt{-c\log(f)-i f}}{2\left(c^2\log(f)^2+f^2\right)}\right) e^{\left(\frac{4 a f^2 \log(f)-\left(b^2 c-4 a c^2\right.}{4}\right.}}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d),x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*(I*c*log(f) + f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x - I*
b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2
 + f^2))*e^(1/4*(4*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 +
 (4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(-I*c*lo
g(f) + f)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + I*b*f*log(f) + (2*c^2*x
 + b*c)*log(f)^2)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*
f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2 + (-4*I*c^2*d - I*b^2*
f)*log(f)^2)/(c^2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

3.97.6 Sympy [F]

$$\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sin(f*x**2+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sin(d + f*x**2), x)
```

3.97.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(145) = 290.

Time = 0.23 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.35

$$\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)\,dx=$$

$$-\frac{\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+2\,f^2}\left(\left(f^a\cos\left(\frac{4\,df^2+\left(4\,c^2d+b^2f\right)\log(f)^2}{4\left(c^2\log(f)^2+f^2\right)}\right)-i\,f^a\sin\left(\frac{4\,df^2+\left(4\,c^2d+b^2f\right)\log(f)^2}{4\left(c^2\log(f)^2+f^2\right)}\right)\right)\mathrm{erf}\left(\frac{2\,(c\,\mathrm{lo}}{2}\right.\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d),x, algorithm="maxima")
```

output
```
-1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((f^a*cos(1/4*(4*d*f^2 + (4*c^
2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) - I*f^a*sin(1/4*(4*d*f^2 + (4
*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*
f)*x + b*log(f))/sqrt(-c*log(f) + I*f)) + (f^a*cos(1/4*(4*d*f^2 + (4*c^2*d
 + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + I*f^a*sin(1/4*(4*d*f^2 + (4*c^
2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*
x + b*log(f))/sqrt(-c*log(f) - I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 +
f^2)) + sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((I*f^a*cos(1/4*(4*d*f^2 + (
4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + f^a*sin(1/4*(4*d*f^2 +
(4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) -
I*f)*x + b*log(f))/sqrt(-c*log(f) + I*f)) + (-I*f^a*cos(1/4*(4*d*f^2 + (4*
c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + f^a*sin(1/4*(4*d*f^2 + (4
*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*
f)*x + b*log(f))/sqrt(-c*log(f) - I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^
2 + f^2)))/(c^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + f^2
*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))
```

3.97.8 Giac [F]

$$\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)\,dx=\int f^{cx^2+bx+a}\sin\left(fx^2+d\right)\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + d), x)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\,\sin\left(f\,x^2+d\right)\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sin(d + f*x^2),x)
```

output
```
int(f^(a + b*x + c*x^2)*sin(d + f*x^2), x)
```

3.98 $\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)\,dx$

3.98.1 Optimal result

Integrand size = 23, antiderivative size = 245

$$\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)\,dx=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}-\frac{e^{2id-\frac{b^2\log^2(f)}{8if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(2if+c\log(f))}{2\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}$$

output
```
1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/
2)/ln(f)^(1/2)+1/8*exp(-2*I*d+b^2*ln(f)^2/(8*I*f-4*c*ln(f)))*f^a*erf(1/2*(
b*ln(f)-2*x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f-c*ln(f
))^(1/2)-1/8*exp(2*I*d-b^2*ln(f)^2/(8*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f
)+2*x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*ln(f))^(1/
2)
```

3.98.2 Mathematica [A] (warning: unable to verify)

Time = 2.23 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.22

$$\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)\,dx=\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2f^{-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{\sqrt{c}\sqrt{\log(f)}}\right.$$
$$+\frac{\sqrt[4]{-1}e^{\frac{b^2\log^2(f)}{8if-4c\log(f)}}\left(\operatorname{erf}\left(\frac{\sqrt[4]{-1}(4fx+i(b+2cx)\log(f))}{2\sqrt{2f+ic\log(f)}}\right)\sqrt{2f+ic\log(f)}(2if+c\log(f))(\cos(2d)-i\sin(2d))+\right.}{4f^2+c^2\log}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sin[d + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*((2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f
^(b^2/(4*c))*Sqrt[Log[f]]) + ((-1)^(1/4)*E^((b^2*Log[f]^2)/((8*I)*f - 4*c*
Log[f]))*(Erf[((-1)^(1/4)*(4*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[2*f + I*
c*Log[f]])]*Sqrt[2*f + I*c*Log[f]]*((2*I)*f + c*Log[f])*(Cos[2*d] - I*Sin[
2*d]) + E^((I*b^2*f*Log[f]^2)/(4*f^2 + c^2*Log[f]^2))*Erf[((-1)^(3/4)*(4*f
*x - I*(b + 2*c*x)*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Sqrt[2*f - I*c*Log
[f]]*(2*f + I*c*Log[f])*(Cos[2*d] + I*Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))
/8
```

3.98.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2\left(d+fx^2\right)f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{4975}$$

$$\int \left(-\frac{1}{4}e^{-2id-2ifx^2}f^{a+bx+cx^2}-\frac{1}{4}e^{2id+2ifx^2}f^{a+bx+cx^2}+\frac{1}{2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi} f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+8if}-2id}\operatorname{erf}\left(\frac{b\log(f)-2x(-c\log(f)+2if)}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}} - \frac{\sqrt{\pi} f^a e^{2id-\frac{b^2\log^2(f)}{4c\log(f)+8if}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2if)}{2\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}} + \frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sin[d + f*x^2]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) + (E^((-2*I)*d + (b^2*Log[f]^2)/((8*I)*f - 4*c*Lo
g[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*((2*I)*f - c*Log[f]))/(2*Sqrt[(2*I
)*f - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) - (E^((2*I)*d - (b^2*Log[f
]^2)/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((2*I)*f +
c*Log[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

3.98.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.98.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.93

method	result
risch	$\frac{\sqrt{\pi}\, f^a \mathrm{e}^{-\frac{\ln(f)^2 b^2+8id\ln(f)c+16df}{4(c\ln(f)-2if)}}\operatorname{erf}\left(-x\sqrt{2if-c\ln(f)}+\frac{\ln(f)b}{2\sqrt{2if-c\ln(f)}}\right)}{8\sqrt{2if-c\ln(f)}} + \frac{\sqrt{\pi}\, f^a \mathrm{e}^{-\frac{\ln(f)^2 b^2-8id\ln(f)c+16df}{4(2if+c\ln(f))}}\operatorname{erf}\left(-\sqrt{-c\ln(f)-2\right.}{8\sqrt{-c\ln(f)-2if}}$

input
```
int(f^(c*x^2+b*x+a)*sin(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+8*I*d*ln(f)*c+16*d*f)/(c*ln(f)-2*I*
f))/(2*I*f-c*ln(f))^(1/2)*erf(-x*(2*I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*I*f-
c*ln(f))^(1/2))+1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-8*I*d*ln(f)*c+16*d*
f)/(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+1
/2*ln(f)*b/(-c*ln(f)-2*I*f)^(1/2))-1/4*Pi^(1/2)*f^(-1/4*b^2/c)*f^a/(-c*ln(
f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

3.98.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(185) = 370.

Time = 0.27 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.64

$$\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)\,dx$$

$$= \frac{\sqrt{\pi}\left(c^2\log\left(f\right)^2-2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-2i\,f}\,\mathrm{erf}\left(\frac{\left(8\,f^2x-2i\,bf\log(f)+\left(2\,c^2x+bc\right)\log(f)^2\right)\sqrt{-c\log(f)-2i\,f}}{2\left(c^2\log(f)^2+4\,f^2\right)}\right)e^{\left(\frac{16}{\cdots}\right.}}{\cdots}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^2,x, algorithm="fricas")
```

output
```
1/8*(sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf(
1/2*(8*f^2*x - 2*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) -
 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*(16*a*f^2*log(f) - (b^2*c - 4*a*c^2
)*log(f)^3 + 32*I*d*f^2 - 2*(-4*I*c^2*d - I*b^2*f)*log(f)^2)/(c^2*log(f)^2
 + 4*f^2)) + sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I
*f)*erf(1/2*(8*f^2*x + 2*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*
log(f) + 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*(16*a*f^2*log(f) - (b^2*c -
 4*a*c^2)*log(f)^3 - 32*I*d*f^2 - 2*(4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*l
og(f)^2 + 4*f^2)) - 2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*erf(
1/2*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log(f)^3
+ 4*c*f^2*log(f))
```

3.98.6 Sympy [F]

$$\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sin(f*x**2+d)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sin(d + f*x**2)**2, x)
```

3.98.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 997, normalized size of antiderivative = 4.07

$$\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^a*f^(1/4*b^2/c)*cos(1/2
*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)) + f^a*f^(
1/4*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 +
 4*f^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*log(f))/sqrt(-c*log(f) + 2*I
*f)) + (-I*f^a*f^(1/4*b^2/c)*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^
2)/(c^2*log(f)^2 + 4*f^2)) + f^a*f^(1/4*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*
d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) + 2*I*f
)*x + b*log(f))/sqrt(-c*log(f) - 2*I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^
2 + 4*f^2))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((f^a*
f^(1/4*b^2/c)*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^
2 + 4*f^2)) - I*f^a*f^(1/4*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*lo
g(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*log(f
))/sqrt(-c*log(f) + 2*I*f)) + (f^a*f^(1/4*b^2/c)*cos(1/2*(16*d*f^2 + (4*c^
2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)) + I*f^a*f^(1/4*b^2/c)*sin(1
/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/
2*(2*(c*log(f) + 2*I*f)*x + b*log(f))/sqrt(-c*log(f) - 2*I*f)))*sqrt(-c*lo
g(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)) - 2*sqrt(pi)*((c^2*f^a*
e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/
4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2)))*erf(-1/2*b*conjugate(1/sqrt(-c*l
og(f)))*log(f) + x*conjugate(sqrt(-c*log(f)))) - (c^2*f^a*e^(1/4*b^2*c*...
```

3.98.8 Giac [F]

$$\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)\,dx=\int f^{cx^2+bx+a}\sin\left(fx^2+d\right)^2\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + d)^2, x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\sin\left(f\,x^2+d\right)^2\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sin(d + f*x^2)^2,x)
```

output
```
int(f^(a + b*x + c*x^2)*sin(d + f*x^2)^2, x)
```

3.99 $\int f^{a+bx+cx^2} \sin^3(d+fx^2)\,dx$

3.99.1 Optimal result

Integrand size = 23, antiderivative size = 386

$$\int f^{a+bx+cx^2}\sin^3(d+fx^2)\,dx = -\frac{3ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}} + \frac{ie^{-3id+\frac{b^2\log^2(f)}{12if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}} - \frac{3ie^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}} + \frac{ie^{3id-\frac{b^2\log^2(f)}{4(3if+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

output
```
-3/16*I*exp(-I*d+b^2*ln(f)^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(
I*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/16*I*exp
(-3*I*d+b^2*ln(f)^2/(12*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(3*I*f-c*
ln(f)))/(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f-c*ln(f))^(1/2)-3/16*I*exp(I
*d-b^2*ln(f)^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(I*f+c*ln(f)))
/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)+1/16*I*exp(3*I*d-1/4*b^
2*ln(f)^2/(3*I*f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(3*I*f+c*ln(f)))/(3*I
*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(f))^(1/2)
```

3.99.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3291 vs. 2(386) = 772.

Time = 6.73 (sec) , antiderivative size = 3291, normalized size of antiderivative = 8.53

$$\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)\,dx = \text{Result too large to show}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sin[d + f*x^2]^3,x]
```

output
```
(f^a*Sqrt[Pi]*(-27*(-1)^(3/4)*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f^
3*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt
[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]] + 27*(-1)^(1/4)*c*E^(((I/4)*b^2*Lo
g[f]^2)/(f - I*c*Log[f]))*f^2*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f]
- (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log[f]]
 - 3*(-1)^(3/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f*Cos[d]*Erf
i[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log
[f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]] + 3*(-1)^(1/4)*c^3*E^(((I/4)*b^2*Log[
f]^2)/(f - I*c*Log[f]))*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I
)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]] + 3
*(-1)^(3/4)*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f^3*Cos[3*d]*Erfi[
((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log
[f]])]*Sqrt[3*f - I*c*Log[f]] - (-1)^(1/4)*c*E^(((I/4)*b^2*Log[f]^2)/(3*f
- I*c*Log[f]))*f^2*Cos[3*d]*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c
*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]] + 3*
(-1)^(3/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f*Cos[3*d]*Erfi
[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Lo
g[f]])]*Log[f]^2*Sqrt[3*f - I*c*Log[f]] - (-1)^(1/4)*c^3*E^(((I/4)*b^2*Log
[f]^2)/(3*f - I*c*Log[f]))*Cos[3*d]*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] -
 (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c...
```

3.99.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3\left(d+fx^2\right) f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{4975}$$

$$\int \left(\frac{3}{8}ie^{-id-ifx^2}f^{a+bx+cx^2}-\frac{3}{8}ie^{id+ifx^2}f^{a+bx+cx^2}-\frac{1}{8}ie^{-3id-3ifx^2}f^{a+bx+cx^2}+\frac{1}{8}ie^{3id+3ifx^2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3i\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\text{erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}+\frac{i\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+12if}-3id}\text{erf}\left(\frac{b\log(f)-2x(-c\log(f)+3if)}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{i\sqrt{\pi}f^a\exp\left(3id-\frac{b^2\log^2(f)}{4(c\log(f)+3if)}\right)\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+3if)}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}-\frac{3i\sqrt{\pi}f^a e^{id-\frac{b^2\log^2(f)}{4c\log(f)+4if}}\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sin[d + f*x^2]^3,x]
```

output
```
(((-3*I)/16)*E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[P
i]*Erf[(b*Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I
*f - c*Log[f]] + ((I/16)*E^((-3*I)*d + (b^2*Log[f]^2)/((12*I)*f - 4*c*Log[
f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*
f - c*Log[f]])])/Sqrt[(3*I)*f - c*Log[f]] - (((3*I)/16)*E^(I*d - (b^2*Log[
f]^2)/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(I*f + c*L
og[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f + c*Log[f]] + ((I/16)*E^((3*I)
*d - (b^2*Log[f]^2)/(4*((3*I)*f + c*Log[f]))))*f^a*Sqrt[Pi]*Erfi[(b*Log[f]
 + 2*x*((3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f + c*Log[f]])])/Sqrt[(3*I)*f +
c*Log[f]]
```

3.99. $\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)dx$

3.99.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.99.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-12 i d \ln(f) c+36 d f}{4(3 i f+c\ln(f))}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-3 i f}\, x+\frac{\ln(f) b}{2\sqrt{-c\ln(f)-3 i f}}\right)}{16\sqrt{-c\ln(f)-3 i f}}+\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+12 i d \ln(f) c+36 d f}{4(c\ln(f)-3 i f)}} \operatorname{erf}\left(-x\sqrt{3\right.}}{16\sqrt{3 i f-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*sin(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-12*I*d*ln(f)*c+36*d*f)/(3*I*f+c
*ln(f)))/(-c*ln(f)-3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1/2)*x+1/2*ln(f)*b/
(-c*ln(f)-3*I*f)^(1/2))+1/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+12*I*d*l
n(f)*c+36*d*f)/(c*ln(f)-3*I*f))/(3*I*f-c*ln(f))^(1/2)*erf(-x*(3*I*f-c*ln(f
))^(1/2)+1/2*ln(f)*b/(3*I*f-c*ln(f))^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(-1/4*(
ln(f)^2*b^2+4*I*d*ln(f)*c+4*d*f)/(c*ln(f)-I*f))/(I*f-c*ln(f))^(1/2)*erf(-x
*(I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(I*f-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*
exp(-1/4*(ln(f)^2*b^2-4*I*d*ln(f)*c+4*d*f)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(
1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-I*f)^(1/2))
```

3.99.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 731 vs. 2(289) = 578.

Time = 0.29 (sec) , antiderivative size = 731, normalized size of antiderivative = 1.89

$$\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)\,dx$$

$$=\frac{\sqrt{\pi}\left(-i\,c^3\log\left(f\right)^3-3\,c^2f\log\left(f\right)^2-i\,cf^2\log\left(f\right)-3\,f^3\right)\sqrt{-c\log\left(f\right)-3i\,f}\operatorname{erf}\left(\frac{\left(18\,f^2x-3i\,bf\log(f)+(2\,c^2x+bc}{2\left(c^2\log(f)^2\right.}\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^3,x, algorithm="fricas")
```

output
```
1/16*(sqrt(pi)*(-I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 - I*c*f^2*log(f) - 3*f^
3)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(18*f^2*x - 3*I*b*f*log(f) + (2*c^2*x +
 b*c)*log(f)^2)*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(36
*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 108*I*d*f^2 - 3*(-4*I*c^2*d -
 I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(I*c^3*log(f)^3 - 3
*c^2*f*log(f)^2 + I*c*f^2*log(f) - 3*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*
(18*f^2*x + 3*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) + 3*
I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(36*a*f^2*log(f) - (b^2*c - 4*a*c^2)*l
og(f)^3 - 108*I*d*f^2 - 3*(4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2 +
9*f^2)) - 3*sqrt(pi)*(-I*c^3*log(f)^3 - c^2*f*log(f)^2 - 9*I*c*f^2*log(f)
- 9*f^3)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x - I*b*f*log(f) + (2*c^2*x
+ b*c)*log(f)^2)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*f
^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 + (4*I*c^2*d + I*b^2*f)
*log(f)^2)/(c^2*log(f)^2 + f^2)) - 3*sqrt(pi)*(I*c^3*log(f)^3 - c^2*f*log(
f)^2 + 9*I*c*f^2*log(f) - 9*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x +
I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) + I*f)/(c^2*log(f)
^2 + f^2))*e^(1/4*(4*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2
 + (-4*I*c^2*d - I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 +
 10*c^2*f^2*log(f)^2 + 9*f^4)
```

3.99.6 Sympy [F]

$$\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sin(f*x**2+d)**3,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sin(d + f*x**2)**3, x)
```

3.99.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2451 vs. 2(289) = 578.

Time = 0.31 (sec) , antiderivative size = 2451, normalized size of antiderivative = 6.35

$$\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*f^a*e^(1/4*b^2*c*log(f
)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*
log(f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log
(f)^2 + 9*f^2)) + (-I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*
log(f)^2 - I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*sin(3/
4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2
*(2*(c*log(f) - 3*I*f)*x + b*log(f))/sqrt(-c*log(f) + 3*I*f)) + ((c^2*f^a*
e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^
2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*
log(f)^2)/(c^2*log(f)^2 + 9*f^2)) + (I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*
log(f)^2 + f^2))*log(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^
2 + f^2)))*sin(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 +
 9*f^2)))*erf(1/2*(2*(c*log(f) + 3*I*f)*x + b*log(f))/sqrt(-c*log(f) - 3*I
*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*
log(f)^2 + 2*f^2)*(((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))
*log(f)^2 + 9*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*cos
(1/4*(4*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + (-I*c^
2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 - 9*I*f^(a +
2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^
2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*...
```

3.99.8 Giac [F]

$$\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)\,dx=\int f^{cx^2+bx+a}\sin\left(fx^2+d\right)^3\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + d)^3, x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\sin\left(f\,x^2+d\right)^3\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sin(d + f*x^2)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)*sin(d + f*x^2)^3, x)
```

3.100 $\int f^{a+bx+cx^2} \sin \left(d+ex+fx^2\right) \, dx$

3.100.1 Optimal result

Integrand size = 24, antiderivative size = 212

$$\int f^{a+bx+cx^2} \sin \left(d+ex+fx^2\right) \, dx = \frac{ie^{-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}} f^a \sqrt{\pi}\,\mathrm{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}} - \frac{ie^{id+\frac{(e-ib\log(f))^2}{4if+4c\log(f)}} f^a \sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

output
```
1/4*I*exp(-I*d-(e+I*b*ln(f))^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e-b*ln(f)
+2*x*(I*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/4*
I*exp(I*d+(e-I*b*ln(f))^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(I*e+b*ln(f)+2*x
*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

3.100.2 Mathematica [A] (warning: unable to verify)

Time = 2.02 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.64

$$\int f^{a+bx+cx^2} \sin \left(d+ex+fx^2\right) \, dx = -\frac{\sqrt[4]{-1} e^{-\frac{1}{4}i\left(\frac{e^2}{f-ic\log(f)}+\frac{b^2\log^2(f)}{f+ic\log(f)}\right)} f^{\frac{f(-be+af)+ac^2\log^2(f)}{f^2+c^2\log^2(f)}} \sqrt{\pi}\left(e^{\frac{ib^2 f\log^2(f)}{2\left(f^2+c^2\log^2(f)\right)}} f^{\frac{be}{2f+2ic\log(f)}}\,\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(e+2fx-i(b+2cx)\log}{2\sqrt{f-ic\log(f)}}\right.\right.}{}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2],x]
```

output
```
-1/4*((-1)^(1/4)*f^((f*(-(b*e) + a*f) + a*c^2*Log[f]^2)/(f^2 + c^2*Log[f]^
2))*Sqrt[Pi]*(E^(((I/2)*b^2*f*Log[f]^2)/(f^2 + c^2*Log[f]^2))*f^((b*e)/(2*
f + (2*I)*c*Log[f]))*Erfi[((-1)^(1/4)*(e + 2*f*x - I*(b + 2*c*x)*Log[f]))/
(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*(f + I*c*Log[f])*(Cos[d] +
I*Sin[d]) + E^(((I/2)*e^2*f)/(f^2 + c^2*Log[f]^2))*f^((b*e)/(2*f - (2*I)*c
*Log[f]))*Erfi[((-1)^(3/4)*(e + 2*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f +
 I*c*Log[f]])]*(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*(I*Cos[d] + Sin[d])))
/(E^((I/4)*(e^2/(f - I*c*Log[f]) + (b^2*Log[f]^2)/(f + I*c*Log[f])))*(f^2
+ c^2*Log[f]^2))
```

3.100.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2} \sin \left(d+ex+fx^2\right) \, dx$$

$$\Big\downarrow \text{4975}$$

$$\int \left(\frac{1}{2} i e^{-id-iex-ifx^2} f^{a+bx+cx^2}-\frac{1}{2} i e^{id+iex+ifx^2} f^{a+bx+cx^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\sqrt{\pi} f^a \exp \left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right) \text{erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi} f^a \exp \left(\frac{(e-ib\log(f))^2}{4c\log(f)+4if}+id\right) \text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2],x]
```

output
```
((I/4)*E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]
*Erf[(I*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sq
rt[I*f - c*Log[f]] - ((I/4)*E^(I*d + (e - I*b*Log[f])^2/((4*I)*f + 4*c*Log
[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*
f + c*Log[f]])])/Sqrt[I*f + c*Log[f]]
```

3.100.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.100.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.02

method	result
risch	$\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+2 i \ln(f) b e-4 i d \ln(f) c+4 d f-e^{2}}{4(i f+c\ln(f))}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-i f}\, x+\frac{i e+b\ln(f)}{2\sqrt{-c\ln(f)-i f}}\right)}{4\sqrt{-c\ln(f)-i f}} - \frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-2 i \ln(f) b e+4 i d \ln(f) c\cdots}{4(c\ln(f)-i f)}}}{4\sqrt{\cdots}}$

input
```
int(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c+4*d*f
-e^2)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*
(I*e+b*ln(f))/(-c*ln(f)-I*f)^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b
^2-2*I*ln(f)*b*e+4*I*d*ln(f)*c+4*d*f-e^2)/(c*ln(f)-I*f))/(I*f-c*ln(f))^(1/
2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-I*e)/(I*f-c*ln(f))^(1/2))
```

3.100.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(155) = 310.

Time = 0.26 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.77

$$\int f^{a+bx+cx^2} \sin \left(d+ex+fx^2\right) \, dx$$

$$= \frac{\sqrt{\pi}(i\,c\log\left(f\right)+f)\sqrt{-c\log\left(f\right)-i\,f}\,\mathrm{erf}\left(\frac{\left(2\,f^2x+\left(2\,c^2x+bc\right)\log(f)^2+ef+\left(i\,ce-i\,bf\right)\log(f)\right)\sqrt{-c\log(f)-i\,f}}{2\left(c^2\log(f)^2+f^2\right)}\right)e^{\left(-\frac{\left(b^2c-4\,ac\right.}{}\right.}}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*(I*c*log(f) + f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x + (2
*c^2*x + b*c)*log(f)^2 + e*f + (I*c*e - I*b*f)*log(f))*sqrt(-c*log(f) - I*
f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*e^2*f - 4
*I*d*f^2 - (4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*log(f)^2 - (c*e^2 - 2*b*e*f +
 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(-I*c*log(f) + f)*sqrt(
-c*log(f) + I*f)*erf(1/2*(2*f^2*x + (2*c^2*x + b*c)*log(f)^2 + e*f + (-I*c
*e + I*b*f)*log(f))*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((
b^2*c - 4*a*c^2)*log(f)^3 - I*e^2*f + 4*I*d*f^2 - (-4*I*c^2*d + 2*I*b*c*e
- I*b^2*f)*log(f)^2 - (c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 +
f^2)))/(c^2*log(f)^2 + f^2)
```

3.100.6 Sympy [F]

$$\int f^{a+bx+cx^2} \sin \left(d+ex+fx^2\right) \, dx = \int f^{a+bx+cx^2} \sin \left(d+ex+fx^2\right) dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sin(f*x**2+e*x+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sin(d + e*x + f*x**2), x)
```

3.100.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1007 vs. $2(155) = 310$.

Time = 0.25 (sec) , antiderivative size = 1007, normalized size of antiderivative = 4.75

$$\int f^{a+bx+cx^2} \sin\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((f^(1/4*c*e^2/(c^2*log(f)^2 +
f^2))*f^a*cos(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2
)/(c^2*log(f)^2 + f^2)) - I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(-1/
4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 +
 f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f) - I*e)*sqrt(-c*log(f) + I
*f)/(c*log(f) - I*f)) + (f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(-1/4*(
e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^
2)) + I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(-1/4*(e^2*f - 4*d*f^2 -
 (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(
c*log(f) + I*f)*x + b*log(f) + I*e)*sqrt(-c*log(f) - I*f)/(c*log(f) + I*f)
))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) + sqrt(pi)*sqrt(2*c^2*log(f)^
2 + 2*f^2)*((I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(-1/4*(e^2*f - 4*
d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + f^(1
/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d -
2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I
*f)*x + b*log(f) - I*e)*sqrt(-c*log(f) + I*f)/(c*log(f) - I*f)) + (-I*f^(1
/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d -
2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + f^(1/4*c*e^2/(c^2*log(f
)^2 + f^2))*f^a*sin(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*lo
g(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f)...
```

3.100.8 Giac [F]

$$\int f^{a+bx+cx^2} \sin\left(d+ex+fx^2\right)\,dx = \int f^{cx^2+bx+a} \sin\left(fx^2+ex+d\right)\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + e*x + d), x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \sin\left(d+ex+fx^2\right)\,dx = \int f^{c\,x^2+b\,x+a} \sin\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2),x)
```

output
```
int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2), x)
```

3.101 $\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx$

3.101.1 Optimal result

Integrand size = 26, antiderivative size = 268

$$\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id-\frac{(2e+ib\log(f))^2}{8if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{2ie-b\log(f)+2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}-\frac{e^{2id+\frac{(2e-ib\log(f))^2}{8if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2ie+b\log(f)+2x(2if+c\log(f))}{2\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}$$

output
```
1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/
2)/ln(f)^(1/2)-1/8*exp(-2*I*d-(2*e+I*b*ln(f))^2/(8*I*f-4*c*ln(f)))*f^a*erf
(1/2*(2*I*e-b*ln(f)+2*x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(
2*I*f-c*ln(f))^(1/2)-1/8*exp(2*I*d+(2*e-I*b*ln(f))^2/(8*I*f+4*c*ln(f)))*f^
a*erfi(1/2*(2*I*e+b*ln(f)+2*x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(
1/2)/(2*I*f+c*ln(f))^(1/2)
```

3.101.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1120 vs. 2(268) = 536.

Time = 6.53 (sec) , antiderivative size = 1120, normalized size of antiderivative = 4.18

$$\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx$$

$$= \frac{f^a\sqrt{\pi}\left(8\sqrt{c}f^{2-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\sqrt{\log(f)}+2c^{5/2}f^{-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\log^{\frac{5}{2}}(f)+2\sqrt[4]{-1}ce^{\frac{i\left(-4e^2+4ibe\right.}{4(2f}}\right.}{}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*(8*Sqrt[c]*f^(2 - b^2/(4*c))*Erfi[((b + 2*c*x)*Sqrt[Log[f]])
/(2*Sqrt[c])]*Sqrt[Log[f]] + (2*c^(5/2)*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2
*Sqrt[c])]*Log[f]^(5/2))/f^(b^2/(4*c)) + 2*(-1)^(1/4)*c*E^(((I/4)*(-4*e^2
+ (4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*f*Cos[2*d]*Erf[((-
1)^(3/4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*
Log[f]])]*Log[f]*Sqrt[2*f - I*c*Log[f]] + (-1)^(3/4)*c^2*E^(((I/4)*(-4*e^2
 + (4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*Cos[2*d]*Erf[((-1
)^(3/4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*L
og[f]])]*Log[f]^2*Sqrt[2*f - I*c*Log[f]] + (2*(-1)^(3/4)*c*f*Cos[2*d]*Erf[
((-1)^(1/4)*(2*e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I
*c*Log[f]])]*Log[f]*Sqrt[2*f + I*c*Log[f]])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*
Log[f] + b^2*Log[f]^2))/(2*f + I*c*Log[f])) + ((-1)^(1/4)*c^2*Cos[2*d]*Erf
[((-1)^(1/4)*(2*e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f +
I*c*Log[f]])]*Log[f]^2*Sqrt[2*f + I*c*Log[f]])/E^(((I/4)*(-4*e^2 - (4*I)*b
*e*Log[f] + b^2*Log[f]^2))/(2*f + I*c*Log[f])) + 2*(-1)^(3/4)*c*E^(((I/4)*
(-4*e^2 + (4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*f*Erf[((-1
)^(3/4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*L
og[f]])]*Log[f]*Sqrt[2*f - I*c*Log[f]]*Sin[2*d] - (-1)^(1/4)*c^2*E^(((I/4)
*(-4*e^2 + (4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*Erf[((-1)
^(3/4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c...
```

3.101.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)dx$$

$$\Big\downarrow \text{4975}$$

$$\int \left(-\frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+bx+cx^2}-\frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+bx+cx^2}+\frac{1}{2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a\exp\left(-\frac{(2e+ib\log(f))^2}{-4c\log(f)+8if}-2id\right)\text{erf}\left(\frac{-b\log(f)+2x(-c\log(f)+2if)+2ie}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}-\frac{\sqrt{\pi}f^a\exp\left(\frac{(2e-ib\log(f))^2}{4c\log(f)+8if}+2id\right)\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2if)+2ie}{2\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) - (E^((-2*I)*d - (2*e + I*b*Log[f])^2/((8*I)*f -
4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[((2*I)*e - b*Log[f] + 2*x*((2*I)*f - c*Log[f
]))/(2*Sqrt[(2*I)*f - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) - (E^((2*I
)*d + (2*e - I*b*Log[f])^2/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I
)*e + b*Log[f] + 2*x*((2*I)*f + c*Log[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/
(8*Sqrt[(2*I)*f + c*Log[f]])
```

3.101.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.101.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.98

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-4 i \ln(f) b e+8 i d \ln(f) c+16 d f-4 e^{2}}{4 (c \ln(f)-2 i f)}} \operatorname{erf}\left(-x \sqrt{2 i f-c \ln(f)}+\frac{b \ln(f)-2 i e}{2 \sqrt{2 i f-c \ln(f)}}\right)}{8 \sqrt{2 i f-c \ln(f)}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4 i \ln(f) b e-8 i d \ln(f) c+}{4 (2 i f+c \ln(f))}}}{8}$

input
```
int(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*ln(f)*b*e+8*I*d*ln(f)*c+16*d*f-
4*e^2)/(c*ln(f)-2*I*f))/(2*I*f-c*ln(f))^(1/2)*erf(-x*(2*I*f-c*ln(f))^(1/2)
+1/2*(b*ln(f)-2*I*e)/(2*I*f-c*ln(f))^(1/2))+1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(
f)^2*b^2+4*I*ln(f)*b*e-8*I*d*ln(f)*c+16*d*f-4*e^2)/(2*I*f+c*ln(f)))/(-c*ln
(f)-2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-c*ln(
f)-2*I*f)^(1/2))-1/4*Pi^(1/2)*f^(-1/4*b^2/c)*f^a/(-c*ln(f))^(1/2)*erf(-(-c
*ln(f))^(1/2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

3.101.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(199) = 398.

Time = 0.26 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.75

$$\int f^{a+bx+cx^2} \sin^2\left(d+ex+fx^2\right)\,dx$$

$$= \frac{\sqrt{\pi}\left(c^2\log(f)^2 - 2i\,cf\log(f)\right)\sqrt{-c\log(f)-2i\,f}\,\operatorname{erf}\left(\frac{\left(8\,f^2x+\left(2\,c^2x+bc\right)\log(f)^2+4\,ef-2\left(-i\,ce+i\,bf\right)\log(f)\right)\sqrt{-c\log(f}}{2\left(c^2\log(f)^2+4\,f^2\right)}\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
1/8*(sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf(
1/2*(8*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 4*e*f - 2*(-I*c*e + I*b*f)*log(f
))*sqrt(-c*log(f) - 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(-1/4*((b^2*c - 4*a*c
^2)*log(f)^3 + 8*I*e^2*f - 32*I*d*f^2 + 2*(-4*I*c^2*d + 2*I*b*c*e - I*b^2*
f)*log(f)^2 - 4*(c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)
) + sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf(1
/2*(8*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 4*e*f - 2*(I*c*e - I*b*f)*log(f))
*sqrt(-c*log(f) + 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(-1/4*((b^2*c - 4*a*c^2
)*log(f)^3 - 8*I*e^2*f + 32*I*d*f^2 + 2*(4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*
log(f)^2 - 4*(c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) -
 2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqr
t(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log(f)^3 + 4*c*f^2*log(f))
```

3.101.6 Sympy [F]

$$\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx=\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sin(f*x**2+e*x+d)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sin(d + e*x + f*x**2)**2, x)
```

3.101.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 1487, normalized size of antiderivative = 5.55

$$\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^a*cos(-1/2*(4*e^2*f - 16
*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2))*e^(
c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c) + f^a*e^(c*e^2*log
(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c)*sin(-1/2*(4*e^2*f - 16*d*f^
2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2
*(2*(c*log(f) - 2*I*f)*x + b*log(f) - 2*I*e)*sqrt(-c*log(f) + 2*I*f)/(c*lo
g(f) - 2*I*f)) + (-I*f^a*cos(-1/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e
 + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2))*e^(c*e^2*log(f)/(c^2*log(f)^2
 + 4*f^2) + 1/4*b^2*log(f)/c) + f^a*e^(c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2)
 + 1/4*b^2*log(f)/c)*sin(-1/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e + b^
2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) + 2*I*f)*x +
b*log(f) + 2*I*e)*sqrt(-c*log(f) - 2*I*f)/(c*log(f) + 2*I*f)))*sqrt(c*log(
f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2*log
(f)^2 + 8*f^2)*((f^a*cos(-1/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e + b
^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2))*e^(c*e^2*log(f)/(c^2*log(f)^2 + 4*
f^2) + 1/4*b^2*log(f)/c) - I*f^a*e^(c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2) +
1/4*b^2*log(f)/c)*sin(-1/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*
f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*
log(f) - 2*I*e)*sqrt(-c*log(f) + 2*I*f)/(c*log(f) - 2*I*f)) + (f^a*cos(-1/
2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(...
```

3.101.8 Giac [F]

$$\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx=\int f^{cx^2+bx+a}\sin\left(fx^2+ex+d\right)^2\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + e*x + d)^2, x)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\sin\left(f\,x^2+e\,x+d\right)^2\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2)^2,x)
```

output
```
int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2)^2, x)
```

3.102 $\int f^{a+bx+cx^2} \sin^3(d+ex+fx^2)\,dx$

3.102.1 Optimal result

Integrand size = 26, antiderivative size = 430

$$\int f^{a+bx+cx^2}\sin^3(d+ex+fx^2)\,dx = \frac{3ie^{-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}} - \frac{ie^{-3id-\frac{(3e+ib\log(f))^2}{4(3if-c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{3ie-b\log(f)+2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}} - \frac{3ie^{id+\frac{(e-ib\log(f))^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}} + \frac{ie^{3id-\frac{(3ie+b\log(f))^2}{4(3if+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3ie+b\log(f)+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

output
```
3/16*I*exp(-I*d-(e+I*b*ln(f))^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e-b*ln(f
)+2*x*(I*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/1
6*I*exp(-3*I*d-1/4*(3*e+I*b*ln(f))^2/(3*I*f-c*ln(f)))*f^a*erf(1/2*(3*I*e-b
*ln(f)+2*x*(3*I*f-c*ln(f)))/(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f-c*ln(f)
)^(1/2)-3/16*I*exp(I*d+(e-I*b*ln(f))^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(I*
e+b*ln(f)+2*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(
1/2)+1/16*I*exp(3*I*d-1/4*(3*I*e+b*ln(f))^2/(3*I*f+c*ln(f)))*f^a*erfi(1/2*
(3*I*e+b*ln(f)+2*x*(3*I*f+c*ln(f)))/(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f
+c*ln(f))^(1/2)
```

3.102.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3835 vs. 2(430) = 860.

Time = 7.20 (sec) , antiderivative size = 3835, normalized size of antiderivative = 8.92

$$\int f^{a+bx+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx=\text{Result too large to show}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2]^3,x]
```

output
```
(f^a*Sqrt[Pi]*(-27*(-1)^(3/4)*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log
[f]^2))/(f - I*c*Log[f]))*f^3*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log
[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]] +
27*(-1)^(1/4)*c*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I
*c*Log[f]))*f^2*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*
x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log[f]] - 3*(-1)^
(3/4)*c^2*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log
[f]))*f*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]
))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]] + 3*(-1)^(1/4)*
c^3*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*
Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sq
rt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]] + 3*(-1)^(3/4)*E^(((I/4
)*(-9*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^3*Cos[
3*d]*Erfi[((-1)^(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sq
rt[3*f - I*c*Log[f]])]*Sqrt[3*f - I*c*Log[f]] - (-1)^(1/4)*c*E^(((I/4)*(-9
*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^2*Cos[3*d]*
Erfi[((-1)^(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*
f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]] + 3*(-1)^(3/4)*c^2*E^(((I/
4)*(-9*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f*Cos[3
*d]*Erfi[((-1)^(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*...
```

3.102.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx$$

$\downarrow$ 4975

$$\int\left(\frac{3}{8}if^{a+bx+cx^2}\exp\left(-3i\left(d+ex+fx^2\right)+2id+2iex+2ifx^2\right)-\frac{3}{8}if^{a+bx+cx^2}\exp\left(-3i\left(d+ex+fx^2\right)+4id+\right.\right.$$

$\downarrow$ 2009

$$\frac{3i\sqrt{\pi}f^a\exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\operatorname{erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}f^a\exp\left(-\frac{(3e+ib\log(f))^2}{4(-c\log(f)+3if)}-3id\right)\operatorname{erf}\left(\frac{-b\log(f)+2x(-c\log(f)+3if)+3ie}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}-\frac{3i\sqrt{\pi}f^a\exp\left(\frac{(e-ib\log(f))^2}{4c\log(f)+4if}+id\right)\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}}+\frac{i\sqrt{\pi}f^a\exp\left(3id-\frac{(b\log(f)+3ie)^2}{4(c\log(f)+3if)}\right)\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+3if)+3ie}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2]^3,x]
```

output
```
(((3*I)/16)*E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqr
t[Pi]*Erf[(I*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])
])/Sqrt[I*f - c*Log[f]] - ((I/16)*E^((-3*I)*d - (3*e + I*b*Log[f])^2/(4*((
3*I)*f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[((3*I)*e - b*Log[f] + 2*x*((3*I)*f -
 c*Log[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])])/Sqrt[(3*I)*f - c*Log[f]] - (((3
*I)/16)*E^(I*d + (e - I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*E
rfi[(I*e + b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqr
t[I*f + c*Log[f]] + ((I/16)*E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4*((3*I)*
f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[((3*I)*e + b*Log[f] + 2*x*((3*I)*f + c*L
og[f]))/(2*Sqrt[(3*I)*f + c*Log[f]])])/Sqrt[(3*I)*f + c*Log[f]]
```

3.102. $\int f^{a+bx+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx$

3.102.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.102.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+6i\ln(f) b e-12 i d\ln(f) c+36 d f-9 e^{2}}{4(3 i f+c\ln(f))}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-3 i f}\, x+\frac{3 i e+b\ln(f)}{2\sqrt{-c\ln(f)-3 i f}}\right)}{16\sqrt{-c\ln(f)-3 i f}}+\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-6i\ln(f) b e+12}{4(c\ln(f)}}}{}$

input
```
int(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6*I*ln(f)*b*e-12*I*d*ln(f)*c+36
*d*f-9*e^2)/(3*I*f+c*ln(f)))/(-c*ln(f)-3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^
(1/2)*x+1/2*(3*I*e+b*ln(f))/(-c*ln(f)-3*I*f)^(1/2))+1/16*I*Pi^(1/2)*f^a*ex
p(-1/4*(ln(f)^2*b^2-6*I*ln(f)*b*e+12*I*d*ln(f)*c+36*d*f-9*e^2)/(c*ln(f)-3*
I*f))/(3*I*f-c*ln(f))^(1/2)*erf(-x*(3*I*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-3*I*
e)/(3*I*f-c*ln(f))^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*I*ln
(f)*b*e+4*I*d*ln(f)*c+4*d*f-e^2)/(c*ln(f)-I*f))/(I*f-c*ln(f))^(1/2)*erf(-x
*(I*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-I*e)/(I*f-c*ln(f))^(1/2))+3/16*I*Pi^(1/2
)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c+4*d*f-e^2)/(I*f+c*
ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))
/(-c*ln(f)-I*f)^(1/2))
```

3.102.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(312) = 624$.

Time = 0.29 (sec) , antiderivative size = 865, normalized size of antiderivative = 2.01

$$\int f^{a+bx+cx^2} \sin^3\left(d+ex+fx^2\right) dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
1/16*(sqrt(pi)*(-I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 - I*c*f^2*log(f) - 3*f^
3)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(18*f^2*x + (2*c^2*x + b*c)*log(f)^2 +
9*e*f - 3*(-I*c*e + I*b*f)*log(f))*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 +
 9*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + 27*I*e^2*f - 108*I*d*f^2 +
3*(-4*I*c^2*d + 2*I*b*c*e - I*b^2*f)*log(f)^2 - 9*(c*e^2 - 2*b*e*f + 4*a*f
^2)*log(f))/(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*(-I*c^3*log(f)^3 - c^2*f*
log(f)^2 - 9*I*c*f^2*log(f) - 9*f^3)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*
x + (2*c^2*x + b*c)*log(f)^2 + e*f + (I*c*e - I*b*f)*log(f))*sqrt(-c*log(f
) - I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*e^2
*f - 4*I*d*f^2 - (4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*log(f)^2 - (c*e^2 - 2*b
*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) - 3*sqrt(pi)*(I*c^3*log(f)^3
 - c^2*f*log(f)^2 + 9*I*c*f^2*log(f) - 9*f^3)*sqrt(-c*log(f) + I*f)*erf(1/
2*(2*f^2*x + (2*c^2*x + b*c)*log(f)^2 + e*f + (-I*c*e + I*b*f)*log(f))*sqr
t(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)
^3 - I*e^2*f + 4*I*d*f^2 - (-4*I*c^2*d + 2*I*b*c*e - I*b^2*f)*log(f)^2 - (
c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(I*c^3
*log(f)^3 - 3*c^2*f*log(f)^2 + I*c*f^2*log(f) - 3*f^3)*sqrt(-c*log(f) + 3*
I*f)*erf(1/2*(18*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 9*e*f - 3*(I*c*e - I*b
*f)*log(f))*sqrt(-c*log(f) + 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(-1/4*((b^2*
c - 4*a*c^2)*log(f)^3 - 27*I*e^2*f + 108*I*d*f^2 + 3*(4*I*c^2*d - 2*I*b...
```

3.102.6 Sympy [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \sin^3\left(d+ex+fx^2\right) dx = \text{Timed out}$$

input
```
integrate(f**(c*x**2+b*x+a)*sin(f*x**2+e*x+d)**3,x)
```

output
```
Timed out
```

3.102.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4343 vs. 2(312) = 624.

Time = 0.32 (sec) , antiderivative size = 4343, normalized size of antiderivative = 10.10

$$\int f^{a+bx+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
-1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*f^a*e^(1/4*b^2*c*log(
f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*
b*e*f*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f
)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b
*e*f*log(f)/(c^2*log(f)^2 + f^2)))*cos(-3/4*(9*e^2*f - 36*d*f^2 - (4*c^2*d
 - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) + (-I*c^2*f^a*e^(1/4
*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*
f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 - I*f^(a + 2)*e^(1/
4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9
*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2)))*sin(-3/4*(9*e^2*f - 36*d*f
^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/
2*(2*(c*log(f) - 3*I*f)*x + b*log(f) - 3*I*e)*sqrt(-c*log(f) + 3*I*f)/(c*l
og(f) - 3*I*f)) + ((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9
/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 +
f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/
4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f
^2)))*cos(-3/4*(9*e^2*f - 36*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)
/(c^2*log(f)^2 + 9*f^2)) + (I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2
+ f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*l
og(f)^2 + f^2))*log(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f...
```

3.102.8 Giac [F]

$$\int f^{a+bx+cx^2} \sin^3\left(d+ex+fx^2\right) dx = \int f^{cx^2+bx+a} \sin\left(fx^2+ex+d\right)^3 dx$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + e*x + d)^3, x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \sin^3\left(d+ex+fx^2\right) dx = \int f^{c\,x^2+b\,x+a} \sin\left(f\,x^2+e\,x+d\right)^3 dx$$

input
```
int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2)^3, x)
```

3.103 $\int f^{a+bx+cx^2} \sin\left(a+bx+ex^2\right)\,dx$

3.103.1 Optimal result

Integrand size = 24, antiderivative size = 213

$$\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)\,dx = \frac{ie^{-\left((i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)}\sqrt{\pi}\,\mathrm{erf}\left(\frac{b(i-\log(f))+2x(ie-c\log(f))}{2\sqrt{ie-c\log(f)}}\right)}{4\sqrt{ie-c\log(f)}} - \frac{ie^{(i+\log(f))\left(a-\frac{b^2(i+\log(f))}{4ie+4c\log(f)}\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b(i+\log(f))+2x(ie+c\log(f))}{2\sqrt{ie+c\log(f)}}\right)}{4\sqrt{ie+c\log(f)}}$$

output
```
-1/4*I*erf(1/2*(-b*(I-ln(f))-2*x*(I*e-c*ln(f)))/(I*e-c*ln(f))^(1/2))*Pi^(1
/2)/exp((I-ln(f))*(a-b^2*(I-ln(f))/(4*I*e-4*c*ln(f))))/(I*e-c*ln(f))^(1/2)
-1/4*I*exp((I+ln(f))*(a-b^2*(I+ln(f))/(4*I*e+4*c*ln(f))))*erfi(1/2*(b*(I+l
n(f))+2*x*(I*e+c*ln(f)))/(I*e+c*ln(f))^(1/2))*Pi^(1/2)/(I*e+c*ln(f))^(1/2)
```

3.103.2 Mathematica [A] (warning: unable to verify)

Time = 1.61 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.52

$$\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)\,dx$$
$$= \frac{e^{-\frac{b^2c\log^3(f)}{2\left(e^2+c^2\log^2(f)\right)}}f^{a-\frac{b^2}{2(e-ic\log(f))}}\sqrt{\pi}\left(-e^{\frac{1}{4}b^2\left(\frac{1}{-ie+c\log(f)}+\frac{\log^2(f)}{ie+c\log(f)}\right)}f^{\frac{ib^2c\log(f)}{e^2+c^2\log^2(f)}}\operatorname{erfi}\left(\frac{-i(b+2ex)+(b+2cx)\log(f)}{2\sqrt{-ie+c\log(f)}}\right)(e-ic\right.}{}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sin[a + b*x + e*x^2],x]
```

output
```
(f^(a - b^2/(2*(e - I*c*Log[f])))*Sqrt[Pi]*(-(E^((b^2*(((-I)*e + c*Log[f])
^(-1) + Log[f]^2/(I*e + c*Log[f])))/4)*f^((I*b^2*c*Log[f])/(e^2 + c^2*Log[
f]^2))*Erfi[((-I)*(b + 2*e*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[(-I)*e + c*Log
[f]])]*(e - I*c*Log[f])*Sqrt[(-I)*e + c*Log[f]]*(Cos[a] - I*Sin[a])) + E^(
(b^2*(Log[f]^2/((-I)*e + c*Log[f]) + (I*e + c*Log[f])^(-1)))/4)*Erfi[((-I)
*(b + 2*e*x) - (b + 2*c*x)*Log[f])/(2*Sqrt[I*e + c*Log[f]])]*(e + I*c*Log[
f])*Sqrt[I*e + c*Log[f]]*(Cos[a] + I*Sin[a])))/(4*E^((b^2*c*Log[f]^3)/(2*(
e^2 + c^2*Log[f]^2)))*(e^2 + c^2*Log[f]^2))
```

3.103.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)\,dx$$
$$\Big\downarrow \; 4975$$
$$\int \left(\frac{1}{2}ie^{-ia-ibx-iex^2}f^{a+bx+cx^2}-\frac{1}{2}ie^{ia+ibx+iex^2}f^{a+bx+cx^2}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{i\sqrt{\pi}\exp\left(-\left((-\log(f)+i)\left(a-\frac{b^2(-\log(f)+i)}{-4c\log(f)+4ie}\right)\right)\right)\operatorname{erf}\left(\frac{b(-\log(f)+i)+2x(-c\log(f)+ie)}{2\sqrt{-c\log(f)+ie}}\right)}{4\sqrt{-c\log(f)+ie}} - \frac{i\sqrt{\pi}\exp\left((\log(f)+i)\left(a-\frac{b^2(\log(f)+i)}{4c\log(f)+4ie}\right)\right)\operatorname{erfi}\left(\frac{b(\log(f)+i)+2x(c\log(f)+ie)}{2\sqrt{c\log(f)+ie}}\right)}{4\sqrt{c\log(f)+ie}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sin[a + b*x + e*x^2],x]
```

output
```
((I/4)*Sqrt[Pi]*Erf[(b*(I - Log[f]) + 2*x*(I*e - c*Log[f]))/(2*Sqrt[I*e -
c*Log[f]])])/(E^((I - Log[f])*(a - (b^2*(I - Log[f]))/((4*I)*e - 4*c*Log[f
])))*Sqrt[I*e - c*Log[f]]) - ((I/4)*E^((I + Log[f])*(a - (b^2*(I + Log[f])
)/((4*I)*e + 4*c*Log[f])))*Sqrt[Pi]*Erfi[(b*(I + Log[f]) + 2*x*(I*e + c*Lo
g[f]))/(2*Sqrt[I*e + c*Log[f]])])/Sqrt[I*e + c*Log[f]]
```

3.103.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.103.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02

method	result
risch	$\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{-\ln(f)^2 b^2+4i\ln(f)ac-2i\ln(f)b^2-4ae+b^2}{4ie+4c\ln(f)}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-ie}\, x+\frac{b\ln(f)+ib}{2\sqrt{-c\ln(f)-ie}}\right)}{4\sqrt{-c\ln(f)-ie}} - \frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^2 b^2+4i\ln(f)ac-2i\ln(f)b^2}{4(c\ln(f)-ie)}}}{4\sqrt{}}$

input
```
int(f^(c*x^2+b*x+a)*sin(e*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*Pi^(1/2)*f^a*exp(1/4*(-ln(f)^2*b^2+4*I*ln(f)*a*c-2*I*ln(f)*b^2-4*a*e
+b^2)/(I*e+c*ln(f)))/(-c*ln(f)-I*e)^(1/2)*erf(-(-c*ln(f)-I*e)^(1/2)*x+1/2*
(b*ln(f)+I*b)/(-c*ln(f)-I*e)^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b
^2+4*I*ln(f)*a*c-2*I*ln(f)*b^2+4*a*e-b^2)/(c*ln(f)-I*e))/(I*e-c*ln(f))^(1/
2)*erf(-(I*e-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-I*b)/(I*e-c*ln(f))^(1/2))
```

3.103.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(153) = 306.

Time = 0.26 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.78

$$\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)\,dx$$

$$= \frac{\sqrt{\pi}(i\,c\log\left(f\right)+e)\sqrt{-c\log\left(f\right)-i\,e}\,\mathrm{erf}\left(\frac{\left(2\,e^2x+\left(2\,c^2x+bc\right)\log\left(f\right)^2+be+\left(i\,bc-i\,be\right)\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-i\,e}}{2\left(c^2\log\left(f\right)^2+e^2\right)}\right)e^{\left(-\frac{\left(b^2c-4\,ac^2\right)\cdots}{\cdots}\right.}}{\cdots}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(e*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*(I*c*log(f) + e)*sqrt(-c*log(f) - I*e)*erf(1/2*(2*e^2*x + (2
*c^2*x + b*c)*log(f)^2 + b*e + (I*b*c - I*b*e)*log(f))*sqrt(-c*log(f) - I*
e)/(c^2*log(f)^2 + e^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*b^2*e - 4
*I*a*e^2 - (-2*I*b^2*c + 4*I*a*c^2 + I*b^2*e)*log(f)^2 - (b^2*c - 2*b^2*e
+ 4*a*e^2)*log(f))/(c^2*log(f)^2 + e^2)) + sqrt(pi)*(-I*c*log(f) + e)*sqrt
(-c*log(f) + I*e)*erf(1/2*(2*e^2*x + (2*c^2*x + b*c)*log(f)^2 + b*e + (-I*
b*c + I*b*e)*log(f))*sqrt(-c*log(f) + I*e)/(c^2*log(f)^2 + e^2))*e^(-1/4*(
(b^2*c - 4*a*c^2)*log(f)^3 - I*b^2*e + 4*I*a*e^2 - (2*I*b^2*c - 4*I*a*c^2
- I*b^2*e)*log(f)^2 - (b^2*c - 2*b^2*e + 4*a*e^2)*log(f))/(c^2*log(f)^2 +
e^2)))/(c^2*log(f)^2 + e^2)
```

3.103.6 Sympy [F]

$$\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)\,dx=\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sin(e*x**2+b*x+a),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sin(a + b*x + e*x**2), x)
```

3.103.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 1054, normalized size of antiderivative = 4.95

$$\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(e*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*((f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*(-I*cos(1/2*arctan2(
e, -c*log(f))) + sin(1/2*arctan2(e, -c*log(f))))*cos(-1/4*(b^2*e - 4*a*e^2
 + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2)/(c^2*log(f)^2 + e^2)) + f^(1/4*b^
2*c/(c^2*log(f)^2 + e^2))*f^a*(cos(1/2*arctan2(e, -c*log(f))) + I*sin(1/2*
arctan2(e, -c*log(f))))*sin(-1/4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b
^2*e)*log(f)^2)/(c^2*log(f)^2 + e^2)))*erf(x*conjugate(sqrt(-c*log(f) + I*
e)) - 1/2*(b*log(f) + I*b)*conjugate(1/sqrt(-c*log(f) + I*e))) + (f^(1/4*b
^2*c/(c^2*log(f)^2 + e^2))*f^a*(I*cos(1/2*arctan2(e, -c*log(f))) + sin(1/2
*arctan2(e, -c*log(f))))*cos(-1/4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 -
b^2*e)*log(f)^2)/(c^2*log(f)^2 + e^2)) + f^(1/4*b^2*c/(c^2*log(f)^2 + e^2)
)*f^a*(cos(1/2*arctan2(e, -c*log(f))) - I*sin(1/2*arctan2(e, -c*log(f))))*
sin(-1/4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2)/(c^2*log
(f)^2 + e^2)))*erf(x*conjugate(sqrt(-c*log(f) - I*e)) - 1/2*(b*log(f) - I*
b)*conjugate(1/sqrt(-c*log(f) - I*e))) + (f^(1/4*b^2*c/(c^2*log(f)^2 + e^2
))*f^a*(I*cos(1/2*arctan2(e, -c*log(f))) + sin(1/2*arctan2(e, -c*log(f))))
*cos(-1/4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2)/(c^2*lo
g(f)^2 + e^2)) + f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*(cos(1/2*arctan2(e
, -c*log(f))) - I*sin(1/2*arctan2(e, -c*log(f))))*sin(-1/4*(b^2*e - 4*a*e^
2 + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2)/(c^2*log(f)^2 + e^2)))*erf(1/2*(
2*(c*log(f) - I*e)*x + b*log(f) - I*b)*sqrt(-c*log(f) + I*e)/(c*log(f) ...
```

3.103.8 Giac [F]

$$\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)\,dx=\int f^{cx^2+bx+a}\sin\left(ex^2+bx+a\right)\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*sin(e*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*sin(e*x^2 + b*x + a), x)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\,\sin\left(e\,x^2+b\,x+a\right)\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sin(a + b*x + e*x^2),x)
```

output
```
int(f^(a + b*x + c*x^2)*sin(a + b*x + e*x^2), x)
```

3.104 $\int e^x \cos(a + bx)\, dx$

3.104.1 Optimal result

Integrand size = 10, antiderivative size = 36

$$\int e^x \cos(a + bx)\, dx = \frac{e^x \cos(a + bx)}{1 + b^2} + \frac{be^x \sin(a + bx)}{1 + b^2}$$

output
```
exp(x)*cos(b*x+a)/(b^2+1)+b*exp(x)*sin(b*x+a)/(b^2+1)
```

3.104.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int e^x \cos(a + bx)\, dx = \frac{e^x(\cos(a + bx) + b\sin(a + bx))}{1 + b^2}$$

input
```
Integrate[E^x*Cos[a + b*x],x]
```

output
```
(E^x*(Cos[a + b*x] + b*Sin[a + b*x]))/(1 + b^2)
```

3.104.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cos(a+bx)\,dx$$

$$\Big\downarrow \text{4933}$$

$$\frac{be^x \sin(a+bx)}{b^2+1}+\frac{e^x \cos(a+bx)}{b^2+1}$$

input
```
Int[E^x*Cos[a + b*x],x]
```

output
```
(E^x*Cos[a + b*x])/(1 + b^2) + (b*E^x*Sin[a + b*x])/(1 + b^2)
```

3.104.3.1 Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.104.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

method	result	size
parallelrisch	$\frac{e^{x}\left(b\sin\left(xb+a\right)+\cos\left(xb+a\right)\right)}{b^{2}+1}$	26
default	$\frac{e^{x}\cos\left(xb+a\right)}{b^{2}+1}+\frac{b\,e^{x}\sin\left(xb+a\right)}{b^{2}+1}$	35
risch	$\frac{ie^{x}\left(2i\cos\left(xb+a\right)+2ib\sin\left(xb+a\right)\right)}{2\left(-b+i\right)\left(i+b\right)}$	40
norman	$\frac{\frac{e^{x}}{b^{2}+1}-\frac{e^{x}\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}{b^{2}+1}+\frac{2b\,e^{x}\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{b^{2}+1}}{1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}$	71

input
```
int(exp(x)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
exp(x)/(b^2+1)*(b*sin(b*x+a)+cos(b*x+a))
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int e^x \cos(a+bx)\,dx = \frac{be^x \sin(bx+a) + \cos(bx+a)\,e^x}{b^2+1}$$

input
```
integrate(exp(x)*cos(b*x+a),x, algorithm="fricas")
```

output
```
(b*e^x*sin(b*x + a) + cos(b*x + a)*e^x)/(b^2 + 1)
```

3.104.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.17

$$\int e^x \cos(a+bx)\,dx = \begin{cases} -\frac{ixe^x \sin(a-ix)}{2} + \frac{xe^x \cos(a-ix)}{2} + \frac{ie^x \sin(a-ix)}{2} & \text{for } b=-i \\ \frac{ixe^x \sin(a+ix)}{2} + \frac{xe^x \cos(a+ix)}{2} + \frac{e^x \cos(a+ix)}{2} & \text{for } b=i \\ \frac{be^x \sin(a+bx)}{b^2+1} + \frac{e^x \cos(a+bx)}{b^2+1} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(x)*cos(b*x+a),x)
```

output
```
Piecewise((-I*x*exp(x)*sin(a - I*x)/2 + x*exp(x)*cos(a - I*x)/2 + I*exp(x)
*sin(a - I*x)/2, Eq(b, -I)), (I*x*exp(x)*sin(a + I*x)/2 + x*exp(x)*cos(a +
 I*x)/2 + exp(x)*cos(a + I*x)/2, Eq(b, I)), (b*exp(x)*sin(a + b*x)/(b**2 +
 1) + exp(x)*cos(a + b*x)/(b**2 + 1), True))
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int e^x \cos(a+bx)\,dx = \frac{(b\sin(bx+a)+\cos(bx+a))e^x}{b^2+1}$$

input
```
integrate(exp(x)*cos(b*x+a),x, algorithm="maxima")
```

output
```
(b*sin(b*x + a) + cos(b*x + a))*e^x/(b^2 + 1)
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int e^x \cos(a+bx)\,dx = \left(\frac{b\sin(bx+a)}{b^2+1}+\frac{\cos(bx+a)}{b^2+1}\right)e^x$$

input
```
integrate(exp(x)*cos(b*x+a),x, algorithm="giac")
```

output
```
(b*sin(b*x + a)/(b^2 + 1) + cos(b*x + a)/(b^2 + 1))*e^x
```

3.104.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int e^x \cos(a+bx)\,dx = \frac{e^x(\cos(a+bx)+b\sin(a+bx))}{b^2+1}$$

input
```
int(cos(a + b*x)*exp(x),x)
```

output
```
(exp(x)*(cos(a + b*x) + b*sin(a + b*x)))/(b^2 + 1)
```

3.105 $\int e^x \cos(a + cx^2) \, dx$

3.105.1 Optimal result

Integrand size = 12, antiderivative size = 115

$$\int e^x \cos(a + cx^2) \, dx = -\frac{\sqrt[4]{-1} e^{\frac{1}{4} i \left(4a + \frac{1}{c}\right)} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt[4]{-1}(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{\sqrt[4]{-1} e^{-\frac{1}{4} i \left(4a + \frac{1}{c}\right)} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt[4]{-1}(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

output
```
-1/4*(-1)^(1/4)*exp(1/4*I*(4*a+1/c))*erf(1/2*(-1)^(1/4)*(1+2*I*c*x)/c^(1/2
))*Pi^(1/2)/c^(1/2)+1/4*(-1)^(1/4)*erfi(1/2*(-1)^(1/4)*(1-2*I*c*x)/c^(1/2)
)*Pi^(1/2)/exp(1/4*I*(4*a+1/c))/c^(1/2)
```

3.105.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int e^x \cos(a + cx^2) \, dx = \frac{\sqrt[4]{-1} e^{-\frac{i}{4}/c} \sqrt{\pi} \left(-\operatorname{erfi}\left(\frac{(-1)^{3/4}(i+2cx)}{2\sqrt{c}}\right) (\cos(a) - i \sin(a)) + e^{\frac{i}{2}/c} \operatorname{erfi}\left(\frac{\sqrt[4]{-1}(-i+2cx)}{2\sqrt{c}}\right) (-i \cos(a) + \sin(a))\right)}{4\sqrt{c}}$$

input
```
Integrate[E^x*Cos[a + c*x^2],x]
```

output
```
((-1)^(1/4)*Sqrt[Pi]*(-(Erfi[((-1)^(3/4)*(I + 2*c*x))/(2*Sqrt[c])]*(Cos[a]
 - I*Sin[a])) + E^((I/2)/c)*Erfi[((-1)^(1/4)*(-I + 2*c*x))/(2*Sqrt[c])]*((
-I)*Cos[a] + Sin[a])))/(4*Sqrt[c]*E^((I/4)/c))
```

3.105.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cos\left(a + cx^2\right) dx$$

$$\Big\downarrow \text{ 4976}$$

$$\int \left(\frac{1}{2}e^{-ia-icx^2+x} + \frac{1}{2}e^{ia+icx^2+x}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt[4]{-1}\sqrt{\pi}e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\text{erfi}\left(\frac{\sqrt[4]{-1}(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}} - \frac{\sqrt[4]{-1}\sqrt{\pi}e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\text{erf}\left(\frac{\sqrt[4]{-1}(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

input
```
Int[E^x*Cos[a + c*x^2],x]
```

output
```
-1/4*((-1)^(1/4)*E^((I/4)*(4*a + c^(-1)))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 + (2
*I)*c*x))/(2*Sqrt[c])])/Sqrt[c] + ((-1)^(1/4)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1
 - (2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]*E^((I/4)*(4*a + c^(-1))))
```

3.105.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.105.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{i(4ac+1)}{4c}}\,\mathrm{erf}\left(\sqrt{ic}\,x-\frac{1}{2\sqrt{ic}}\right)}{4\sqrt{ic}}+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{i(4ac+1)}{4c}}\,\mathrm{erf}\left(\sqrt{-ic}\,x-\frac{1}{2\sqrt{-ic}}\right)}{4\sqrt{-ic}}$	86

input
```
int(exp(x)*cos(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*exp(-1/4*I*(4*a*c+1)/c)/(I*c)^(1/2)*erf((I*c)^(1/2)*x-1/2/(I*
c)^(1/2))+1/4*Pi^(1/2)*exp(1/4*I*(4*a*c+1)/c)/(-I*c)^(1/2)*erf((-I*c)^(1/2
)*x-1/2/(-I*c)^(1/2))
```

3.105.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(73) = 146.

Time = 0.26 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.26

$$\int e^x \cos\left(a+cx^2\right)\,dx$$
$$= \frac{\sqrt{2}\left(\pi\cos\left(\frac{4\,ac+1}{4\,c}\right)-i\,\pi\sin\left(\frac{4\,ac+1}{4\,c}\right)\right)\sqrt{\frac{c}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}(2\,cx+i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)-\sqrt{2}\left(\pi\cos\left(\frac{4\,ac+1}{4\,c}\right)+i\,\pi\sin\left(\frac{4\,ac+1}{4\,c}\right)\right)\sqrt{\frac{c}{\pi}}\,\mathrm{C}\left(-\right.}{}$$

input
```
integrate(exp(x)*cos(c*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*(pi*cos(1/4*(4*a*c + 1)/c) - I*pi*sin(1/4*(4*a*c + 1)/c))*sqr
t(c/pi)*fresnel_cos(1/2*sqrt(2)*(2*c*x + I)*sqrt(c/pi)/c) - sqrt(2)*(pi*co
s(1/4*(4*a*c + 1)/c) + I*pi*sin(1/4*(4*a*c + 1)/c))*sqrt(c/pi)*fresnel_cos
(-1/2*sqrt(2)*(2*c*x - I)*sqrt(c/pi)/c) + sqrt(2)*(-I*pi*cos(1/4*(4*a*c +
1)/c) - pi*sin(1/4*(4*a*c + 1)/c))*sqrt(c/pi)*fresnel_sin(1/2*sqrt(2)*(2*c
*x + I)*sqrt(c/pi)/c) + sqrt(2)*(-I*pi*cos(1/4*(4*a*c + 1)/c) + pi*sin(1/4
*(4*a*c + 1)/c))*sqrt(c/pi)*fresnel_sin(-1/2*sqrt(2)*(2*c*x - I)*sqrt(c/pi
)/c))/c
```

3.105.6 Sympy [F]

$$\int e^x \cos\left(a+cx^2\right)\,dx = \int e^x \cos\left(a+cx^2\right)dx$$

input
```
integrate(exp(x)*cos(c*x**2+a),x)
```

output
```
Integral(exp(x)*cos(a + c*x**2), x)
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

$$\int e^x \cos\left(a+cx^2\right)\,dx =$$
$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left((i-1)\cos\left(\frac{4\,ac+1}{4\,c}\right)+(i+1)\sin\left(\frac{4\,ac+1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{2i\,cx-1}{2\sqrt{i\,c}}\right)+\left((i+1)\cos\left(\frac{4\,ac+1}{4\,c}\right)+(i-1)\sin\left(\frac{4c}{}\right.\right.\right.}{8\sqrt{c}}$$

input
```
integrate(exp(x)*cos(c*x^2+a),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*sqrt(pi)*(((I - 1)*cos(1/4*(4*a*c + 1)/c) + (I + 1)*sin(1/4*(
4*a*c + 1)/c))*erf(1/2*(2*I*c*x - 1)/sqrt(I*c)) + ((I + 1)*cos(1/4*(4*a*c
+ 1)/c) + (I - 1)*sin(1/4*(4*a*c + 1)/c))*erf(1/2*(2*I*c*x + 1)/sqrt(-I*c)
))/sqrt(c)
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

$$\int e^x \cos\left(a+cx^2\right)\,dx = -\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\frac{1}{4}i\sqrt{2}\left(2\,x+\frac{i}{c}\right)\left(-\frac{i\,c}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{4i\,ac+i}{4\,c}\right)}}{4\left(-\frac{i\,c}{|c|}+1\right)\sqrt{|c|}} + \frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}i\sqrt{2}\left(2\,x-\frac{i}{c}\right)\left(\frac{i\,c}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-4i\,ac-i}{4\,c}\right)}}{4\left(\frac{i\,c}{|c|}+1\right)\sqrt{|c|}}$$

input
```
integrate(exp(x)*cos(c*x^2+a),x, algorithm="giac")
```

output
```
-1/4*I*sqrt(2)*sqrt(pi)*erf(1/4*I*sqrt(2)*(2*x + I/c)*(-I*c/abs(c) + 1)*sq
rt(abs(c)))*e^(-1/4*(4*I*a*c + I)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c))) + 1/
4*I*sqrt(2)*sqrt(pi)*erf(-1/4*I*sqrt(2)*(2*x - I/c)*(I*c/abs(c) + 1)*sqrt(
abs(c)))*e^(-1/4*(-4*I*a*c - I)/c)/((I*c/abs(c) + 1)*sqrt(abs(c)))
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int e^x \cos \left(a+cx^2\right) \, dx=\int \mathrm{e}^x \, \cos \left(c\,x^2+a\right) \, dx$$

input
```
int(exp(x)*cos(a + c*x^2),x)
```

output
```
int(exp(x)*cos(a + c*x^2), x)
```

3.106 $\int e^x \cos(a + bx + cx^2)\,dx$

3.106.1 Optimal result

Integrand size = 15, antiderivative size = 144

$$\int e^x \cos(a + bx + cx^2)\,dx = -\frac{\sqrt[4]{-1}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}(1+ib+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{\sqrt[4]{-1}e^{-ia+\frac{i(i+b)^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(1-ib-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

output
```
-1/4*(-1)^(1/4)*exp(1/4*I*(4*a+(1+I*b)^2/c))*erf(1/2*(-1)^(1/4)*(1+I*b+2*I
*c*x)/c^(1/2))*Pi^(1/2)/c^(1/2)+1/4*(-1)^(1/4)*exp(-I*a+1/4*I*(I+b)^2/c)*e
rfi(1/2*(-1)^(1/4)*(1-I*b-2*I*c*x)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94

$$\int e^x \cos(a + bx + cx^2)\,dx = \frac{\sqrt[4]{-1}e^{-\frac{i(1-2ib+b^2)}{4c}}\sqrt{\pi}\left(-e^{\frac{ib^2}{2c}}\mathrm{erfi}\left(\frac{(-1)^{3/4}(i+b+2cx)}{2\sqrt{c}}\right)(\cos(a)-i\sin(a)) + e^{\frac{i}{2}/c}\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(-i+b+2cx)}{2\sqrt{c}}\right)(-i\cos(a$$

$$4\sqrt{c}$$

input
```
Integrate[E^x*Cos[a + b*x + c*x^2],x]
```

output
```
((-1)^(1/4)*Sqrt[Pi]*(-(E^(((I/2)*b^2)/c)*Erfi[((-1)^(3/4)*(I + b + 2*c*x)
)/(2*Sqrt[c])]*(Cos[a] - I*Sin[a])) + E^((I/2)/c)*Erfi[((-1)^(1/4)*(-I + b
 + 2*c*x))/(2*Sqrt[c])]*((-I)*Cos[a] + Sin[a])))/(4*Sqrt[c]*E^(((I/4)*(1 -
 (2*I)*b + b^2))/c))
```

3.106.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cos\left(a+bx+cx^2\right) dx$$

$$\Big\downarrow \text{ 4976}$$

$$\int \left(\frac{1}{2}e^{-ia+(1-ib)x-icx^2}+\frac{1}{2}e^{ia+(1+ib)x+icx^2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt[4]{-1}\sqrt{\pi}e^{\frac{i(b+i)^2}{4c}-ia}\text{erfi}\left(\frac{\sqrt[4]{-1}(-ib-2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt[4]{-1}\sqrt{\pi}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\text{erf}\left(\frac{\sqrt[4]{-1}(ib+2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

input
```
Int[E^x*Cos[a + b*x + c*x^2],x]
```

output
```
-1/4*((-1)^(1/4)*E^((I/4)*(4*a + (1 + I*b)^2/c))*Sqrt[Pi]*Erf[((-1)^(1/4)*
(1 + I*b + (2*I)*c*x))/(2*Sqrt[c])])/Sqrt[c] + ((-1)^(1/4)*E^((-I)*a + ((I
/4)*(I + b)^2)/c)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 - I*b - (2*I)*c*x))/(2*Sqrt
[c])])/(4*Sqrt[c])
```

3.106.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.106.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{i\left(4ac-b^{2}-2ib+1\right)}{4c}}\operatorname{erf}\left(\sqrt{ic}\,x-\frac{-ib+1}{2\sqrt{ic}}\right)}{4\sqrt{ic}}-\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{i\left(4ac-b^{2}+2ib+1\right)}{4c}}\operatorname{erf}\left(-\sqrt{-ic}\,x+\frac{ib+1}{2\sqrt{-ic}}\right)}{4\sqrt{-ic}}$	117

input
```
int(exp(x)*cos(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*exp(-1/4*I*(-b^2-2*I*b+4*a*c+1)/c)/(I*c)^(1/2)*erf((I*c)^(1/2
)*x-1/2*(-I*b+1)/(I*c)^(1/2))-1/4*Pi^(1/2)*exp(1/4*I*(-b^2+2*I*b+4*a*c+1)/
c)/(-I*c)^(1/2)*erf(-(-I*c)^(1/2)*x+1/2*(1+I*b)/(-I*c)^(1/2))
```

3.106.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(91) = 182.

Time = 0.26 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.59

$$\int e^{x}\cos\left(a+bx+cx^{2}\right)\,dx$$
$$=\frac{\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{i\,b^{2}-4i\,ac-2\,b-i}{4\,c}\right)}\operatorname{C}\left(\frac{\sqrt{2}(2\,cx+b+i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)-\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{-i\,b^{2}+4i\,ac-2\,b+i}{4\,c}\right)}\operatorname{C}\left(-\frac{\sqrt{2}(2\,cx+b-i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)-i\sqrt{2}\pi\sqrt{\frac{c}{\pi}}}{4\,c}$$

input
```
integrate(exp(x)*cos(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(I*b^2 - 4*I*a*c - 2*b - I)/c)*fresnel_c
os(1/2*sqrt(2)*(2*c*x + b + I)*sqrt(c/pi)/c) - sqrt(2)*pi*sqrt(c/pi)*e^(1/
4*(-I*b^2 + 4*I*a*c - 2*b + I)/c)*fresnel_cos(-1/2*sqrt(2)*(2*c*x + b - I)
*sqrt(c/pi)/c) - I*sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(I*b^2 - 4*I*a*c - 2*b - I
)/c)*fresnel_sin(1/2*sqrt(2)*(2*c*x + b + I)*sqrt(c/pi)/c) - I*sqrt(2)*pi*
sqrt(c/pi)*e^(1/4*(-I*b^2 + 4*I*a*c - 2*b + I)/c)*fresnel_sin(-1/2*sqrt(2)
*(2*c*x + b - I)*sqrt(c/pi)/c))/c
```

3.106.6 Sympy [F]

$$\int e^x \cos\left(a+bx+cx^2\right)\,dx = \int e^x \cos\left(a+bx+cx^2\right)dx$$

input
```
integrate(exp(x)*cos(c*x**2+b*x+a),x)
```

output
```
Integral(exp(x)*cos(a + b*x + c*x**2), x)
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91

$$\int e^x \cos\left(a+bx+cx^2\right)\,dx = -\frac{\sqrt{2}\sqrt{\pi}\left(\left(-(i-1)\cos\left(-\frac{b^2-4\,ac-1}{4\,c}\right)-(i+1)\sin\left(-\frac{b^2-4\,ac-1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{i\,(2i\,cx+i\,b-1)\sqrt{i\,c}}{2\,c}\right)+\left((i+1)\cos\left(-\right.\right.}{8\,\sqrt{c}}$$

input
```
integrate(exp(x)*cos(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*sqrt(pi)*((-(I - 1)*cos(-1/4*(b^2 - 4*a*c - 1)/c) - (I + 1)*s
in(-1/4*(b^2 - 4*a*c - 1)/c))*erf(1/2*I*(2*I*c*x + I*b - 1)*sqrt(I*c)/c) +
 ((I + 1)*cos(-1/4*(b^2 - 4*a*c - 1)/c) + (I - 1)*sin(-1/4*(b^2 - 4*a*c -
1)/c))*erf(1/2*I*(2*I*c*x + I*b + 1)*sqrt(-I*c)/c))*e^(-1/2*b/c)/sqrt(c)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

$$\int e^x \cos\left(a+bx+cx^2\right)\,dx$$
$$= \frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{4}i\sqrt{2}\left(2x+\frac{b-i}{c}\right)\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{ib^2-4iac+2b-i}{4c}\right)}}{4\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}}$$
$$-\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{4}i\sqrt{2}\left(2x+\frac{b+i}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-ib^2+4iac+2b+i}{4c}\right)}}{4\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}}$$

input
```
integrate(exp(x)*cos(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/4*I*sqrt(2)*sqrt(pi)*erf(-1/4*I*sqrt(2)*(2*x + (b - I)/c)*(I*c/abs(c) +
1)*sqrt(abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c + 2*b - I)/c)/((I*c/abs(c) + 1)*
sqrt(abs(c))) - 1/4*I*sqrt(2)*sqrt(pi)*erf(1/4*I*sqrt(2)*(2*x + (b + I)/c)
*(-I*c/abs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(-I*b^2 + 4*I*a*c + 2*b + I)/c)/(
(-I*c/abs(c) + 1)*sqrt(abs(c)))
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int e^x \cos\left(a+bx+cx^2\right)\,dx = \int e^x\,\cos\left(c\,x^2+b\,x+a\right)\,dx$$

input
```
int(exp(x)*cos(a + b*x + c*x^2),x)
```

output
```
int(exp(x)*cos(a + b*x + c*x^2), x)
```

3.107 $\int e^{x^2} \cos(a+bx)\,dx$

3.107.1 Optimal result

Integrand size = 12, antiderivative size = 77

$$\int e^{x^2}\cos(a+bx)\,dx=\frac{1}{4}e^{-ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\frac{1}{4}e^{ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)$$

output
```
-1/4*exp(-I*a+1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)+1/4*exp(I*a+1/4*b^2)*erfi(
1/2*I*b+x)*Pi^(1/2)
```

3.107.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int e^{x^2}\cos(a+bx)\,dx=\frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\left(\cos(a)\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\cos(a)\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)-\left(\mathrm{erf}\left(\frac{b}{2}-ix\right)+\mathrm{erf}\left(\frac{b}{2}+ix\right)\right)\sin(a)\right)$$

input
```
Integrate[E^x^2*Cos[a + b*x],x]
```

output
```
(E^(b^2/4)*Sqrt[Pi]*(Cos[a]*Erfi[((-I)*b + 2*x)/2] + Cos[a]*Erfi[(I*b + 2*
x)/2] - (Erf[b/2 - I*x] + Erf[b/2 + I*x])*Sin[a]))/4
```

3.107.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2}\cos(a+bx)\,dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{1}{2}e^{-ia-ibx+x^2}+\frac{1}{2}e^{ia+ibx+x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}-ia}\text{erfi}\left(\frac{1}{2}(2x-ib)\right)+\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}+ia}\text{erfi}\left(\frac{1}{2}(2x+ib)\right)$$

input
```
Int[E^x^2*Cos[a + b*x],x]
```

output
```
(E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2])/4 + (E^(I*a + b^2/4)*
Sqrt[Pi]*Erfi[(I*b + 2*x)/2])/4
```

3.107.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.107.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{e}^{-ia}\,\mathrm{erf}\left(ix+\frac{b}{2}\right)}{4}+\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{e}^{ia}\,\mathrm{erf}\left(-ix+\frac{b}{2}\right)}{4}$	54

input
```
int(exp(x^2)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*Pi^(1/2)*exp(1/4*b^2)*exp(-I*a)*erf(I*x+1/2*b)+1/4*I*Pi^(1/2)*exp(1
/4*b^2)*exp(I*a)*erf(-I*x+1/2*b)
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.60

$$\int e^{x^2}\cos(a+bx)\,dx = \frac{1}{4}\sqrt{\pi}\left(-i\ \mathrm{erf}\left(-\frac{1}{2}b+i\,x\right)e^{\left(\frac{1}{4}b^2+i\,a\right)}-i\ \mathrm{erf}\left(\frac{1}{2}b+i\,x\right)e^{\left(\frac{1}{4}b^2-i\,a\right)}\right)$$

input
```
integrate(exp(x^2)*cos(b*x+a),x, algorithm="fricas")
```

output
```
1/4*sqrt(pi)*(-I*erf(-1/2*b + I*x)*e^(1/4*b^2 + I*a) - I*erf(1/2*b + I*x)*
e^(1/4*b^2 - I*a))
```

3.107.6 Sympy [F]

$$\int e^{x^2}\cos(a+bx)\,dx = \int e^{x^2}\cos(a+bx)\,dx$$

input
```
integrate(exp(x**2)*cos(b*x+a),x)
```

output
```
Integral(exp(x**2)*cos(a + b*x), x)
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int e^{x^2}\cos(a+bx)\,dx = -\frac{1}{4}\sqrt{\pi}\left((i\cos(a)+\sin(a))\operatorname{erf}\left(\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}+(i\cos(a)-\sin(a))\operatorname{erf}\left(-\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}\right)$$

input
```
integrate(exp(x^2)*cos(b*x+a),x, algorithm="maxima")
```

output
```
-1/4*sqrt(pi)*((I*cos(a) + sin(a))*erf(1/2*b + I*x)*e^(1/4*b^2) + (I*cos(a
) - sin(a))*erf(-1/2*b + I*x)*e^(1/4*b^2))
```

3.107.8 Giac [F]

$$\int e^{x^2}\cos(a+bx)\,dx = \int \cos(bx+a)\,e^{\left(x^2\right)}\,dx$$

input
```
integrate(exp(x^2)*cos(b*x+a),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*e^(x^2), x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\cos(a+bx)\,dx = \int \cos(a+b\,x)\,\mathrm{e}^{x^2}\,dx$$

input
```
int(cos(a + b*x)*exp(x^2),x)
```

output
```
int(cos(a + b*x)*exp(x^2), x)
```

3.108 $\int e^{x^2}\cos\left(a+cx^2\right)\,dx$

3.108.1 Optimal result

Integrand size = 14, antiderivative size = 83

$$\int e^{x^2}\cos\left(a+cx^2\right)\,dx=\frac{e^{-ia}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{1-ic}x\right)}{4\sqrt{1-ic}}+\frac{e^{ia}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{1+ic}x\right)}{4\sqrt{1+ic}}$$

output
```
1/4*erfi(x*(1-I*c)^(1/2))*Pi^(1/2)/exp(I*a)/(1-I*c)^(1/2)+1/4*exp(I*a)*erf
i(x*(1+I*c)^(1/2))*Pi^(1/2)/(1+I*c)^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.29

$$\int e^{x^2}\cos\left(a+cx^2\right)\,dx$$
$$=\frac{\sqrt[4]{-1}\sqrt{\pi}\left(-\left((-i+c)\sqrt{i+c}\operatorname{erfi}\left((-1)^{3/4}\sqrt{i+c}x\right)(\cos(a)-i\sin(a))\right)+(1-ic)\sqrt{-i+c}\operatorname{erfi}\left(\sqrt[4]{-1}\sqrt{-i+\right.}{4\left(1+c^2\right)}$$

input
```
Integrate[E^x^2*Cos[a + c*x^2],x]
```

output
```
((-1)^(1/4)*Sqrt[Pi]*(-((-I + c)*Sqrt[I + c]*Erfi[(-1)^(3/4)*Sqrt[I + c]*x
]*(Cos[a] - I*Sin[a])) + (1 - I*c)*Sqrt[-I + c]*Erfi[(-1)^(1/4)*Sqrt[-I +
c]*x]*(Cos[a] + I*Sin[a])))/(4*(1 + c^2))
```

3.108.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} \cos \left(a + cx^2\right) dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{1}{2} e^{(1-ic)x^2 - ia} + \frac{1}{2} e^{ia+(1+ic)x^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi} e^{-ia} \text{erfi}\left(\sqrt{1-ic}x\right)}{4\sqrt{1-ic}} + \frac{\sqrt{\pi} e^{ia} \text{erfi}\left(\sqrt{1+ic}x\right)}{4\sqrt{1+ic}}$$

input
```
Int[E^x^2*Cos[a + c*x^2],x]
```

output
```
(Sqrt[Pi]*Erfi[Sqrt[1 - I*c]*x])/(4*Sqrt[1 - I*c]*E^(I*a)) + (E^(I*a)*Sqrt
[Pi]*Erfi[Sqrt[1 + I*c]*x])/(4*Sqrt[1 + I*c])
```

3.108.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.108.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{\sqrt{\pi}\,e^{-ia}\,\mathrm{erf}\left(\sqrt{ic-1}\,x\right)}{4\sqrt{ic-1}}+\frac{\sqrt{\pi}\,e^{ia}\,\mathrm{erf}\left(\sqrt{-ic-1}\,x\right)}{4\sqrt{-ic-1}}$	60

input
```
int(exp(x^2)*cos(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*exp(-I*a)/(I*c-1)^(1/2)*erf((I*c-1)^(1/2)*x)+1/4*Pi^(1/2)*exp
(I*a)/(-I*c-1)^(1/2)*erf((-I*c-1)^(1/2)*x)
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01

$$\int e^{x^2}\cos\left(a+cx^2\right)\,dx = \frac{\sqrt{\pi}((-i\,c-1)\cos(a)-(c-i)\sin(a))\sqrt{i\,c-1}\,\mathrm{erf}\left(\sqrt{i\,c-1}x\right)+\sqrt{\pi}((i\,c-1)\cos(a)-(c+i)\sin(a))\sqrt{-}}{4\left(c^2+1\right)}$$

input
```
integrate(exp(x^2)*cos(c*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*((-I*c - 1)*cos(a) - (c - I)*sin(a))*sqrt(I*c - 1)*erf(sqrt(
I*c - 1)*x) + sqrt(pi)*((I*c - 1)*cos(a) - (c + I)*sin(a))*sqrt(-I*c - 1)*
erf(sqrt(-I*c - 1)*x))/(c^2 + 1)
```

3.108.6 Sympy [F]

$$\int e^{x^2}\cos\left(a+cx^2\right)\,dx = \int e^{x^2}\cos\left(a+cx^2\right)dx$$

input
```
integrate(exp(x**2)*cos(c*x**2+a),x)
```

output
```
Integral(exp(x**2)*cos(a + c*x**2), x)
```

3.108.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(53) = 106.

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.60

$$\int e^{x^2}\cos\left(a+cx^2\right)\,dx = -\frac{\sqrt{\pi}\sqrt{2\,c^2+2}\left(\left(i\,\cos\left(a\right)+\sin\left(a\right)\right)\operatorname{erf}\left(\sqrt{i\,c-1}x\right)+\left(-i\,\cos\left(a\right)+\sin\left(a\right)\right)\operatorname{erf}\left(\sqrt{-i\,c-1}x\right)\right)\sqrt{\sqrt{c^2+1}}}{8}$$

input
```
integrate(exp(x^2)*cos(c*x^2+a),x, algorithm="maxima")
```

output
```
-1/8*(sqrt(pi)*sqrt(2*c^2 + 2)*((I*cos(a) + sin(a))*erf(sqrt(I*c - 1)*x) +
 (-I*cos(a) + sin(a))*erf(sqrt(-I*c - 1)*x))*sqrt(sqrt(c^2 + 1) + 1) - sqr
t(pi)*sqrt(2*c^2 + 2)*((cos(a) - I*sin(a))*erf(sqrt(I*c - 1)*x) + (cos(a)
+ I*sin(a))*erf(sqrt(-I*c - 1)*x))*sqrt(sqrt(c^2 + 1) - 1))/(c^2 + 1)
```

3.108.8 Giac [F]

$$\int e^{x^2}\cos\left(a+cx^2\right)\,dx = \int \cos\left(cx^2+a\right)e^{\left(x^2\right)}\,dx$$

input
```
integrate(exp(x^2)*cos(c*x^2+a),x, algorithm="giac")
```

output
```
integrate(cos(c*x^2 + a)*e^(x^2), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\cos\left(a+cx^2\right)\,dx = \int e^{x^2}\cos\left(c\,x^2+a\right)\,dx$$

input
```
int(exp(x^2)*cos(a + c*x^2),x)
```

output
```
int(exp(x^2)*cos(a + c*x^2), x)
```

3.109 $\int e^{x^2} \cos\left(a+bx+cx^2\right) dx$

3.109.1 Optimal result

Integrand size = 17, antiderivative size = 151

$$\int e^{x^2}\cos\left(a+bx+cx^2\right)dx=-\frac{e^{-i\left(a-\frac{b^2}{4i+4c}\right)}\sqrt{\pi}\operatorname{erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}+\frac{e^{ia+\frac{b^2}{4(1+ic)}}\sqrt{\pi}\operatorname{erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}$$

output
```
-1/4*erfi(1/2*(I*b-2*(1-I*c)*x)/(1-I*c)^(1/2))*Pi^(1/2)/exp(I*(a-b^2/(4*I+
4*c)))/(1-I*c)^(1/2)+1/4*exp(I*a+1/4*b^2/(1+I*c))*erfi(1/2*(I*b+2*(1+I*c)*
x)/(1+I*c)^(1/2))*Pi^(1/2)/(1+I*c)^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.10

$$\int e^{x^2}\cos\left(a+bx+cx^2\right)dx$$
$$=\frac{\sqrt[4]{-1}e^{\frac{ib^2}{4i-4c}}\sqrt{\pi}\left(-\left((-i+c)\sqrt{i+c}e^{\frac{ib^2c}{2+2c^2}}\operatorname{erfi}\left(\frac{(-1)^{3/4}(b+2(i+c)x)}{2\sqrt{i+c}}\right)(\cos(a)-i\sin(a))\right)+\sqrt{-i+c}(i+c)\operatorname{erfi}\left(\right.\right.}{4\left(1+c^2\right)}$$

input
```
Integrate[E^x^2*Cos[a + b*x + c*x^2],x]
```

output
```
((-1)^(1/4)*E^((I*b^2)/(4*I - 4*c))*Sqrt[Pi]*(-((-I + c)*Sqrt[I + c]*E^((I
*b^2*c)/(2 + 2*c^2))*Erfi[((-1)^(3/4)*(b + 2*(I + c)*x))/(2*Sqrt[I + c])]*
(Cos[a] - I*Sin[a])) + Sqrt[-I + c]*(I + c)*Erfi[((-1)^(1/4)*(b + 2*(-I +
c)*x))/(2*Sqrt[-I + c])]*((-I)*Cos[a] + Sin[a])))/(4*(1 + c^2))
```

3.109.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2}\cos\left(a+bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{1}{2}e^{-ia-ibx+(1-ic)x^2}+\frac{1}{2}e^{ia+ibx+(1+ic)x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}e^{ia+\frac{b^2}{4(1+ic)}}\operatorname{erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}-\frac{\sqrt{\pi}e^{-i\left(a-\frac{b^2}{4c+4i}\right)}\operatorname{erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}$$

input
```
Int[E^x^2*Cos[a + b*x + c*x^2],x]
```

output
```
-1/4*(Sqrt[Pi]*Erfi[(I*b - 2*(1 - I*c)*x)/(2*Sqrt[1 - I*c])])/(Sqrt[1 - I*
c]*E^(I*(a - b^2/(4*I + 4*c)))) + (E^(I*a + b^2/(4*(1 + I*c)))*Sqrt[Pi]*Er
fi[(I*b + 2*(1 + I*c)*x)/(2*Sqrt[1 + I*c])])/(4*Sqrt[1 + I*c])
```

3.109.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.109.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac+4ia-b^{2}}{4ic-4}}\operatorname{erf}\left(\sqrt{ic-1}\,x+\frac{ib}{2\sqrt{ic-1}}\right)}{4\sqrt{ic-1}}-\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-4ia-b^{2}}{4(ic+1)}}\operatorname{erf}\left(-\sqrt{-ic-1}\,x+\frac{ib}{2\sqrt{-ic-1}}\right)}{4\sqrt{-ic-1}}$	127

input
```
int(exp(x^2)*cos(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*exp(1/4*(4*a*c+4*I*a-b^2)/(I*c-1))/(I*c-1)^(1/2)*erf((I*c-1)^
(1/2)*x+1/2*I*b/(I*c-1)^(1/2))-1/4*Pi^(1/2)*exp(-1/4*(4*a*c-4*I*a-b^2)/(1+
I*c))/(-I*c-1)^(1/2)*erf(-(-I*c-1)^(1/2)*x+1/2*I*b/(-I*c-1)^(1/2))
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.09

$$\int e^{x^2}\cos\left(a+bx+cx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}(i\,c+1)\sqrt{i\,c-1}\operatorname{erf}\left(-\frac{(bc+2\,(c^2+1)x-i\,b)\sqrt{i\,c-1}}{2\,(c^2+1)}\right)e^{\left(\frac{i\,b^2c-4i\,ac^2+b^2-4i\,a}{4\,(c^2+1)}\right)}+\sqrt{\pi}(i\,c-1)\sqrt{-i\,c-1}\operatorname{erf}\left(\frac{(bc+2\,(c^2 \ldots}{\ldots}\right.}{4\,(c^2+1)}$$

input
```
integrate(exp(x^2)*cos(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*(I*c + 1)*sqrt(I*c - 1)*erf(-1/2*(b*c + 2*(c^2 + 1)*x - I*b)
*sqrt(I*c - 1)/(c^2 + 1))*e^(1/4*(I*b^2*c - 4*I*a*c^2 + b^2 - 4*I*a)/(c^2
+ 1)) + sqrt(pi)*(I*c - 1)*sqrt(-I*c - 1)*erf(1/2*(b*c + 2*(c^2 + 1)*x + I
*b)*sqrt(-I*c - 1)/(c^2 + 1))*e^(1/4*(-I*b^2*c + 4*I*a*c^2 + b^2 + 4*I*a)/
(c^2 + 1)))/(c^2 + 1)
```

3.109.6 Sympy [F]

$$\int e^{x^2}\cos\left(a+bx+cx^2\right)\,dx=\int e^{x^2}\cos\left(a+bx+cx^2\right)dx$$

input
```
integrate(exp(x**2)*cos(c*x**2+b*x+a),x)
```

output
```
Integral(exp(x**2)*cos(a + b*x + c*x**2), x)
```

3.109.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(101) = 202$.

Time = 0.22 (sec) , antiderivative size = 474, normalized size of antiderivative = 3.14

$$\int e^{x^2}\cos\left(a+bx+cx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}\sqrt{2\,c^2+2}\left(\left(\left(-i\,\cos\left(-\frac{b^2c-4\,ac^2-4\,a}{4\,(c^2+1)}\right)e^{\left(\frac{b^2}{4\,(c^2+1)}\right)}-e^{\left(\frac{b^2}{4\,(c^2+1)}\right)}\sin\left(-\frac{b^2c-4\,ac^2-4\,a}{4\,(c^2+1)}\right)\right)\operatorname{erf}\left(-\frac{2\,(-i\,c+1)x-i\,b}{2\,\sqrt{i\,c-1}}\right.\right.\right.}{}$$

input
```
integrate(exp(x^2)*cos(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/8*(sqrt(pi)*sqrt(2*c^2 + 2)*((-I*cos(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 +
 1))*e^(1/4*b^2/(c^2 + 1)) - e^(1/4*b^2/(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c
^2 - 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-I*c + 1)*x - I*b)/sqrt(I*c - 1)) + (-I
*cos(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1))*e^(1/4*b^2/(c^2 + 1)) + e^(1/
4*b^2/(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)))*erf(-1/2*(2*
(-I*c - 1)*x - I*b)/sqrt(-I*c - 1)))*sqrt(sqrt(c^2 + 1) + 1) + sqrt(pi)*sq
rt(2*c^2 + 2)*((cos(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1))*e^(1/4*b^2/(c^
2 + 1)) - I*e^(1/4*b^2/(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 +
1)))*erf(-1/2*(2*(-I*c + 1)*x - I*b)/sqrt(I*c - 1)) - (cos(-1/4*(b^2*c - 4
*a*c^2 - 4*a)/(c^2 + 1))*e^(1/4*b^2/(c^2 + 1)) + I*e^(1/4*b^2/(c^2 + 1))*s
in(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-I*c - 1)*x - I*b
)/sqrt(-I*c - 1)))*sqrt(sqrt(c^2 + 1) - 1))/(c^2 + 1)
```

3.109.8 Giac [F]

$$\int e^{x^2} \cos\left(a+bx+cx^2\right)\,dx = \int \cos\left(cx^2+bx+a\right)e^{\left(x^2\right)}\,dx$$

input
```
integrate(exp(x^2)*cos(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate(cos(c*x^2 + b*x + a)*e^(x^2), x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int e^{x^2} \cos\left(a+bx+cx^2\right)\,dx = \int e^{x^2}\,\cos\left(c\,x^2+b\,x+a\right)\,dx$$

input
```
int(exp(x^2)*cos(a + b*x + c*x^2),x)
```

output
```
int(exp(x^2)*cos(a + b*x + c*x^2), x)
```

3.110 $\int f^{a+bx} \cos\left(d+fx^2\right) dx$

3.110.1 Optimal result

Integrand size = 16, antiderivative size = 142

$$\begin{aligned}\int f^{a+bx}\cos\left(d+fx^2\right)dx &= -\frac{1}{4}\sqrt[4]{-1}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\mathrm{erf}\left(\frac{\sqrt[4]{-1}(2ifx+b\log(f))}{2\sqrt{f}}\right)\\ &\quad-\frac{1}{4}\sqrt[4]{-1}e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(2ifx-b\log(f))}{2\sqrt{f}}\right)\end{aligned}$$

output
```
-1/4*(-1)^(1/4)*exp(1/4*I*(4*d+b^2*ln(f)^2/f))*f^(-1/2+a)*erf(1/2*(-1)^(1/
4)*(2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-1/4*(-1)^(1/4)*f^(-1/2+a)*erfi(1/2*
(-1)^(1/4)*(2*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)/exp(1/4*I*(4*d+b^2*ln(f)^2/
f))
```

3.110.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

$$\begin{aligned}\int f^{a+bx}\cos\left(d+fx^2\right)dx &= \frac{1}{4}\sqrt[4]{-1}e^{-\frac{ib^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(-\mathrm{erfi}\left(\frac{(-1)^{3/4}(2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d))\right.\\ &\quad\left.+e^{\frac{ib^2\log^2(f)}{2f}}\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(2fx-ib\log(f))}{2\sqrt{f}}\right)(-i\cos(d)+\sin(d))\right)\end{aligned}$$

input
```
Integrate[f^(a + b*x)*Cos[d + f*x^2],x]
```

output
```
((-1)^(1/4)*f^(-1/2 + a)*Sqrt[Pi]*(-(Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f])
)/(2*Sqrt[f])]*(Cos[d] - I*Sin[d])) + E^(((I/2)*b^2*Log[f]^2)/f)*Erfi[((-1
)^(1/4)*(2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*((-I)*Cos[d] + Sin[d])))/(4*E^(
((I/4)*b^2*Log[f]^2)/f))
```

3.110.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx} \cos\left(d+fx^2\right) dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{1}{2} e^{-id-ifx^2} f^{a+bx} + \frac{1}{2} e^{id+ifx^2} f^{a+bx}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi} f^{a-\frac{1}{2}} e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)} \text{erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+2ifx)}{2\sqrt{f}}\right) - \frac{1}{4}\sqrt[4]{-1}\sqrt{\pi} f^{a-\frac{1}{2}} e^{-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)} \text{erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+2ifx)}{2\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Cos[d + f*x^2],x]
```

output
```
-1/4*((-1)^(1/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*
Erf[((-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])]) - ((-1)^(1/4)*f^(-1/
2 + a)*Sqrt[Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(4*
E^((I/4)*(4*d + (b^2*Log[f]^2)/f)))
```

3.110.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.110.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{-\frac{i \left(\ln \left(f \right)^{2} b^{2}+4 d f \right)}{4 f}} \operatorname{erf}\left(-\sqrt{i f}\, x +\frac{\ln \left(f \right) b}{2 \sqrt{i f}}\right)}{4 \sqrt{i f}}-\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{\frac{i \left(\ln \left(f \right)^{2} b^{2}+4 d f \right)}{4 f}} \operatorname{erf}\left(-\sqrt{-i f}\, x +\frac{\ln \left(f \right) b}{2 \sqrt{-i f}}\right)}{4 \sqrt{-i f}}$	114

input
```
int(f^(b*x+a)*cos(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)
^(1/2)*x+1/2*ln(f)*b/(I*f)^(1/2))-1/4*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+
4*d*f)/f)/(-I*f)^(1/2)*erf(-(-I*f)^(1/2)*x+1/2*ln(f)*b/(-I*f)^(1/2))
```

3.110.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(98) = 196.

Time = 0.26 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.87

$$\int f^{a+bx}\cos\left(d+fx^2\right)\,dx$$
$$= \frac{\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log(f)^2+4\,af\log(f)-4i\,df}{4\,f}\right)}\,\mathrm{C}\left(\frac{\sqrt{2}(2\,fx+i\,b\log(f))\sqrt{\frac{f}{\pi}}}{2\,f}\right)-\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2+4\,af\log(f)+4i\,df}{4\,f}\right)}\,\mathrm{C}\left(-\frac{\sqrt{2}(2\,fx\cdots\right.}{\cdots}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+d),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*
f)/f)*fresnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) - sqrt(2)
*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel
_cos(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f) - I*sqrt(2)*pi*sqrt(f
/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel_sin(1/2*
sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) - I*sqrt(2)*pi*sqrt(f/pi)*e^(1/
4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_sin(-1/2*sqrt(2)*(2
*f*x - I*b*log(f))*sqrt(f/pi)/f))/f
```

3.110.6 Sympy [F]

$$\int f^{a+bx}\cos\left(d+fx^2\right)\,dx=\int f^{a+bx}\cos\left(d+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cos(f*x**2+d),x)
```

output
```
Integral(f**(a + b*x)*cos(d + f*x**2), x)
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04

$$\int f^{a+bx}\cos\left(d+fx^2\right)\,dx = -\frac{\sqrt{2}\sqrt{\pi}\left(\left((i-1)\ f^a\cos\left(\frac{b^2\log(f)^2+4\,df}{4\,f}\right)+(i+1)\ f^a\sin\left(\frac{b^2\log(f)^2+4\,df}{4\,f}\right)\right)\operatorname{erf}\left(\frac{2i\,fx-b\log(f)}{2\,\sqrt{i\,f}}\right)+\left((i+1)\ f^a \cdots\right.\right.}{8\,\sqrt{f}}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+d),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*sqrt(pi)*(((I - 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*f)/f) + (I
 + 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1/2*(2*I*f*x - b*log(f))/
sqrt(I*f)) + ((I + 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*f)/f) + (I - 1)*f^a*
sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1/2*(2*I*f*x + b*log(f))/sqrt(-I*f)
))/sqrt(f)
```

3.110.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(98) = 196$.

Time = 0.32 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.11

$$\int f^{a+bx}\cos\left(d+fx^2\right)\,dx$$
$$=\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{8}i\sqrt{2}\left(4x-\frac{\pi b\mathrm{sgn}(f)-\pi b+2ib\log(|f|)}{f}\right)\left(\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(\frac{i\pi^2b^2\mathrm{sgn}(f)}{8f}+\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4f}-\frac{i\pi^2b^2}{8f}-\frac{\pi b^2\log(|f|)}{4f}\right.}}{4\left(\frac{if}{|f|}+1\right)\sqrt{|f|}}$$
$$-\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{8}i\sqrt{2}\left(4x+\frac{\pi b\mathrm{sgn}(f)-\pi b+2ib\log(|f|)}{f}\right)\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(-\frac{i\pi^2b^2\mathrm{sgn}(f)}{8f}-\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4f}+\frac{i\pi^2b^2}{8f}+\frac{\pi b^2\log}{4}\right.}}{4\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+d),x, algorithm="giac")
```

output
```
1/4*I*sqrt(2)*sqrt(pi)*erf(-1/8*I*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I
*b*log(abs(f)))/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(f)
/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(a
bs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*
log(abs(f)) + I*d)/((I*f/abs(f) + 1)*sqrt(abs(f))) - 1/4*I*sqrt(2)*sqrt(pi
)*erf(1/8*I*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(-I
*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*log(
abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I*b^2
*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) - I*d)/(
(-I*f/abs(f) + 1)*sqrt(abs(f)))
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cos\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\,\cos\left(f\,x^2+d\right)\,dx$$

input
```
int(f^(a + b*x)*cos(d + f*x^2),x)
```

output
```
int(f^(a + b*x)*cos(d + f*x^2), x)
```

3.111 $\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx$

3.111.1 Optimal result

Integrand size = 18, antiderivative size = 157

$$\begin{aligned}\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx &= \left(-\frac{1}{16}-\frac{i}{16}\right)e^{2id+\frac{ib^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx+b\log(f))}{\sqrt{f}}\right)\\ &\quad-\left(\frac{1}{16}+\frac{i}{16}\right)e^{-\frac{1}{8}i\left(16d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx-b\log(f))}{\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}\end{aligned}$$

output
```
1/2*f^(b*x+a)/b/ln(f)-(1/16+1/16*I)*exp(2*I*d+1/8*I*b^2*ln(f)^2/f)*f^(-1/2
+a)*erf((1/4+1/4*I)*(4*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-(1/16+1/16*I)*f^(-
1/2+a)*erfi((1/4+1/4*I)*(4*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)/exp(1/8*I*(16*
d+b^2*ln(f)^2/f))
```

3.111.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01

$$\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx$$
$$=\frac{1}{16}f^a\left(\frac{8f^{bx}}{b\log(f)}+\frac{(1-i)e^{-\frac{ib^2\log^2(f)}{8f}}\sqrt{\pi}\text{erf}\left(\frac{(4+4i)fx-(1-i)b\log(f)}{4\sqrt{f}}\right)(\cos(d)-i\sin(d))^2}{\sqrt{f}}\right.$$
$$\left.+\frac{(1+i)e^{\frac{ib^2\log^2(f)}{8f}}\sqrt{\pi}\text{erfi}\left(\frac{(4+4i)fx+(1-i)b\log(f)}{4\sqrt{f}}\right)(-i\cos(2d)+\sin(2d))}{\sqrt{f}}\right)$$

input
```
Integrate[f^(a + b*x)*Cos[d + f*x^2]^2,x]
```

output
```
(f^a*((8*f^(b*x))/(b*Log[f]) + ((1 - I)*Sqrt[Pi]*Erf[((4 + 4*I)*f*x - (1 -
 I)*b*Log[f])/(4*Sqrt[f])]*(Cos[d] - I*Sin[d])^2)/(E^(((I/8)*b^2*Log[f]^2)
/f)*Sqrt[f]) + ((1 + I)*E^(((I/8)*b^2*Log[f]^2)/f)*Sqrt[Pi]*Erfi[((4 + 4*I
)*f*x + (1 - I)*b*Log[f])/(4*Sqrt[f])]*((-I)*Cos[2*d] + Sin[2*d]))/Sqrt[f]
))/16
```

3.111.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx$$
$$\Big\downarrow \text{4976}$$
$$\int \left(\frac{1}{4}e^{-2id-2ifx^2}f^{a+bx}+\frac{1}{4}e^{2id+2ifx^2}f^{a+bx}+\frac{1}{2}f^{a+bx}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\left(-\frac{1}{16}-\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{8f}+2id}\operatorname{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+4ifx)}{\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{8}i\left(\frac{b^2\log^2(f)}{f}+16d\right)}\operatorname{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-b\log(f)+4ifx)}{\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

input
```
Int[f^(a + b*x)*Cos[d + f*x^2]^2,x]
```

output
```
(-1/16 - I/16)*E^((2*I)*d + ((I/8)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*
Erf[((1/4 + I/4)*((4*I)*f*x + b*Log[f]))/Sqrt[f]] - ((1/16 + I/16)*f^(-1/2
 + a)*Sqrt[Pi]*Erfi[((1/4 + I/4)*((4*I)*f*x - b*Log[f]))/Sqrt[f]])/E^((I/8
)*(16*d + (b^2*Log[f]^2)/f)) + f^(a + b*x)/(2*b*Log[f])
```

3.111.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.111.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{-\frac{i\left(\ln(f)^{2} b^{2}+16 d f\right)}{8 f}} \sqrt{2}\, \operatorname{erf}\left(-\sqrt{2}\, \sqrt{i f}\, x+\frac{b \ln(f) \sqrt{2}}{4 \sqrt{i f}}\right)}{16 \sqrt{i f}}-\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{\frac{i\left(\ln(f)^{2} b^{2}+16 d f\right)}{8 f}} \operatorname{erf}\left(-\sqrt{-2 i f}\, x+\frac{b \ln(f)}{2 \sqrt{-2 i f}}\right)}{8 \sqrt{-2 i f}}+\frac{f^{x b+a}}{2 b \ln(f)}$

input
```
int(f^(b*x+a)*cos(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16*Pi^(1/2)*f^a*exp(-1/8*I*(ln(f)^2*b^2+16*d*f)/f)*2^(1/2)/(I*f)^(1/2)*
erf(-2^(1/2)*(I*f)^(1/2)*x+1/4*b*ln(f)*2^(1/2)/(I*f)^(1/2))-1/8*Pi^(1/2)*f
^a*exp(1/8*I*(ln(f)^2*b^2+16*d*f)/f)/(-2*I*f)^(1/2)*erf(-(-2*I*f)^(1/2)*x+
1/2*b*ln(f)/(-2*I*f)^(1/2))+1/2*f^(b*x+a)/b/ln(f)
```

3.111.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(103) = 206$.

Time = 0.24 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.72

$$\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx$$
$$= \frac{\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log(f)^2+8\,af\log(f)-16i\,df}{8\,f}\right)}\operatorname{C}\left(\frac{(4\,fx+i\,b\log(f))\sqrt{\frac{f}{\pi}}}{2\,f}\right)\log(f)-\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2+8\,af\log(f)+16i\,df}{8\,f}\right)}\operatorname{C}\left(-\frac{(4\,fx-\cdots}{}\right.}{}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+d)^2,x, algorithm="fricas")
```

output
```
1/8*(pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 8*a*f*log(f) - 16*I*d*f)/f)
*fresnel_cos(1/2*(4*f*x + I*b*log(f))*sqrt(f/pi)/f)*log(f) - pi*b*sqrt(f/p
i)*e^(1/8*(I*b^2*log(f)^2 + 8*a*f*log(f) + 16*I*d*f)/f)*fresnel_cos(-1/2*(
4*f*x - I*b*log(f))*sqrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^
2*log(f)^2 + 8*a*f*log(f) - 16*I*d*f)/f)*fresnel_sin(1/2*(4*f*x + I*b*log(
f))*sqrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 + 8*a*
f*log(f) + 16*I*d*f)/f)*fresnel_sin(-1/2*(4*f*x - I*b*log(f))*sqrt(f/pi)/f
)*log(f) + 4*f*f^(b*x + a))/(b*f*log(f))
```

3.111.6 Sympy [F]

$$\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx=\int f^{a+bx}\cos^2\left(d+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cos(f*x**2+d)**2,x)
```

output
```
Integral(f**(a + b*x)*cos(d + f*x**2)**2, x)
```

3.111.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.18

$$\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx = -\frac{4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left((i-1)\,bf^a\cos\left(\frac{b^2\log(f)^2+16\,df}{8\,f}\right)\log(f)+(i+1)\,bf^a\log(f)\sin\left(\frac{b^2\log(f)^2+16\,df}{8\,f}\right)\right)\operatorname{erf}\left(\frac{4i\,fx-b}{2\sqrt{2}}\right.}{}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*b*f^a*cos(1/8*(b^2*log(f)^2 + 16
*d*f)/f)*log(f) + (I + 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 + 16*d*f)/f))
*erf(1/2*(4*I*f*x - b*log(f))/sqrt(2*I*f)) + ((I + 1)*b*f^a*cos(1/8*(b^2*l
og(f)^2 + 16*d*f)/f)*log(f) + (I - 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 +
 16*d*f)/f))*erf(1/2*(4*I*f*x + b*log(f))/sqrt(-2*I*f)))*f^(3/2) - 16*f^(b
*x)*f^(a + 2))/(b*f^2*log(f))
```

3.111.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(103) = 206.

Time = 0.34 (sec) , antiderivative size = 521, normalized size of antiderivative = 3.32

$$\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+d)^2,x, algorithm="giac")
```

output
```
(2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log
(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) - (pi*b*sgn(f) - p
i*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*
b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(f)) + a*log(ab
s(f))) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f) -
1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) - I*e^(-1/2*I*p
i*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*s
gn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(abs(f))) +
 1/8*I*sqrt(pi)*erf(-1/8*I*sqrt(f)*(8*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(
abs(f)))/f)*(I*f/abs(f) + 1))*e^(1/16*I*pi^2*b^2*sgn(f)/f + 1/8*pi*b^2*log
(abs(f))*sgn(f)/f - 1/16*I*pi^2*b^2/f - 1/8*pi*b^2*log(abs(f))/f + 1/8*I*b
^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) + 2*I*
d)/(sqrt(f)*(I*f/abs(f) + 1)) - 1/8*I*sqrt(pi)*erf(1/8*I*sqrt(f)*(8*x + (p
i*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(-I*f/abs(f) + 1))*e^(-1/16*I*pi
^2*b^2*sgn(f)/f - 1/8*pi*b^2*log(abs(f))*sgn(f)/f + 1/16*I*pi^2*b^2/f + 1/
8*pi*b^2*log(abs(f))/f - 1/8*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1
/2*I*pi*a + a*log(abs(f)) - 2*I*d)/(sqrt(f)*(-I*f/abs(f) + 1))
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\cos\left(f\,x^2+d\right)^2\,dx$$

input
```
int(f^(a + b*x)*cos(d + f*x^2)^2,x)
```

output
```
int(f^(a + b*x)*cos(d + f*x^2)^2, x)
```

3.112 $\int f^{a+bx}\cos^3(d+fx^2)\,dx$

3.112.1 Optimal result

Integrand size = 18, antiderivative size = 298

$$\begin{aligned}\int f^{a+bx}\cos^3(d+fx^2)\,dx &= -\frac{3}{16}\sqrt[4]{-1}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(2ifx+b\log(f))}{2\sqrt{f}}\right)\\ &\quad-\left(\frac{1}{16}+\frac{i}{16}\right)e^{3id+\frac{ib^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(6ifx+b\log(f))}{\sqrt{6}\sqrt{f}}\right)\\ &\quad-\frac{3}{16}\sqrt[4]{-1}e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(2ifx-b\log(f))}{2\sqrt{f}}\right)\\ &\quad-\left(\frac{1}{16}+\frac{i}{16}\right)e^{-\frac{1}{12}i\left(36d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(6ifx-b\log(f))}{\sqrt{6}\sqrt{f}}\right)\end{aligned}$$

output
```
(-1/96-1/96*I)*exp(3*I*d+1/12*I*b^2*ln(f)^2/f)*f^(-1/2+a)*erf((1/12+1/12*I
)*(6*I*f*x+b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)-(1/96+1/96*I)*f^(-1/
2+a)*erfi((1/12+1/12*I)*(6*I*f*x-b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2
)/exp(1/12*I*(36*d+b^2*ln(f)^2/f))-3/16*(-1)^(1/4)*exp(1/4*I*(4*d+b^2*ln(f
)^2/f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4)*(2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-
3/16*(-1)^(1/4)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(2*I*f*x-b*ln(f))/f^(1/2))*
Pi^(1/2)/exp(1/4*I*(4*d+b^2*ln(f)^2/f))
```

3.112.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.90

$$\int f^{a+bx}\cos^3\left(d+fx^2\right)\,dx$$
$$=\frac{1}{48}\sqrt[4]{-1}e^{-\frac{ib^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(-9\text{erfi}\left(\frac{(-1)^{3/4}(2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d))\right.$$
$$+9e^{\frac{ib^2\log^2(f)}{2f}}\text{erfi}\left(\frac{\sqrt[4]{-1}(2fx-ib\log(f))}{2\sqrt{f}}\right)(-i\cos(d)+\sin(d))$$
$$+\sqrt{3}e^{\frac{ib^2\log^2(f)}{6f}}\left(-\text{erfi}\left(\frac{(-1)^{3/4}(6fx+ib\log(f))}{2\sqrt{3}\sqrt{f}}\right)(\cos(3d)-i\sin(3d))+e^{\frac{ib^2\log^2(f)}{6f}}\text{erfi}\left(\frac{(6+6i)fx+(1-\right.\right.$$

input
```
Integrate[f^(a + b*x)*Cos[d + f*x^2]^3,x]
```

output
```
((-1)^(1/4)*f^(-1/2 + a)*Sqrt[Pi]*(-9*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f]
))/(2*Sqrt[f])]*(Cos[d] - I*Sin[d]) + 9*E^(((I/2)*b^2*Log[f]^2)/f)*Erfi[((
-1)^(1/4)*(2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*((-I)*Cos[d] + Sin[d]) + Sqrt
[3]*E^(((I/6)*b^2*Log[f]^2)/f)*(-(Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f]))/(
2*Sqrt[3]*Sqrt[f])]*(Cos[3*d] - I*Sin[3*d])) + E^(((I/6)*b^2*Log[f]^2)/f)*
Erfi[((6 + 6*I)*f*x + (1 - I)*b*Log[f])/(2*Sqrt[6]*Sqrt[f])]*((-I)*Cos[3*d
] + Sin[3*d]))))/(48*E^(((I/4)*b^2*Log[f]^2)/f))
```

3.112.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\cos^3\left(d+fx^2\right)\,dx$$
$$\Big\downarrow \text{4976}$$
$$\int \left(\frac{3}{8}e^{-id-ifx^2}f^{a+bx}+\frac{3}{8}e^{id+ifx^2}f^{a+bx}+\frac{1}{8}e^{-3id-3ifx^2}f^{a+bx}+\frac{1}{8}e^{3id+3ifx^2}f^{a+bx}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{3}{16}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+2ifx)}{2\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{12f}+3id}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\log(f)+6ifx)}{\sqrt{6}\sqrt{f}}\right)-\frac{3}{16}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+2ifx)}{2\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{-\frac{1}{12}i\left(\frac{b^2\log^2(f)}{f}+36d\right)}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-b\log(f)+6ifx)}{\sqrt{6}\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Cos[d + f*x^2]^3,x]
```

output
```
(-3*(-1)^(1/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Er
f[((-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 - (1/16 + I/16)*E^(
(3*I)*d + ((I/12)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6]*Erf[((1/2 + I/2
)*((6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (3*(-1)^(1/4)*f^(-1/2 + a)*
Sqrt[Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(16*E^((I/
4)*(4*d + (b^2*Log[f]^2)/f))) - ((1/16 + I/16)*f^(-1/2 + a)*Sqrt[Pi/6]*Erf
i[((1/2 + I/2)*((6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])])/E^((I/12)*(36*d
 + (b^2*Log[f]^2)/f))
```

3.112.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.112.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+36 d f\right)}{12 f}} \sqrt{3}\, \operatorname{erf}\left(-\sqrt{3}\, \sqrt{i f}\, x+\frac{\ln(f) b \sqrt{3}}{6 \sqrt{i f}}\right)}{48 \sqrt{i f}}-\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+4 d f\right)}{4 f}} \operatorname{erf}\left(-\sqrt{i f}\, x+\frac{\ln(f) b}{2 \sqrt{i f}}\right)}{16 \sqrt{i f}}-\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{i\left(\ln(f\right.}{}}}{}$

input
```
int(f^(b*x+a)*cos(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/48*Pi^(1/2)*f^a*exp(-1/12*I*(ln(f)^2*b^2+36*d*f)/f)*3^(1/2)/(I*f)^(1/2)
*erf(-3^(1/2)*(I*f)^(1/2)*x+1/6*ln(f)*b*3^(1/2)/(I*f)^(1/2))-3/16*Pi^(1/2)
*f^a*exp(-1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*
ln(f)*b/(I*f)^(1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(-
I*f)^(1/2)*erf(-(-I*f)^(1/2)*x+1/2*ln(f)*b/(-I*f)^(1/2))-1/16*Pi^(1/2)*f^a
*exp(1/12*I*(ln(f)^2*b^2+36*d*f)/f)/(-3*I*f)^(1/2)*erf(-(-3*I*f)^(1/2)*x+1
/2*ln(f)*b/(-3*I*f)^(1/2))
```

3.112.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(196) = 392.

Time = 0.27 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.76

$$\int f^{a+bx}\cos^{3}\left(d+fx^{2}\right)\,dx = \frac{\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^{2}\log(f)^{2}+12\,af\log(f)-36i\,df}{12\,f}\right)}\mathrm{C}\left(\frac{\sqrt{6}(6\,fx+i\,b\log(f))\sqrt{\frac{f}{\pi}}}{6\,f}\right)-\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^{2}\log(f)^{2}+12\,af\log(f)+36i\,df}{12\,f}\right)}\mathrm{C}\left(-\frac{\sqrt{6}(6}{}\right.}{}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+d)^3,x, algorithm="fricas")
```

output
```
1/48*(sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 12*a*f*log(f) - 36*
I*d*f)/f)*fresnel_cos(1/6*sqrt(6)*(6*f*x + I*b*log(f))*sqrt(f/pi)/f) - sqr
t(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 + 12*a*f*log(f) + 36*I*d*f)/f)*
fresnel_cos(-1/6*sqrt(6)*(6*f*x - I*b*log(f))*sqrt(f/pi)/f) + 9*sqrt(2)*pi
*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel_c
os(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) - 9*sqrt(2)*pi*sqrt(f/pi
)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_cos(-1/2*sqr
t(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f) - I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*
(-I*b^2*log(f)^2 + 12*a*f*log(f) - 36*I*d*f)/f)*fresnel_sin(1/6*sqrt(6)*(6
*f*x + I*b*log(f))*sqrt(f/pi)/f) - I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*
log(f)^2 + 12*a*f*log(f) + 36*I*d*f)/f)*fresnel_sin(-1/6*sqrt(6)*(6*f*x -
I*b*log(f))*sqrt(f/pi)/f) - 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f
)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I*b*log(
f))*sqrt(f/pi)/f) - 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a
*f*log(f) + 4*I*d*f)/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqrt
(f/pi)/f))/f
```

3.112.6 Sympy [F]

$$\int f^{a+bx}\cos^3\left(d+fx^2\right)\,dx=\int f^{a+bx}\cos^3\left(d+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cos(f*x**2+d)**3,x)
```

output
```
Integral(f**(a + b*x)*cos(d + f*x**2)**3, x)
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.01

$$\int f^{a+bx}\cos^3\left(d+fx^2\right)\,dx=$$
$$-\frac{9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left((i-1)\,f^a\cos\left(\frac{b^2\log(f)^2+36\,df}{12\,f}\right)+(i+1)\,f^a\sin\left(\frac{b^2\log(f)^2+36\,df}{12\,f}\right)\right)\operatorname{erf}\left(\frac{6i\,fx-b\log(f)}{2\sqrt{3i\,f}}\right)+\left((i+1\right.}{}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/96*(9^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*f^a*cos(1/12*(b^2*log(f)^2 + 36*
d*f)/f) + (I + 1)*f^a*sin(1/12*(b^2*log(f)^2 + 36*d*f)/f))*erf(1/2*(6*I*f*
x - b*log(f))/sqrt(3*I*f)) + ((I + 1)*f^a*cos(1/12*(b^2*log(f)^2 + 36*d*f)
/f) + (I - 1)*f^a*sin(1/12*(b^2*log(f)^2 + 36*d*f)/f))*erf(1/2*(6*I*f*x +
b*log(f))/sqrt(-3*I*f)))*f^(3/2) - 9*sqrt(2)*sqrt(pi)*((-(I - 1)*f^a*cos(1
/4*(b^2*log(f)^2 + 4*d*f)/f) - (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/
f))*erf(1/2*(2*I*f*x - b*log(f))/sqrt(I*f)) + (-(I + 1)*f^a*cos(1/4*(b^2*l
og(f)^2 + 4*d*f)/f) - (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1
/2*(2*I*f*x + b*log(f))/sqrt(-I*f)))*f^(3/2))/f^2
```

3.112.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(196) = 392.

Time = 0.38 (sec) , antiderivative size = 595, normalized size of antiderivative = 2.00

$$\int f^{a+bx} \cos^3\left(d+fx^2\right) dx = \text{Too large to display}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+d)^3,x, algorithm="giac")
```

output
```
3/16*I*sqrt(2)*sqrt(pi)*erf(-1/8*I*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*
I*b*log(abs(f)))/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(f
)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(
abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a
*log(abs(f)) + I*d)/((I*f/abs(f) + 1)*sqrt(abs(f))) + 1/48*I*sqrt(6)*sqrt(
pi)*erf(-1/24*I*sqrt(6)*sqrt(f)*(12*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(ab
s(f)))/f)*(I*f/abs(f) + 1))*e^(1/24*I*pi^2*b^2*sgn(f)/f + 1/12*pi*b^2*log(
abs(f))*sgn(f)/f - 1/24*I*pi^2*b^2/f - 1/12*pi*b^2*log(abs(f))/f + 1/12*I*
b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) + 3*I
*d)/(sqrt(f)*(I*f/abs(f) + 1)) - 1/48*I*sqrt(6)*sqrt(pi)*erf(1/24*I*sqrt(6
)*sqrt(f)*(12*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(-I*f/abs(f)
 + 1))*e^(-1/24*I*pi^2*b^2*sgn(f)/f - 1/12*pi*b^2*log(abs(f))*sgn(f)/f + 1
/24*I*pi^2*b^2/f + 1/12*pi*b^2*log(abs(f))/f - 1/12*I*b^2*log(abs(f))^2/f
- 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) - 3*I*d)/(sqrt(f)*(-I*f/a
bs(f) + 1)) - 3/16*I*sqrt(2)*sqrt(pi)*erf(1/8*I*sqrt(2)*(4*x + (pi*b*sgn(f
) - pi*b + 2*I*b*log(abs(f)))/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I
*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f +
1/4*pi*b^2*log(abs(f))/f - 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) +
 1/2*I*pi*a + a*log(abs(f)) - I*d)/((-I*f/abs(f) + 1)*sqrt(abs(f)))
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cos^3\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\cos\left(f\,x^2+d\right)^3\,dx$$

input
```
int(f^(a + b*x)*cos(d + f*x^2)^3,x)
```

output
```
int(f^(a + b*x)*cos(d + f*x^2)^3, x)
```

3.113 $\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx$

3.113.1 Optimal result

Integrand size = 19, antiderivative size = 162

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx = -\frac{1}{4}\sqrt[4]{-1}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(ie+2ifx+b\log(f))}{2\sqrt{f}}\right) - \frac{1}{4}\sqrt[4]{-1}e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(ie+2ifx-b\log(f))}{2\sqrt{f}}\right)$$

output
```
-1/4*(-1)^(1/4)*exp(1/4*I*(4*d+(I*e+b*ln(f))^2/f))*f^(-1/2+a)*erf(1/2*(-1)
^(1/4)*(I*e+2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-1/4*(-1)^(1/4)*exp(-I*d+1/4
*I*(e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(I*e+2*I*f*x-b*ln(f))
/f^(1/2))*Pi^(1/2)
```

3.113.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx = \frac{1}{4}\sqrt[4]{-1}e^{-\frac{i\left(e^2+b^2\log^2(f)\right)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(-e^{\frac{ie^2}{2f}}\operatorname{erf}\left(\frac{(-1)^{3/4}(e+2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d)) + e^{\frac{ib^2\log^2(f)}{2f}}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(e+2fx-ib\log(f))}{2\sqrt{f}}\right)(-i\cos(d)+\sin(d))\right)$$

input
```
Integrate[f^(a + b*x)*Cos[d + e*x + f*x^2],x]
```

output
```
((-1)^(1/4)*f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(-(E^(((I/2)*e^2)/f)*Erfi[((-
1)^(3/4)*(e + 2*f*x + I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] - I*Sin[d])) + E^(
((I/2)*b^2*Log[f]^2)/f)*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f]))/(2*Sqrt
[f])]*((-I)*Cos[d] + Sin[d])))/(4*E^(((I/4)*(e^2 + b^2*Log[f]^2))/f))
```

3.113.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{1}{2}f^{a+bx}e^{-id-iex-ifx^2}+\frac{1}{2}f^{a+bx}e^{id+iex+ifx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\text{erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(e+ib\log(f))^2}{4f}-id}\text{erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Cos[d + e*x + f*x^2],x]
```

output
```
-1/4*((-1)^(1/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[
Pi]*Erf[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])]) - ((-1)^(1
/4)*E^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(
(-1)^(1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/4
```

3.113.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.113.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+4df-e^{2}\right)}{4f}} \operatorname{erf}\left(-\sqrt{if}\, x+\frac{b\ln(f)-ie}{2\sqrt{if}}\right)}{4\sqrt{if}}-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+4df-e^{2}\right)}{4f}} \operatorname{erf}\left(-\sqrt{-if}\, x+\frac{ie+b\ln(f)}{2\sqrt{-if}}\right)}{4\sqrt{-if}}$

input
```
int(f^(b*x+a)*cos(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(-1/4*I*(ln(f)^2*b^2+4*d*f-e^2)/f)/(I*
f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*(b*ln(f)-I*e)/(I*f)^(1/2))-1/4*Pi^(1/2)*f^
a*f^(-1/2/f*b*e)*exp(1/4*I*(ln(f)^2*b^2+4*d*f-e^2)/f)/(-I*f)^(1/2)*erf(-(-
I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-I*f)^(1/2))
```

3.113.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(109) = 218.

Time = 0.25 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.93

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx$$
$$= \frac{\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^{2}\log(f)^{2}+i\,e^{2}-4i\,df-2\,(be-2\,af)\log(f)}{4\,f}\right)}\,\mathrm{C}\left(\frac{\sqrt{2}(2\,fx+i\,b\log(f)+e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)-\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^{2}\log(f)^{2}-i\,e^{2}+4i\,df-2\,(be-2\,af)}{4\,f}\right.}}{}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + I*e^2 - 4*I*d*f - 2*(
b*e - 2*a*f)*log(f))/f)*fresnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f) + e)*s
qrt(f/pi)/f) - sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 - I*e^2 + 4*I*
d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/2*sqrt(2)*(2*f*x - I*b*log
(f) + e)*sqrt(f/pi)/f) - I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 +
 I*e^2 - 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*sqrt(2)*(2*f
*x + I*b*log(f) + e)*sqrt(f/pi)/f) - I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2
*log(f)^2 - I*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/2*
sqrt(2)*(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f))/f
```

3.113.6 Sympy [F]

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\cos\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cos(f*x**2+e*x+d),x)
```

output
```
Integral(f**(a + b*x)*cos(d + e*x + f*x**2), x)
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.17

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx=$$
$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(-(i-1)\,f^a\cos\left(\frac{b^2\log(f)^2-e^2+4\,df}{4\,f}\right)-(i+1)\,f^a\sin\left(\frac{b^2\log(f)^2-e^2+4\,df}{4\,f}\right)\right)\operatorname{erf}\left(\frac{i\,(2i\,fx-b\log(f)+i\,e)\sqrt{i\,f}}{2\,f}\right)\cdots}{8\sqrt{f}\cdots}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*sqrt(pi)*((-(I - 1)*f^a*cos(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/
f) - (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f))*erf(1/2*I*(2*I*f
*x - b*log(f) + I*e)*sqrt(I*f)/f) + ((I + 1)*f^a*cos(1/4*(b^2*log(f)^2 - e
^2 + 4*d*f)/f) + (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f))*erf(
1/2*I*(2*I*f*x + b*log(f) + I*e)*sqrt(-I*f)/f))/(sqrt(f)*f^(1/2*b*e/f))
```

3.113.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(109) = 218.

Time = 0.34 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.33

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx$$
$$= \frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{8}i\sqrt{2}\left(4x-\frac{\pi b\mathrm{sgn}(f)-\pi b+2ib\log(|f|)-2e}{f}\right)\left(\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(\frac{i\pi^2b^2\mathrm{sgn}(f)}{8f}+\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4f}-\frac{i\pi^2b^2}{8f}-\frac{\pi b^2\log\ldots}{4f}\ldots\right.}}{4\left(\frac{if}{|f|}+1\right)\sqrt{|f|}}$$
$$-\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{8}i\sqrt{2}\left(4x+\frac{\pi b\mathrm{sgn}(f)-\pi b+2ib\log(|f|)+2e}{f}\right)\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(-\frac{i\pi^2b^2\mathrm{sgn}(f)}{8f}-\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4f}+\frac{i\pi^2b^2}{8f}+\ldots\right.}}{4\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/4*I*sqrt(2)*sqrt(pi)*erf(-1/8*I*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I
*b*log(abs(f)) - 2*e)/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*
sgn(f)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2
*log(abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*
b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(a
bs(f))/f + I*d - 1/4*I*e^2/f)/((I*f/abs(f) + 1)*sqrt(abs(f))) - 1/4*I*sqrt
(2)*sqrt(pi)*erf(1/8*I*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(
f)) + 2*e)/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f
- 1/4*pi*b^2*log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(
f))/f - 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f
)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f
- I*d + 1/4*I*e^2/f)/((-I*f/abs(f) + 1)*sqrt(abs(f)))
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx = \int f^{a+b\,x}\,\cos\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int(f^(a + b*x)*cos(d + e*x + f*x^2),x)
```

output
```
int(f^(a + b*x)*cos(d + e*x + f*x^2), x)
```

3.113. $\int f^{a+bx} \cos\left(d + ex + fx^2\right)\, dx$

3.114 $\int f^{a+bx} \cos^2(d+ex+fx^2)\, dx$

3.114.1 Optimal result

Integrand size = 21, antiderivative size = 179

$$\begin{aligned}\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx &= \left(-\frac{1}{16}-\frac{i}{16}\right)e^{2id+\frac{i(2ie+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2ie+4ifx+b\log(f))}{\sqrt{f}}\right)\\ &\quad-\left(\frac{1}{16}+\frac{i}{16}\right)e^{-2id+\frac{i(2e+ib\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2ie+4ifx-b\log(f))}{\sqrt{f}}\right)\\ &\quad+\frac{f^{a+bx}}{2b\log(f)}\end{aligned}$$

output
```
1/2*f^(b*x+a)/b/ln(f)-(1/16+1/16*I)*exp(2*I*d+1/8*I*(2*I*e+b*ln(f))^2/f)*f
^(-1/2+a)*erf((1/4+1/4*I)*(2*I*e+4*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-(1/16+
1/16*I)*exp(-2*I*d+1/8*I*(2*e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi((1/4+1/4*I)*
(2*I*e+4*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)
```

3.114.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.37

$$\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx = \frac{e^{-\frac{i\left(4e^2+b^2\log^2(f)\right)}{8f}}f^{a-\frac{be+f}{2f}}\left(8e^{\frac{i\left(4e^2+b^2\log^2(f)\right)}{8f}}f^{\frac{1}{2}+b\left(\frac{e}{2f}+x\right)}+\sqrt[4]{-1}be^{\frac{ib^2\log^2(f)}{4f}}\sqrt{2\pi}\,\mathrm{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2e+4fx-ib\log(f))}{\sqrt{f}}\right)\log(f\cdots}{16b\log(f\cdots}$$

input
```
Integrate[f^(a + b*x)*Cos[d + e*x + f*x^2]^2,x]
```

output
```
(f^(a - (b*e + f)/(2*f))*(8*E^(((I/8)*(4*e^2 + b^2*Log[f]^2))/f)*f^(1/2 +
b*(e/(2*f) + x)) + (-1)^(1/4)*b*E^(((I/4)*b^2*Log[f]^2)/f)*Sqrt[2*Pi]*Erfi
[((1/4 + I/4)*(2*e + 4*f*x - I*b*Log[f]))/Sqrt[f]]*Log[f]*((-I)*Cos[2*d] +
 Sin[2*d]) - (-1)^(1/4)*b*E^((I*e^2)/f)*Sqrt[2*Pi]*Erf[((1/4 + I/4)*(2*e +
 4*f*x + I*b*Log[f]))/Sqrt[f]]*Log[f]*(I*Cos[2*d] + Sin[2*d])))/(16*b*E^((
(I/8)*(4*e^2 + b^2*Log[f]^2))/f)*Log[f])
```

3.114.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{4976}$$

$$\int\left(\frac{1}{4}f^{a+bx}e^{-2id-2iex-2ifx^2}+\frac{1}{4}f^{a+bx}e^{2id+2iex+2ifx^2}+\frac{1}{2}f^{a+bx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\left(-\frac{1}{16}-\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+2ie)^2}{8f}+2id}\mathrm{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+2ie+4ifx)}{\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(2e+ib\log(f))^2}{8f}-2id}\mathrm{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-b\log(f)+2ie+4ifx)}{\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

input
```
Int[f^(a + b*x)*Cos[d + e*x + f*x^2]^2,x]
```

output
```
(-1/16 - I/16)*E^((2*I)*d + ((I/8)*((2*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)
*Sqrt[Pi]*Erf[((1/4 + I/4)*((2*I)*e + (4*I)*f*x + b*Log[f]))/Sqrt[f]] - (1
/16 + I/16)*E^((-2*I)*d + ((I/8)*(2*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqr
t[Pi]*Erfi[((1/4 + I/4)*((2*I)*e + (4*I)*f*x - b*Log[f]))/Sqrt[f]] + f^(a
+ b*x)/(2*b*Log[f])
```

3.114.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.114.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+16df-4e^{2}\right)}{8f}} \sqrt{2}\, \mathrm{erf}\left(-\sqrt{2}\,\sqrt{if}\,x+\frac{(b\ln(f)-2ie)\sqrt{2}}{4\sqrt{if}}\right)}{16\sqrt{if}}-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+16df-4e^{2}\right)}{8f}} \mathrm{erf}\left(-\sqrt{-2if}\,x\right.}{8\sqrt{-2if}}$

input
```
int(f^(b*x+a)*cos(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(-1/8*I*(ln(f)^2*b^2+16*d*f-4*e^2)/f)
*2^(1/2)/(I*f)^(1/2)*erf(-2^(1/2)*(I*f)^(1/2)*x+1/4*(b*ln(f)-2*I*e)*2^(1/2
)/(I*f)^(1/2))-1/8*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(1/8*I*(ln(f)^2*b^2+16*d
*f-4*e^2)/f)/(-2*I*f)^(1/2)*erf(-(-2*I*f)^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-2*
I*f)^(1/2))+1/2*f^(b*x+a)/b/ln(f)
```

3.114.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(116) = 232.

Time = 0.26 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.82

$$\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx$$
$$= \frac{\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log(f)^2+4i\,e^2-16i\,df-4\,(be-2\,af)\log(f)}{8\,f}\right)}\mathrm{C}\left(\frac{(4\,fx+i\,b\log(f)+2\,e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)\log(f)-\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2-4i\,e^2+16i\,df-4\,(be}{8\,f}\right.}}{}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
1/8*(pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 4*I*e^2 - 16*I*d*f - 4*(b*e
 - 2*a*f)*log(f))/f)*fresnel_cos(1/2*(4*f*x + I*b*log(f) + 2*e)*sqrt(f/pi)
/f)*log(f) - pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 - 4*I*e^2 + 16*I*d*f -
 4*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/2*(4*f*x - I*b*log(f) + 2*e)*sq
rt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 4*I*e^2 -
 16*I*d*f - 4*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*(4*f*x + I*b*log(f)
 + 2*e)*sqrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 -
4*I*e^2 + 16*I*d*f - 4*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/2*(4*f*x -
I*b*log(f) + 2*e)*sqrt(f/pi)/f)*log(f) + 4*f*f^(b*x + a))/(b*f*log(f))
```

3.114.6 Sympy [F]

$$\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cos(f*x**2+e*x+d)**2,x)
```

output
```
Integral(f**(a + b*x)*cos(d + e*x + f*x**2)**2, x)
```

3.114.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(116) = 232.

Time = 0.32 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.34

$$\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(-(i-1)\,bf^a\cos\left(\frac{b^2\log(f)^2-4\,e^2+16\,df}{8\,f}\right)\log(f)-(i+1)\,bf^a\log(f)\sin\left(\frac{b^2\log(f)^2-4\,e^2+16\,df}{8\,f}\right)\right)e$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
-1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*((-(I - 1)*b*f^a*cos(1/8*(b^2*log(f)^2 - 4
*e^2 + 16*d*f)/f)*log(f) - (I + 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 - 4*
e^2 + 16*d*f)/f))*erf(1/4*I*(4*I*f*x - b*log(f) + 2*I*e)*sqrt(2*I*f)/f) +
((I + 1)*b*f^a*cos(1/8*(b^2*log(f)^2 - 4*e^2 + 16*d*f)/f)*log(f) + (I - 1)
*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 - 4*e^2 + 16*d*f)/f))*erf(1/4*I*(4*I*f
*x + b*log(f) + 2*I*e)*sqrt(-2*I*f)/f))*f^(3/2) - 16*f^(a + 2)*e^(b*x*log(
f) + 1/2*b*e*log(f)/f))/(b*f^2*f^(1/2*b*e/f)*log(f))
```

3.114.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(116) = 232.

Time = 0.37 (sec) , antiderivative size = 599, normalized size of antiderivative = 3.35

$$\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
(2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log
(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) - (pi*b*sgn(f) - p
i*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*
b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(f)) + a*log(ab
s(f))) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f) -
1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) - I*e^(-1/2*I*p
i*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*s
gn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(abs(f))) +
 1/8*I*sqrt(pi)*erf(-1/8*I*sqrt(f)*(8*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(
abs(f)) - 4*e)/f)*(I*f/abs(f) + 1))*e^(1/16*I*pi^2*b^2*sgn(f)/f + 1/8*pi*b
^2*log(abs(f))*sgn(f)/f - 1/16*I*pi^2*b^2/f - 1/8*pi*b^2*log(abs(f))/f + 1
/8*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2
*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f + 2*I*d -
 1/2*I*e^2/f)/(sqrt(f)*(I*f/abs(f) + 1)) - 1/8*I*sqrt(pi)*erf(1/8*I*sqrt(f
)*(8*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 4*e)/f)*(-I*f/abs(f) +
1))*e^(-1/16*I*pi^2*b^2*sgn(f)/f - 1/8*pi*b^2*log(abs(f))*sgn(f)/f + 1/16*
I*pi^2*b^2/f + 1/8*pi*b^2*log(abs(f))/f - 1/8*I*b^2*log(abs(f))^2/f - 1/2*
I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*lo
g(abs(f)) - 1/2*b*e*log(abs(f))/f - 2*I*d + 1/2*I*e^2/f)/(sqrt(f)*(-I*f/ab
s(f) + 1))
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx=\int f^{a+b\,x}\cos\left(f\,x^2+e\,x+d\right)^2\,dx$$

input
```
int(f^(a + b*x)*cos(d + e*x + f*x^2)^2,x)
```

output
```
int(f^(a + b*x)*cos(d + e*x + f*x^2)^2, x)
```

3.115 $\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx$

3.115.1 Optimal result

Integrand size = 21, antiderivative size = 340

$$\begin{aligned}
&\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx\\
&=-\frac{3}{16}\sqrt[4]{-1}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}(ie+2ifx+b\log(f))}{2\sqrt{f}}\right)\\
&\quad-\left(\frac{1}{16}+\frac{i}{16}\right)e^{3id+\frac{i(3ie+b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\,\mathrm{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(3ie+6ifx+b\log(f))}{\sqrt{6}\sqrt{f}}\right)\\
&\quad-\frac{3}{16}\sqrt[4]{-1}e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(ie+2ifx-b\log(f))}{2\sqrt{f}}\right)\\
&\quad-\left(\frac{1}{16}+\frac{i}{16}\right)e^{-3id+\frac{i(3e+ib\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\,\mathrm{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(3ie+6ifx-b\log(f))}{\sqrt{6}\sqrt{f}}\right)
\end{aligned}$$

output
```
(-1/96-1/96*I)*exp(3*I*d+1/12*I*(3*I*e+b*ln(f))^2/f)*f^(-1/2+a)*erf((1/12+
1/12*I)*(3*I*e+6*I*f*x+b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)-(1/96+1/
96*I)*exp(-3*I*d+1/12*I*(3*e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi((1/12+1/12*I)
*(3*I*e+6*I*f*x-b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)-3/16*(-1)^(1/4)
*exp(1/4*I*(4*d+(I*e+b*ln(f))^2/f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4)*(I*e+2*I
*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-3/16*(-1)^(1/4)*exp(-I*d+1/4*I*(e+I*b*ln(f
))^2/f)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(I*e+2*I*f*x-b*ln(f))/f^(1/2))*Pi^(
1/2)
```

3.115.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.95

$$\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{48}\sqrt[4]{-1}e^{-\frac{i\left(3e^2+b^2\log^2(f)\right)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(9e^{\frac{i\left(e^2+b^2\log^2(f)\right)}{2f}}\text{erfi}\left(\frac{\sqrt[4]{-1}(e+2fx-ib\log(f))}{2\sqrt{f}}\right)(-i\cos(d)+\sin(d))\right.$$
$$+e^{\frac{ie^2}{f}}\left(-9\text{erfi}\left(\frac{(-1)^{3/4}(e+2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d))-\sqrt{3}e^{\frac{i\left(3e^2+b^2\log^2(f)\right)}{6f}}\text{erfi}\left(\frac{(-1)^{3/4}(3e+6f}{2\sqrt{3}}\right.\right.$$

input
```
Integrate[f^(a + b*x)*Cos[d + e*x + f*x^2]^3,x]
```

output
```
((-1)^(1/4)*f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(9*E^(((I/2)*(e^2 + b^2*Log[f
]^2))/f)*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*((-I)*Cos
[d] + Sin[d]) + E^((I*e^2)/f)*(-9*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f]
))/(2*Sqrt[f])]*(Cos[d] - I*Sin[d]) - Sqrt[3]*E^(((I/6)*(3*e^2 + b^2*Log[f
]^2))/f)*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f]))/(2*Sqrt[3]*Sqrt[f])]
*(Cos[3*d] - I*Sin[3*d])) + Sqrt[3]*E^(((I/3)*b^2*Log[f]^2)/f)*Erfi[((1/2
+ I/2)*(3*e + 6*f*x - I*b*Log[f]))/(Sqrt[6]*Sqrt[f])]*((-I)*Cos[3*d] + Sin
[3*d])))/(48*E^(((I/4)*(3*e^2 + b^2*Log[f]^2))/f))
```

3.115.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx$$
$$\Big\downarrow \text{4976}$$
$$\int \left(\frac{3}{8}f^{a+bx}\exp\left(-3i\left(d+ex+fx^2\right)+2id+2iex+2ifx^2\right)+\frac{3}{8}f^{a+bx}\exp\left(-3i\left(d+ex+fx^2\right)+4id+4iex+4if\right.\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{3}{16}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+3ie)^2}{12f}+3id}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\log(f)+3ie+6ifx)}{\sqrt{6}\sqrt{f}}\right)-\frac{3}{16}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(e+ib\log(f))^2}{4f}-id}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{i(3e+ib\log(f))^2}{12f}-3id}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-b\log(f)+3ie+6ifx)}{\sqrt{6}\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Cos[d + e*x + f*x^2]^3,x]
```

output
```
(-3*(-1)^(1/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi
]*Erf[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 - (1/16 +
 I/16)*E^((3*I)*d + ((I/12)*((3*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[P
i/6]*Erf[((1/2 + I/2)*((3*I)*e + (6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])]
 - (3*(-1)^(1/4)*E^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sq
rt[Pi]*Erfi[((-1)^(1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/16 - (
1/16 + I/16)*E^((-3*I)*d + ((I/12)*(3*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*S
qrt[Pi/6]*Erfi[((1/2 + I/2)*((3*I)*e + (6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqr
t[f])]
```

3.115.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.115.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}}\, \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+36df-9e^{2}\right)}{12f}} \sqrt{3}\, \mathrm{erf}\left(-\sqrt{3}\,\sqrt{if}\,x+\frac{(b\ln(f)-3ie)\sqrt{3}}{6\sqrt{if}}\right)}{48\sqrt{if}}-\frac{3\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}}\, \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+4df-e^{2}\right)}{4f}}\, \mathrm{erf}\left(-\sqrt{if}\,x+\right.}{16\sqrt{if}}$

input
```
int(f^(b*x+a)*cos(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/48*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(-1/12*I*(ln(f)^2*b^2+36*d*f-9*e^2)/f
)*3^(1/2)/(I*f)^(1/2)*erf(-3^(1/2)*(I*f)^(1/2)*x+1/6*(b*ln(f)-3*I*e)*3^(1/
2)/(I*f)^(1/2))-3/16*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(-1/4*I*(ln(f)^2*b^2+4
*d*f-e^2)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*(b*ln(f)-I*e)/(I*f)^(1/2))
-3/16*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(1/4*I*(ln(f)^2*b^2+4*d*f-e^2)/f)/(-I
*f)^(1/2)*erf(-(-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-I*f)^(1/2))-1/16*Pi^(1/2
)*f^a*f^(-1/2/f*b*e)*exp(1/12*I*(ln(f)^2*b^2+36*d*f-9*e^2)/f)/(-3*I*f)^(1/
2)*erf(-(-3*I*f)^(1/2)*x+1/2*(3*I*e+b*ln(f))/(-3*I*f)^(1/2))
```

3.115.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(220) = 440.

Time = 0.27 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.85

$$\int f^{a+bx}\cos^{3}\left(d+ex+fx^{2}\right)\,dx$$
$$=\frac{\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^{2}\log(f)^{2}+9i\,e^{2}-36i\,df-6\,(be-2\,af)\log(f)}{12\,f}\right)}\mathrm{C}\left(\frac{\sqrt{6}(6\,fx+i\,b\log(f)+3\,e)\sqrt{\frac{f}{\pi}}}{6\,f}\right)-\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^{2}\log(f)^{2}-9i\,e^{2}+36i\,df-6\,(be}{12\,f}\right.}}{}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
1/48*(sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 9*I*e^2 - 36*I*d*f
- 6*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(1/6*sqrt(6)*(6*f*x + I*b*log(f) +
 3*e)*sqrt(f/pi)/f) - sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 - 9*I*
e^2 + 36*I*d*f - 6*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/6*sqrt(6)*(6*f*
x - I*b*log(f) + 3*e)*sqrt(f/pi)/f) + 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b
^2*log(f)^2 + I*e^2 - 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(1/2
*sqrt(2)*(2*f*x + I*b*log(f) + e)*sqrt(f/pi)/f) - 9*sqrt(2)*pi*sqrt(f/pi)*
e^(1/4*(I*b^2*log(f)^2 - I*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fres
nel_cos(-1/2*sqrt(2)*(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f) - I*sqrt(6)*pi
*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 9*I*e^2 - 36*I*d*f - 6*(b*e - 2*a*f
)*log(f))/f)*fresnel_sin(1/6*sqrt(6)*(6*f*x + I*b*log(f) + 3*e)*sqrt(f/pi)
/f) - I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 - 9*I*e^2 + 36*I*d*f
 - 6*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/6*sqrt(6)*(6*f*x - I*b*log(f)
 + 3*e)*sqrt(f/pi)/f) - 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2
+ I*e^2 - 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*sqrt(2)*(2*
f*x + I*b*log(f) + e)*sqrt(f/pi)/f) - 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*
b^2*log(f)^2 - I*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1
/2*sqrt(2)*(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f))/f
```

3.115.6 Sympy [F]

$$\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cos(f*x**2+e*x+d)**3,x)
```

output
```
Integral(f**(a + b*x)*cos(d + e*x + f*x**2)**3, x)
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.11

$$\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx=$$

$$-\frac{9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(-(i-1)\ f^a\cos\left(\frac{b^2\log(f)^2-9\,e^2+36\,df}{12\,f}\right)-(i+1)\ f^a\sin\left(\frac{b^2\log(f)^2-9\,e^2+36\,df}{12\,f}\right)\right)\mathrm{erf}\left(\frac{i\,(6i\,fx-b\log(f)}{6\,f}\right.}{}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
-1/96*(9^(1/4)*sqrt(2)*sqrt(pi)*((-(I - 1)*f^a*cos(1/12*(b^2*log(f)^2 - 9*
e^2 + 36*d*f)/f) - (I + 1)*f^a*sin(1/12*(b^2*log(f)^2 - 9*e^2 + 36*d*f)/f)
)*erf(1/6*I*(6*I*f*x - b*log(f) + 3*I*e)*sqrt(3*I*f)/f) + ((I + 1)*f^a*cos
(1/12*(b^2*log(f)^2 - 9*e^2 + 36*d*f)/f) + (I - 1)*f^a*sin(1/12*(b^2*log(f
)^2 - 9*e^2 + 36*d*f)/f))*erf(1/6*I*(6*I*f*x + b*log(f) + 3*I*e)*sqrt(-3*I
*f)/f))*f^(3/2) - 9*sqrt(2)*sqrt(pi)*(((I - 1)*f^a*cos(1/4*(b^2*log(f)^2 -
 e^2 + 4*d*f)/f) + (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f))*er
f(1/2*I*(2*I*f*x - b*log(f) + I*e)*sqrt(I*f)/f) + (-(I + 1)*f^a*cos(1/4*(b
^2*log(f)^2 - e^2 + 4*d*f)/f) - (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 - e^2 +
4*d*f)/f))*erf(1/2*I*(2*I*f*x + b*log(f) + I*e)*sqrt(-I*f)/f))*f^(3/2))/(f
^2*f^(1/2*b*e/f))
```

3.115.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 751 vs. 2(220) = 440.

Time = 0.49 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.21

$$\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
3/16*I*sqrt(2)*sqrt(pi)*erf(-1/8*I*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*
I*b*log(abs(f)) - 2*e)/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2
*sgn(f)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^
2*log(abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi
*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(
abs(f))/f + I*d - 1/4*I*e^2/f)/((I*f/abs(f) + 1)*sqrt(abs(f))) + 1/48*I*sq
rt(6)*sqrt(pi)*erf(-1/24*I*sqrt(6)*sqrt(f)*(12*x - (pi*b*sgn(f) - pi*b + 2
*I*b*log(abs(f)) - 6*e)/f)*(I*f/abs(f) + 1))*e^(1/24*I*pi^2*b^2*sgn(f)/f +
 1/12*pi*b^2*log(abs(f))*sgn(f)/f - 1/24*I*pi^2*b^2/f - 1/12*pi*b^2*log(ab
s(f))/f + 1/12*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sg
n(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))
/f + 3*I*d - 3/4*I*e^2/f)/(sqrt(f)*(I*f/abs(f) + 1)) - 1/48*I*sqrt(6)*sqrt
(pi)*erf(1/24*I*sqrt(6)*sqrt(f)*(12*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(ab
s(f)) + 6*e)/f)*(-I*f/abs(f) + 1))*e^(-1/24*I*pi^2*b^2*sgn(f)/f - 1/12*pi*
b^2*log(abs(f))*sgn(f)/f + 1/24*I*pi^2*b^2/f + 1/12*pi*b^2*log(abs(f))/f -
 1/12*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f +
1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f - 3*I*
d + 3/4*I*e^2/f)/(sqrt(f)*(-I*f/abs(f) + 1)) - 3/16*I*sqrt(2)*sqrt(pi)*erf
(1/8*I*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 2*e)/f)*(-
I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*...
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx} \cos^3\left(d+ex+fx^2\right)\,dx = \int f^{a+b\,x}\cos\left(f\,x^2+e\,x+d\right)^3\,dx$$

input
```
int(f^(a + b*x)*cos(d + e*x + f*x^2)^3,x)
```

output
```
int(f^(a + b*x)*cos(d + e*x + f*x^2)^3, x)
```

3.116 $\int f^{a+cx^2}\cos(d+ex)\,dx$

3.116.1 Optimal result

Integrand size = 16, antiderivative size = 147

$$\int f^{a+cx^2}\cos(d+ex)\,dx=-\frac{e^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/4*exp(-I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-I*e+2*c*x*ln(f))/c^(1/2)/ln(f
)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(I*d+1/4*e^2/c/ln(f))*f^a*erf
i(1/2*(I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

$$\int f^{a+cx^2}\cos(d+ex)\,dx=\frac{e^{\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(\text{erfi}\left(\frac{-ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)-i\sin(d))+\text{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + c*x^2)*Cos[d + e*x],x]
```

output
```
(E^(e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*(Erfi[((-I)*e + 2*c*x*Log[f])/(2*Sqrt[c
]*Sqrt[Log[f]])]*(Cos[d] - I*Sin[d]) + Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c
]*Sqrt[Log[f]])]*(Cos[d] + I*Sin[d])))/(4*Sqrt[c]*Sqrt[Log[f]])
```

3.116.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cos(d+ex)\,dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{1}{2}e^{-id-iex}f^{a+cx^2}+\frac{1}{2}e^{id+iex}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}+id}\operatorname{erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}-id}\operatorname{erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Cos[d + e*x],x]
```

output
```
-1/4*(E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*Log[f])
/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) + (E^(I*d + e^2/(4*c*Lo
g[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(
4*Sqrt[c]*Sqrt[Log[f]])
```

3.116.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.116.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{4 i d \ln(f) c-e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x+\frac{i e}{2 \sqrt{-c \ln(f)}}\right)}{4 \sqrt{-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{4 i d \ln(f) c+e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{i e}{2 \sqrt{-c \ln(f)}}\right)}{4 \sqrt{-c \ln(f)}}$	121

input
```
int(f^(c*x^2+a)*cos(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*er
f((-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(1/4*(4
*I*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*I*
e/(-c*ln(f))^(1/2))
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\int f^{a+cx^2} \cos(d+ex)\, dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{(2\,cx\log(f)+i\,e)\sqrt{-c\log(f)}}{2\,c\log(f)}\right)e^{\left(\frac{4\,ac\log(f)^2+4i\,cd\log(f)+e^2}{4\,c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{(2\,cx\log(f)-i\,e)}{2\,c\log(}\right.}{4\,c\log(f)}$$

input
```
integrate(f^(c*x^2+a)*cos(e*x+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + I*e)*sqrt(-c*log(f)
)/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 4*I*c*d*log(f) + e^2)/(c*log(f))) +
 sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - I*e)*sqrt(-c*log(f))/(c*
log(f)))*e^(1/4*(4*a*c*log(f)^2 - 4*I*c*d*log(f) + e^2)/(c*log(f))))/(c*lo
g(f))
```

3.116.6 Sympy [F]

$$\int f^{a+cx^2}\cos(d+ex)\,dx=\int f^{a+cx^2}\cos{(d+ex)}\,dx$$

input
```
integrate(f**(c*x**2+a)*cos(e*x+d),x)
```

output
```
Integral(f**(a + c*x**2)*cos(d + e*x), x)
```

3.116.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.39

$$\int f^{a+cx^2}\cos(d+ex)\,dx=$$

$$-\frac{\sqrt{\pi}\left(f^a(\cos{(d)}-i\sin{(d)})\operatorname{erf}\left(x\overline{\sqrt{-c\log{(f)}}}+\frac{1}{2}i\,e\overline{\frac{1}{\sqrt{-c\log{(f)}}}}\right)e^{\left(\frac{e^2}{4c\log(f)}\right)}+f^a(\cos{(d)}+i\sin{(d)})\operatorname{erf}\left(\right.\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cos(e*x+d),x, algorithm="maxima")
```

output
```
-1/8*sqrt(pi)*(f^a*(cos(d) - I*sin(d))*erf(x*conjugate(sqrt(-c*log(f))) +
1/2*I*e*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(cos(d)
 + I*sin(d))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*I*e*conjugate(1/sqrt(-
c*log(f))))*e^(1/4*e^2/(c*log(f))) - f^a*(cos(d) + I*sin(d))*erf(1/2*(2*c*
x*log(f) + I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f))) - f^a*(cos(d) - I*
sin(d))*erf(1/2*(2*c*x*log(f) - I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f)
)))*sqrt(-c*log(f))/(c*log(f))
```

3.116.8 Giac [F]

$$\int f^{a+cx^2}\cos(d+ex)\,dx=\int f^{cx^2+a}\cos(ex+d)\,dx$$

input
```
integrate(f^(c*x^2+a)*cos(e*x+d),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*cos(e*x + d), x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cos(d+ex)\,dx=\int f^{c\,x^2+a}\cos(d+e\,x)\,dx$$

input
```
int(f^(a + c*x^2)*cos(d + e*x),x)
```

output
```
int(f^(a + c*x^2)*cos(d + e*x), x)
```

3.117 $\int f^{a+cx^2}\cos^2(d+ex)\,dx$

3.117.1 Optimal result

Integrand size = 18, antiderivative size = 171

$$\int f^{a+cx^2}\cos^2(d+ex)\,dx=\frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{ie-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{e^{2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{ie+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/8*exp(-2*I*d+e^2/c/ln(f))*f^a*erfi((-I*e+c*x*ln(f))/c^(1/2)/ln(f)^(1/2))
*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*I*d+e^2/c/ln(f))*f^a*erfi((I*e+c*x
*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^a*erfi(x*c
^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

3.117.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.77

$$\int f^{a+cx^2}\cos^2(d+ex)\,dx$$
$$=\frac{f^a\sqrt{\pi}\left(2\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)+e^{\frac{e^2}{c\log(f)}}\left(\text{erfi}\left(\frac{-ie+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)(\cos(2d)-i\sin(2d))+\text{erfi}\left(\frac{ie+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)(\cos(2d)\right.\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + c*x^2)*Cos[d + e*x]^2,x]
```

output
```
(f^a*Sqrt[Pi]*(2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] + E^(e^2/(c*Log[f]))*(Erfi[(
(-I)*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] - I*Sin[2*d]) + Erf
i[(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] + I*Sin[2*d]))))/(8
*Sqrt[c]*Sqrt[Log[f]])
```

3.117.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cos^2(d+ex)\,dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{1}{4}e^{-2id-2iex}f^{a+cx^2}+\frac{1}{4}e^{2id+2iex}f^{a+cx^2}+\frac{1}{2}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}f^a e^{\frac{e^2}{c\log(f)}-2id}\text{erfi}\left(\frac{-cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{\frac{e^2}{c\log(f)}+2id}\text{erfi}\left(\frac{cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Cos[d + e*x]^2,x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^
((-2*I)*d + e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - c*x*Log[f])/(Sqrt[c]*
Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^((2*I)*d + e^2/(c*Log[f]))*f
^a*Sqrt[Pi]*Erfi[(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sq
rt[Log[f]])
```

3.117.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.117.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{2 i d \ln(f) c-e^{2}}{\ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x+\frac{i e}{\sqrt{-c \ln(f)}}\right)}{8 \sqrt{-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{2 i d \ln(f) c+e^{2}}{\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{i e}{\sqrt{-c \ln(f)}}\right)}{8 \sqrt{-c \ln(f)}}+\frac{f^{a} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-c}\right.}{4 \sqrt{-c \ln(}}$

input
```
int(f^(c*x^2+a)*cos(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*Pi^(1/2)*f^a*exp(-(2*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-
c*ln(f))^(1/2)*x+I*e/(-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp((2*I*d*ln(f)*c
+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+I*e/(-c*ln(f))^(1/
2))+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.93

$$\int f^{a+cx^2} \cos^2(d+ex)\, dx =$$

$$-\frac{2\sqrt{\pi}\sqrt{-c\log(f)}f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)}x\right)+\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(cx\log(f)+ie)\sqrt{-c\log(f)}}{c\log(f)}\right)e^{\left(\frac{ac\log(f)^{2}+2icd\log(f}{c\log(f)}\right.}}{8\,c\log(f)}$$

input
```
integrate(f^(c*x^2+a)*cos(e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(pi)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log(f))*x) + sqrt(pi)*sqr
t(-c*log(f))*erf((c*x*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^((a*c*lo
g(f)^2 + 2*I*c*d*log(f) + e^2)/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf(
(c*x*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^((a*c*log(f)^2 - 2*I*c*d*
log(f) + e^2)/(c*log(f))))/(c*log(f))
```

3.117.6 Sympy [F]

$$\int f^{a+cx^2}\cos^2(d+ex)\,dx=\int f^{a+cx^2}\cos^2{(d+ex)}\,dx$$

input
```
integrate(f**(c*x**2+a)*cos(e*x+d)**2,x)
```

output
```
Integral(f**(a + c*x**2)*cos(d + e*x)**2, x)
```

3.117.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.38

$$\int f^{a+cx^2}\cos^2(d+ex)\,dx$$
$$=\frac{\sqrt{\pi}\left(f^a(\cos(2d)-i\sin(2d))\,\mathrm{erf}\left(x\overline{\sqrt{-c\log(f)}}+ie\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^2}{c\log(f)}\right)}+f^a(\cos(2d)+i\sin(2d))\,\mathrm{erf}\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cos(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*(f^a*(cos(2*d) - I*sin(2*d))*erf(x*conjugate(sqrt(-c*log(f))
) + I*e*conjugate(1/sqrt(-c*log(f))))*e^(e^2/(c*log(f))) + f^a*(cos(2*d) +
 I*sin(2*d))*erf(x*conjugate(sqrt(-c*log(f))) - I*e*conjugate(1/sqrt(-c*lo
g(f))))*e^(e^2/(c*log(f))) - f^a*(cos(2*d) + I*sin(2*d))*erf((c*x*log(f) +
 I*e)/sqrt(-c*log(f)))*e^(e^2/(c*log(f))) - f^a*(cos(2*d) - I*sin(2*d))*er
f((c*x*log(f) - I*e)/sqrt(-c*log(f)))*e^(e^2/(c*log(f))) + 2*f^a*erf(x*con
jugate(sqrt(-c*log(f)))) + 2*f^a*erf(sqrt(-c*log(f))*x))/sqrt(-c*log(f))
```

3.117.8 Giac [F]

$$\int f^{a+cx^2}\cos^2(d+ex)\,dx = \int f^{cx^2+a}\cos(ex+d)^2\,dx$$

input
```
integrate(f^(c*x^2+a)*cos(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*cos(e*x + d)^2, x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cos^2(d+ex)\,dx = \int f^{c\,x^2+a}\cos(d+e\,x)^2\,dx$$

input
```
int(f^(a + c*x^2)*cos(d + e*x)^2,x)
```

output
```
int(f^(a + c*x^2)*cos(d + e*x)^2, x)
```

3.118 $\int f^{a+cx^2}\cos^3(d+ex)\,dx$

3.118.1 Optimal result

Integrand size = 18, antiderivative size = 293

$$\begin{aligned}\int f^{a+cx^2}\cos^3(d+ex)\,dx = &-\frac{3e^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} \\ &-\frac{e^{-3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} \\ &+\frac{3e^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} \\ &+\frac{e^{3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

output
```
3/16*exp(-I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-I*e+2*c*x*ln(f))/c^(1/2)/ln(
f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(-3*I*d+9/4*e^2/c/ln(f))*f^
a*erfi(1/2*(-3*I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f
)^(1/2)+3/16*exp(I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(I*e+2*c*x*ln(f))/c^(1/
2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(3*I*d+9/4*e^2/c/ln(f
))*f^a*erfi(1/2*(3*I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/
ln(f)^(1/2)
```

3.118.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.74

$$\int f^{a+cx^2}\cos^3(d+ex)\,dx$$
$$=\frac{e^{\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(3\mathrm{erfi}\left(\frac{-ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)-i\sin(d))+3\mathrm{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))+e^{\frac{2e^2}{c\log(f)}}\left(\mathrm{er}\right.\right.}{16\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + c*x^2)*Cos[d + e*x]^3,x]
```

output
```
(E^(e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*(3*Erfi[((-I)*e + 2*c*x*Log[f])/(2*Sqrt
[c]*Sqrt[Log[f]])]*(Cos[d] - I*Sin[d]) + 3*Erfi[(I*e + 2*c*x*Log[f])/(2*Sq
rt[c]*Sqrt[Log[f]])]*(Cos[d] + I*Sin[d]) + E^((2*e^2)/(c*Log[f]))*(Erfi[((
-3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[3*d] - I*Sin[3*d])
+ Erfi[((3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[3*d] + I*Si
n[3*d]))))/(16*Sqrt[c]*Sqrt[Log[f]])
```

3.118.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cos^3(d+ex)\,dx$$
$$\Big\downarrow \text{4976}$$
$$\int\left(\frac{3}{8}e^{-id-iex}f^{a+cx^2}+\frac{3}{8}e^{id+iex}f^{a+cx^2}+\frac{1}{8}e^{-3id-3iex}f^{a+cx^2}+\frac{1}{8}e^{3id+3iex}f^{a+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{3\sqrt{\pi}f^ae^{\frac{e^2}{4c\log(f)}-id}\mathrm{erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^ae^{\frac{9e^2}{4c\log(f)}-3id}\mathrm{erfi}\left(\frac{-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3\sqrt{\pi}f^ae^{\frac{e^2}{4c\log(f)}+id}\mathrm{erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^ae^{\frac{9e^2}{4c\log(f)}+3id}\mathrm{erfi}\left(\frac{2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Cos[d + e*x]^3,x]
```

output
```
(-3*E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*Log[f])/(
2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E^((-3*I)*d + (9*e^
2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqr
t[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (3*E^(I*d + e^2/(4*c*Log[f]))*f^a
*Sqrt[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]
*Sqrt[Log[f]]) + (E^((3*I)*d + (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3
*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

3.118.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.118.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.83

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4 i d \ln(f) c-3 e^{2}\right)}{4 \ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x+\frac{3 i e}{2 \sqrt{-c \ln(f)}}\right)}{16 \sqrt{-c \ln(f)}}+\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{4 i d \ln(f) c-e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x+\frac{i e}{2 \sqrt{-c \ln(f)}}\right)}{16 \sqrt{-c \ln(f)}}-\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}}{}$

input
```
int(f^(c*x^2+a)*cos(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*Pi^(1/2)*f^a*exp(-3/4*(4*I*d*ln(f)*c-3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)
*erf((-c*ln(f))^(1/2)*x+3/2*I*e/(-c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1
/4*(4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/
2*I*e/(-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c+e^2)/ln(f
)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2))-1/
16*Pi^(1/2)*f^a*exp(3/4*(4*I*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*er
f(-(-c*ln(f))^(1/2)*x+3/2*I*e/(-c*ln(f))^(1/2))
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.96

$$\int f^{a+cx^2}\cos^3(d+ex)\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{(2\,cx\log(f)+3i\,e)\sqrt{-c\log(f)}}{2\,c\log(f)}\right)e^{\left(\frac{4\,ac\log(f)^2+12i\,cd\log(f)+9\,e^2}{4\,c\log(f)}\right)}+3\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{(2\,cx\log(f}{2}\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cos(e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + 3*I*e)*sqrt(-c*log
(f))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 12*I*c*d*log(f) + 9*e^2)/(c*log(
f))) + 3*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + I*e)*sqrt(-c*log
(f))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 4*I*c*d*log(f) + e^2)/(c*log(f))
) + 3*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - I*e)*sqrt(-c*log(f)
)/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 - 4*I*c*d*log(f) + e^2)/(c*log(f))) +
 sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - 3*I*e)*sqrt(-c*log(f))/(
c*log(f)))*e^(1/4*(4*a*c*log(f)^2 - 12*I*c*d*log(f) + 9*e^2)/(c*log(f))))/
(c*log(f))
```

3.118.6 Sympy [F]

$$\int f^{a+cx^2}\cos^3(d+ex)\,dx = \int f^{a+cx^2}\cos^3(d+ex)\,dx$$

input
```
integrate(f**(c*x**2+a)*cos(e*x+d)**3,x)
```

output
```
Integral(f**(a + c*x**2)*cos(d + e*x)**3, x)
```

3.118.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.39

$$\int f^{a+cx^2} \cos^3(d+ex)\,dx =$$

$$-\frac{\sqrt{\pi}\left(f^a(\cos(3d) - i\sin(3d))\,\mathrm{erf}\left(x\overline{\sqrt{-c\log(f)}} + \frac{3}{2}i\,e\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{9e^2}{4c\log(f)}\right)} + f^a(\cos(3d) + i\sin(3d$$

input
```
integrate(f^(c*x^2+a)*cos(e*x+d)^3,x, algorithm="maxima")
```

output
```
-1/32*sqrt(pi)*(f^a*(cos(3*d) - I*sin(3*d))*erf(x*conjugate(sqrt(-c*log(f)
)) + 3/2*I*e*conjugate(1/sqrt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*(c
os(3*d) + I*sin(3*d))*erf(x*conjugate(sqrt(-c*log(f))) - 3/2*I*e*conjugate
(1/sqrt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) - f^a*(cos(3*d) + I*sin(3*d))*
erf(1/2*(2*c*x*log(f) + 3*I*e)/sqrt(-c*log(f)))*e^(9/4*e^2/(c*log(f))) - f
^a*(cos(3*d) - I*sin(3*d))*erf(1/2*(2*c*x*log(f) - 3*I*e)/sqrt(-c*log(f)))
*e^(9/4*e^2/(c*log(f))) + 3*f^a*(cos(d) - I*sin(d))*erf(x*conjugate(sqrt(-
c*log(f))) + 1/2*I*e*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f)))
+ 3*f^a*(cos(d) + I*sin(d))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*I*e*con
jugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) - 3*f^a*(cos(d) + I*sin(
d))*erf(1/2*(2*c*x*log(f) + I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f))) -
 3*f^a*(cos(d) - I*sin(d))*erf(1/2*(2*c*x*log(f) - I*e)/sqrt(-c*log(f)))*e
^(1/4*e^2/(c*log(f))))*sqrt(-c*log(f))/(c*log(f))
```

3.118.8 Giac [F]

$$\int f^{a+cx^2} \cos^3(d+ex)\,dx = \int f^{cx^2+a}\cos(ex+d)^3\,dx$$

input
```
integrate(f^(c*x^2+a)*cos(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*cos(e*x + d)^3, x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2} \cos^3(d+ex)\,dx = \int f^{c\,x^2+a} \cos\left(d+e\,x\right)^3\,dx$$

input
```
int(f^(a + c*x^2)*cos(d + e*x)^3,x)
```

output
```
int(f^(a + c*x^2)*cos(d + e*x)^3, x)
```

3.119 $\int f^{a+cx^2} \cos(d+fx^2)\,dx$

3.119.1 Optimal result

Integrand size = 18, antiderivative size = 103

$$\int f^{a+cx^2}\cos(d+fx^2)\,dx = \frac{e^{-id}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{if-c\log(f)}\right)}{4\sqrt{if-c\log(f)}}+\frac{e^{id}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{if+c\log(f)}\right)}{4\sqrt{if+c\log(f)}}$$

output
```
1/4*f^a*erf(x*(I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(I*d)/(I*f-c*ln(f))^(1/2)+1
/4*exp(I*d)*f^a*erfi(x*(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

3.119.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.65

$$\int f^{a+cx^2}\cos(d+fx^2)\,dx = -\frac{(-1)^{3/4}f^a\sqrt{\pi}\left(\operatorname{erfi}\left(\sqrt[4]{-1}x\sqrt{f-ic\log(f)}\right)\sqrt{f-ic\log(f)}(f+ic\log(f))(\cos(d)+i\sin(d))+\sqrt{f+ic}\right.}{4\,(}$$

input
```
Integrate[f^(a + c*x^2)*Cos[d + f*x^2],x]
```

output
```
-1/4*((-1)^(3/4)*f^a*Sqrt[Pi]*(Erfi[(-1)^(1/4)*x*Sqrt[f - I*c*Log[f]]]*Sqr
t[f - I*c*Log[f]]*(f + I*c*Log[f])*(Cos[d] + I*Sin[d]) + Sqrt[f + I*c*Log[
f]]*(f*Cos[d]*Erf[((1 + I)*x*Sqrt[f + I*c*Log[f]])/Sqrt[2]] - Erfi[(-1)^(3
/4)*x*Sqrt[f + I*c*Log[f]]]*(c*Cos[d]*Log[f] + (f - I*c*Log[f])*Sin[d]))))
/(f^2 + c^2*Log[f]^2)
```

3.119.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2} \cos\left(d+fx^2\right) dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{1}{2}e^{-id-ifx^2}f^{a+cx^2}+\frac{1}{2}e^{id+ifx^2}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}e^{-id}f^a\text{erf}\left(x\sqrt{-c\log(f)+if}\right)}{4\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}e^{id}f^a\text{erfi}\left(x\sqrt{c\log(f)+if}\right)}{4\sqrt{c\log(f)+if}}$$

input
```
Int[f^(a + c*x^2)*Cos[d + f*x^2],x]
```

output
```
(f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(4*E^(I*d)*Sqrt[I*f - c*Log[f]]
) + (E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[I*f + c*Log[f]]])/(4*Sqrt[I*f + c*Lo
g[f]])
```

3.119.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.119.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{-i d} \operatorname{erf}\left(x \sqrt{i f -c \ln \left(f \right)}\right)}{4 \sqrt{i f -c \ln \left(f \right)}}+\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{i d} \operatorname{erf}\left(\sqrt{-c \ln \left(f \right)-i f}\, x \right)}{4 \sqrt{-c \ln \left(f \right)-i f}}$	82

input
```
int(f^(c*x^2+a)*cos(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*f^a*exp(-I*d)/(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2))+
1/4*Pi^(1/2)*f^a*exp(I*d)/(-c*ln(f)-I*f)^(1/2)*erf((-c*ln(f)-I*f)^(1/2)*x)
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.06

$$\int f^{a+cx^2} \cos\left(d+fx^2\right)\,dx =$$
$$-\frac{\sqrt{\pi}(c\log(f)-i\,f)\sqrt{-c\log(f)-i\,f}\,\operatorname{erf}\left(\sqrt{-c\log(f)-i\,f}x\right)e^{(a\log(f)+i\,d)}+\sqrt{\pi}(c\log(f)+i\,f)\sqrt{-c\,\mathrm{l}}}{4\left(c^2\log(f)^2+f^2\right)}$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*(c*log(f) - I*f)*sqrt(-c*log(f) - I*f)*erf(sqrt(-c*log(f) -
 I*f)*x)*e^(a*log(f) + I*d) + sqrt(pi)*(c*log(f) + I*f)*sqrt(-c*log(f) + I
*f)*erf(sqrt(-c*log(f) + I*f)*x)*e^(a*log(f) - I*d))/(c^2*log(f)^2 + f^2)
```

3.119.6 Sympy [F]

$$\int f^{a+cx^2}\cos\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\cos\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*cos(f*x**2+d),x)
```

output
```
Integral(f**(a + c*x**2)*cos(d + f*x**2), x)
```

3.119.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(73) = 146$.

Time = 0.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.99

$$\int f^{a+cx^2}\cos\left(d+fx^2\right)\,dx=$$

$$-\frac{\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+2\,f^2}\left(f^a(i\,\cos\left(d\right)+\sin\left(d\right))\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+i\,f}x\right)+f^a(-i\,\cos\left(d\right)+\sin\left(d\right))\operatorname{er}\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+d),x, algorithm="maxima")
```

output
```
-1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(f^a*(I*cos(d) + sin(d))*erf(s
qrt(-c*log(f) + I*f)*x) + f^a*(-I*cos(d) + sin(d))*erf(sqrt(-c*log(f) - I*
f)*x))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) - sqrt(pi)*sqrt(2*c^2*log
(f)^2 + 2*f^2)*(f^a*(cos(d) - I*sin(d))*erf(sqrt(-c*log(f) + I*f)*x) + f^a
*(cos(d) + I*sin(d))*erf(sqrt(-c*log(f) - I*f)*x))*sqrt(-c*log(f) + sqrt(c
^2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

3.119.8 Giac [F]

$$\int f^{a+cx^2}\cos\left(d+fx^2\right)\,dx=\int f^{cx^2+a}\cos\left(fx^2+d\right)\,dx$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+d),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*cos(f*x^2 + d), x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cos\left(d+fx^2\right)\,dx=\int f^{c\,x^2+a}\,\cos\left(f\,x^2+d\right)\,dx$$

input
```
int(f^(a + c*x^2)*cos(d + f*x^2),x)
```

output
```
int(f^(a + c*x^2)*cos(d + f*x^2), x)
```

3.120 $\int f^{a+cx^2}\cos^2\left(d+fx^2\right)\,dx$

3.120.1 Optimal result

Integrand size = 20, antiderivative size = 140

$$\int f^{a+cx^2}\cos^2\left(d+fx^2\right)\,dx=\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{2if-c\log(f)}\right)}{8\sqrt{2if-c\log(f)}}+\frac{e^{2id}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{2if+c\log(f)}\right)}{8\sqrt{2if+c\log(f)}}$$

output
```
1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*f^a*e
rf(x*(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(2*I*d)/(2*I*f-c*ln(f))^(1/2)+1/8*
exp(2*I*d)*f^a*erfi(x*(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*ln(f))^(1/2
)
```

3.120.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.35

$$\int f^{a+cx^2}\cos^2\left(d+fx^2\right)\,dx=\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt[4]{-1}\left(-\operatorname{erfi}\left((-1)^{3/4}x\sqrt{2f+ic\log(f)}\right)(2f-ic\log(f))\sqrt{2f+ic\log(f)}(\cos(2d)-i\sin(2d))+\operatorname{erfi}\left(\sqrt[4]{\cdots}\right.\right.}{4f^2+c^2\log^2(f)}\right.$$

input
```
Integrate[f^(a + c*x^2)*Cos[d + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*((2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + (
(-1)^(1/4)*(-(Erfi[(-1)^(3/4)*x*Sqrt[2*f + I*c*Log[f]]]*(2*f - I*c*Log[f])
*Sqrt[2*f + I*c*Log[f]]*(Cos[2*d] - I*Sin[2*d])) + Erfi[(-1)^(1/4)*x*Sqrt[
2*f - I*c*Log[f]]]*Sqrt[2*f - I*c*Log[f]]*((-2*I)*f + c*Log[f])*(Cos[2*d]
+ I*Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/8
```

3.120.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2} \cos^2\left(d+fx^2\right) dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{1}{4}e^{-2id-2ifx^2}f^{a+cx^2}+\frac{1}{4}e^{2id+2ifx^2}f^{a+cx^2}+\frac{1}{2}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}e^{-2id}f^a\text{erf}\left(x\sqrt{-c\log(f)+2if}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}e^{2id}f^a\text{erfi}\left(x\sqrt{c\log(f)+2if}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}f^a\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Cos[d + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (f^
a*Sqrt[Pi]*Erf[x*Sqrt[(2*I)*f - c*Log[f]]])/(8*E^((2*I)*d)*Sqrt[(2*I)*f -
c*Log[f]]) + (E^((2*I)*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[(2*I)*f + c*Log[f]]])/(
8*Sqrt[(2*I)*f + c*Log[f]])
```

3.120.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.120.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{-2 i d} \operatorname{erf}\left(x \sqrt{2 i f -c \ln \left(f \right)}\right)}{8 \sqrt{2 i f -c \ln \left(f \right)}}+\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{2 i d} \operatorname{erf}\left(\sqrt{-c \ln \left(f \right)-2 i f}\, x \right)}{8 \sqrt{-c \ln \left(f \right)-2 i f}}+\frac{f^{a} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-c \ln \left(f \right)}\, x \right)}{4 \sqrt{-c \ln \left(f \right)}}$	107

input
```
int(f^(c*x^2+a)*cos(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*Pi^(1/2)*f^a*exp(-2*I*d)/(2*I*f-c*ln(f))^(1/2)*erf(x*(2*I*f-c*ln(f))^(
1/2))+1/8*Pi^(1/2)*f^a*exp(2*I*d)/(-c*ln(f)-2*I*f)^(1/2)*erf((-c*ln(f)-2*I
*f)^(1/2)*x)+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.19

$$\int f^{a+cx^2} \cos^2\left(d+fx^2\right)\,dx =$$

$$-\frac{2\sqrt{\pi}\left(c^2\log\left(f\right)^2+4\,f^2\right)\sqrt{-c\log\left(f\right)}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x\right)+\sqrt{\pi}\left(c^2\log\left(f\right)^2-2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(\right.}}{}$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*lo
g(f))*x) + sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f
)*erf(sqrt(-c*log(f) - 2*I*f)*x)*e^(a*log(f) + 2*I*d) + sqrt(pi)*(c^2*log(
f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf(sqrt(-c*log(f) + 2*I*f)
*x)*e^(a*log(f) - 2*I*d))/(c^3*log(f)^3 + 4*c*f^2*log(f))
```

3.120.6 Sympy [F]

$$\int f^{a+cx^2}\cos^2\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\cos^2\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*cos(f*x**2+d)**2,x)
```

output
```
Integral(f**(a + c*x**2)*cos(d + f*x**2)**2, x)
```

3.120.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.25

$$\int f^{a+cx^2}\cos^2\left(d+fx^2\right)\,dx=$$

$$-\frac{\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+8\,f^2}\left(f^a(i\,\cos\left(2\,d\right)+\sin\left(2\,d\right))\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+2i\,f}x\right)+f^a(-i\,\cos\left(2\,d\right)+\sin\left(\right.\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*(f^a*(I*cos(2*d) + sin(2*d))*
erf(sqrt(-c*log(f) + 2*I*f)*x) + f^a*(-I*cos(2*d) + sin(2*d))*erf(sqrt(-c*
log(f) - 2*I*f)*x))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*lo
g(f)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*(f^a*(cos(2*d) - I*sin(2*d))
*erf(sqrt(-c*log(f) + 2*I*f)*x) + f^a*(cos(2*d) + I*sin(2*d))*erf(sqrt(-c*
log(f) - 2*I*f)*x))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*l
og(f)) - 2*sqrt(pi)*((c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(x*conjugate(sqrt
(-c*log(f)))) + (c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(sqrt(-c*log(f))*x)))/
((c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f)))
```

3.120.8 Giac [F]

$$\int f^{a+cx^2} \cos^2 \left(d+fx^2\right) \, dx = \int f^{cx^2+a} \cos \left(fx^2+d\right)^2 \, dx$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*cos(f*x^2 + d)^2, x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2} \cos^2 \left(d+fx^2\right) \, dx = \int f^{c\,x^2+a} \cos \left(f\,x^2+d\right)^2 \, dx$$

input
```
int(f^(a + c*x^2)*cos(d + f*x^2)^2,x)
```

output
```
int(f^(a + c*x^2)*cos(d + f*x^2)^2, x)
```

3.121 $\int f^{a+cx^2}\cos^3\left(d+fx^2\right)\,dx$

3.121.1 Optimal result

Integrand size = 20, antiderivative size = 205

$$\int f^{a+cx^2}\cos^3\left(d+fx^2\right)\,dx=\frac{3e^{-id}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{if-c\log(f)}\right)}{16\sqrt{if-c\log(f)}}+\frac{e^{-3id}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{3if-c\log(f)}\right)}{16\sqrt{3if-c\log(f)}}+\frac{3e^{id}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{if+c\log(f)}\right)}{16\sqrt{if+c\log(f)}}+\frac{e^{3id}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{3if+c\log(f)}\right)}{16\sqrt{3if+c\log(f)}}$$

output
```
3/16*f^a*erf(x*(I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(I*d)/(I*f-c*ln(f))^(1/2)+
1/16*f^a*erf(x*(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(3*I*d)/(3*I*f-c*ln(f))^
(1/2)+3/16*exp(I*d)*f^a*erfi(x*(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))
^(1/2)+1/16*exp(3*I*d)*f^a*erfi(x*(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f+c
*ln(f))^(1/2)
```

3.121.2 Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.90

$$\int f^{a+cx^2}\cos^3\left(d+fx^2\right)\,dx$$
$$=\frac{\sqrt[4]{-1}f^a\sqrt{\pi}\left(3\operatorname{erfi}\left(\sqrt[4]{-1}x\sqrt{f-ic\log(f)}\right)\sqrt{f-ic\log(f)}\left(-9if^3+9cf^2\log(f)-ic^2f\log^2(f)+c^3\log^3(f\right.\right.}{}$$

input
```
Integrate[f^(a + c*x^2)*Cos[d + f*x^2]^3,x]
```

output
```
((-1)^(1/4)*f^a*Sqrt[Pi]*(3*Erfi[(-1)^(1/4)*x*Sqrt[f - I*c*Log[f]]]*Sqrt[f
 - I*c*Log[f]]*((-9*I)*f^3 + 9*c*f^2*Log[f] - I*c^2*f*Log[f]^2 + c^3*Log[f
]^3)*(Cos[d] + I*Sin[d]) + (f - I*c*Log[f])*(-((3*f - I*c*Log[f])*(9*f*Erf
[((1 + I)*x*Sqrt[f + I*c*Log[f]])/Sqrt[2]]*Sqrt[f + I*c*Log[f]]*Sin[d] + 3
*Erfi[(-1)^(3/4)*x*Sqrt[f + I*c*Log[f]]]*Sqrt[f + I*c*Log[f]]*(Cos[d]*(3*f
 + I*c*Log[f]) + c*Log[f]*Sin[d]) + Erfi[(-1)^(3/4)*x*Sqrt[3*f + I*c*Log[f
]]]*(f + I*c*Log[f])*Sqrt[3*f + I*c*Log[f]]*(Cos[3*d] - I*Sin[3*d]))) + Er
fi[(-1)^(1/4)*x*Sqrt[3*f - I*c*Log[f]]]*Sqrt[3*f - I*c*Log[f]]*((-3*I)*f^2
 + 4*c*f*Log[f] + I*c^2*Log[f]^2)*(Cos[3*d] + I*Sin[3*d]))))/(16*(9*f^4 +
10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

3.121.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cos^3\left(d+fx^2\right)dx$$
$$\Big\downarrow \text{4976}$$
$$\int\left(\frac{3}{8}e^{-id-ifx^2}f^{a+cx^2}+\frac{3}{8}e^{id+ifx^2}f^{a+cx^2}+\frac{1}{8}e^{-3id-3ifx^2}f^{a+cx^2}+\frac{1}{8}e^{3id+3ifx^2}f^{a+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\pi}e^{-id}f^{a}\text{erf}\left(x\sqrt{-c\log(f)+if}\right)}{16\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}e^{-3id}f^{a}\text{erf}\left(x\sqrt{-c\log(f)+3if}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{3\sqrt{\pi}e^{id}f^{a}\text{erfi}\left(x\sqrt{c\log(f)+if}\right)}{16\sqrt{c\log(f)+if}}+\frac{\sqrt{\pi}e^{3id}f^{a}\text{erfi}\left(x\sqrt{c\log(f)+3if}\right)}{16\sqrt{c\log(f)+3if}}$$

input
```
Int[f^(a + c*x^2)*Cos[d + f*x^2]^3,x]
```

output
```
(3*f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(16*E^(I*d)*Sqrt[I*f - c*Log[
f]]) + (f^a*Sqrt[Pi]*Erf[x*Sqrt[(3*I)*f - c*Log[f]]])/(16*E^((3*I)*d)*Sqrt
[(3*I)*f - c*Log[f]]) + (3*E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[I*f + c*Log[f]
]])/(16*Sqrt[I*f + c*Log[f]]) + (E^((3*I)*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[(3*I
)*f + c*Log[f]]])/(16*Sqrt[(3*I)*f + c*Log[f]])
```

3.121.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.121.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.79

method	result
risch	$\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-3id}\,\mathrm{erf}\left(x\sqrt{3if-c\ln(f)}\right)}{16\sqrt{3if-c\ln(f)}}+\frac{3\sqrt{\pi}\,f^{a}\mathrm{e}^{-id}\,\mathrm{erf}\left(x\sqrt{if-c\ln(f)}\right)}{16\sqrt{if-c\ln(f)}}+\frac{3\sqrt{\pi}\,f^{a}\mathrm{e}^{id}\,\mathrm{erf}\left(\sqrt{-c\ln(f)-if}\,x\right)}{16\sqrt{-c\ln(f)-if}}+\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{3id}\,\mathrm{erf}\left(\sqrt{-\right.}{16\sqrt{-c\ln(}$

input
```
int(f^(c*x^2+a)*cos(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*Pi^(1/2)*f^a*exp(-3*I*d)/(3*I*f-c*ln(f))^(1/2)*erf(x*(3*I*f-c*ln(f))^
(1/2))+3/16*Pi^(1/2)*f^a*exp(-I*d)/(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))
^(1/2))+3/16*Pi^(1/2)*f^a*exp(I*d)/(-c*ln(f)-I*f)^(1/2)*erf((-c*ln(f)-I*f)
^(1/2)*x)+1/16*Pi^(1/2)*f^a*exp(3*I*d)/(-c*ln(f)-3*I*f)^(1/2)*erf((-c*ln(f
)-3*I*f)^(1/2)*x)
```

3.121.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(145) = 290.

Time = 0.26 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.52

$$\int f^{a+cx^2}\cos^3\left(d+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\left(c^3\log\left(f\right)^3-3i\,c^2f\log\left(f\right)^2+cf^2\log\left(f\right)-3i\,f^3\right)\sqrt{-c\log\left(f\right)-3i\,f}\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-3i\,f}x\right)e^{(a}}{}$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+d)^3,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(pi)*(c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + c*f^2*log(f) - 3*I*f^
3)*sqrt(-c*log(f) - 3*I*f)*erf(sqrt(-c*log(f) - 3*I*f)*x)*e^(a*log(f) + 3*
I*d) + 3*sqrt(pi)*(c^3*log(f)^3 - I*c^2*f*log(f)^2 + 9*c*f^2*log(f) - 9*I*
f^3)*sqrt(-c*log(f) - I*f)*erf(sqrt(-c*log(f) - I*f)*x)*e^(a*log(f) + I*d)
 + 3*sqrt(pi)*(c^3*log(f)^3 + I*c^2*f*log(f)^2 + 9*c*f^2*log(f) + 9*I*f^3)
*sqrt(-c*log(f) + I*f)*erf(sqrt(-c*log(f) + I*f)*x)*e^(a*log(f) - I*d) + s
qrt(pi)*(c^3*log(f)^3 + 3*I*c^2*f*log(f)^2 + c*f^2*log(f) + 3*I*f^3)*sqrt(
-c*log(f) + 3*I*f)*erf(sqrt(-c*log(f) + 3*I*f)*x)*e^(a*log(f) - 3*I*d))/(c
^4*log(f)^4 + 10*c^2*f^2*log(f)^2 + 9*f^4)
```

3.121.6 Sympy [F]

$$\int f^{a+cx^2}\cos^3\left(d+fx^2\right)\,dx = \int f^{a+cx^2}\cos^3\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*cos(f*x**2+d)**3,x)
```

output
```
Integral(f**(a + c*x**2)*cos(d + f*x**2)**3, x)
```

3.121.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(145) = 290$.

Time = 0.25 (sec) , antiderivative size = 667, normalized size of antiderivative = 3.25

$$\int f^{a+cx^2} \cos^3\left(d+fx^2\right) dx$$

$$= \frac{\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+18\,f^2}\left(\left(\left(-i\,c^2\cos\left(3\,d\right)-c^2\sin\left(3\,d\right)\right)f^a\log\left(f\right)^2+f^{a+2}\left(-i\,\cos\left(3\,d\right)-\sin\left(3\,d\right)\right)\right)\operatorname{erf}\left(\right.\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((-I*c^2*cos(3*d) - c^2*sin(
3*d))*f^a*log(f)^2 + f^(a + 2)*(-I*cos(3*d) - sin(3*d)))*erf(sqrt(-c*log(f
) + 3*I*f)*x) + ((I*c^2*cos(3*d) - c^2*sin(3*d))*f^a*log(f)^2 + f^(a + 2)*
(I*cos(3*d) - sin(3*d)))*erf(sqrt(-c*log(f) - 3*I*f)*x))*sqrt(c*log(f) + s
qrt(c^2*log(f)^2 + 9*f^2)) + 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((-I
*c^2*cos(d) - c^2*sin(d))*f^a*log(f)^2 + 9*f^(a + 2)*(-I*cos(d) - sin(d)))
*erf(sqrt(-c*log(f) + I*f)*x) + ((I*c^2*cos(d) - c^2*sin(d))*f^a*log(f)^2
+ 9*f^(a + 2)*(I*cos(d) - sin(d)))*erf(sqrt(-c*log(f) - I*f)*x))*sqrt(c*lo
g(f) + sqrt(c^2*log(f)^2 + f^2)) + sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*
(((c^2*cos(3*d) - I*c^2*sin(3*d))*f^a*log(f)^2 + f^(a + 2)*(cos(3*d) - I*s
in(3*d)))*erf(sqrt(-c*log(f) + 3*I*f)*x) + ((c^2*cos(3*d) + I*c^2*sin(3*d)
)*f^a*log(f)^2 + f^(a + 2)*(cos(3*d) + I*sin(3*d)))*erf(sqrt(-c*log(f) - 3
*I*f)*x))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) + 3*sqrt(pi)*sqrt(2
*c^2*log(f)^2 + 2*f^2)*(((c^2*cos(d) - I*c^2*sin(d))*f^a*log(f)^2 + 9*f^(a
 + 2)*(cos(d) - I*sin(d)))*erf(sqrt(-c*log(f) + I*f)*x) + ((c^2*cos(d) + I
*c^2*sin(d))*f^a*log(f)^2 + 9*f^(a + 2)*(cos(d) + I*sin(d)))*erf(sqrt(-c*l
og(f) - I*f)*x))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4
 + 10*c^2*f^2*log(f)^2 + 9*f^4)
```

3.121.8 Giac [F]

$$\int f^{a+cx^2}\cos^3\left(d+fx^2\right)\,dx=\int f^{cx^2+a}\cos\left(fx^2+d\right)^3\,dx$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*cos(f*x^2 + d)^3, x)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cos^3\left(d+fx^2\right)\,dx=\int f^{c\,x^2+a}\cos\left(f\,x^2+d\right)^3\,dx$$

input
```
int(f^(a + c*x^2)*cos(d + f*x^2)^3,x)
```

output
```
int(f^(a + c*x^2)*cos(d + f*x^2)^3, x)
```

3.122 $\int f^{a+cx^2} \cos(d+ex+fx^2)\,dx$

3.122.1 Optimal result

Integrand size = 21, antiderivative size = 183

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)\,dx=\frac{e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}+\frac{e^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

output
```
1/4*exp(-I*d-e^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e+2*x*(I*f-c*ln(f)))/(I
*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/4*exp(I*d+e^2/(4*I*f+4*c
*ln(f)))*f^a*erfi(1/2*(I*e+2*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2
)/(I*f+c*ln(f))^(1/2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.19

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt[4]{-1}e^{\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\left(-e^{\frac{ie^2f}{2\left(f^2+c^2\log^2(f)\right)}}\mathrm{erfi}\left(\frac{(-1)^{3/4}(e+2fx+2icx\log(f))}{2\sqrt{f+ic\log(f)}}\right)(f-ic\log(f))\sqrt{f+ic\log(f)}(\cos(d)}{4\left(f^2+c^2\log^2(\right.}$$

input
```
Integrate[f^(a + c*x^2)*Cos[d + e*x + f*x^2],x]
```

output
```
((-1)^(1/4)*E^(e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*(-(E^(((I/2)*e^2*f
)/(f^2 + c^2*Log[f]^2))*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(
2*Sqrt[f + I*c*Log[f]])]*(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*(Cos[d] - I
*Sin[d])) + Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I
*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*((-I)*f + c*Log[f])*(Cos[d] + I*Sin[d]))
)/(4*(f^2 + c^2*Log[f]^2))
```

3.122.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{1}{2}f^{a+cx^2}e^{-id-iex-ifx^2}+\frac{1}{2}f^{a+cx^2}e^{id+iex+ifx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{-4c\log(f)+4if}-id}\text{erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)+4if}+id}\text{erfi}\left(\frac{2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

input
```
Int[f^(a + c*x^2)*Cos[d + e*x + f*x^2],x]
```

output
```
(E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + 2*x*(I*f
- c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(4*Sqrt[I*f - c*Log[f]]) + (E^(I*d
 + e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*x*(I*f + c*Log[f
]))/(2*Sqrt[I*f + c*Log[f]])])/(4*Sqrt[I*f + c*Log[f]])
```

3.122.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.122.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} e^{-\frac{4 i d \ln(f) c+4 d f-e^{2}}{4(c\ln(f)-i f)}} \operatorname{erf}\left(x\sqrt{i f-c\ln(f)}+\frac{i e}{2\sqrt{i f-c\ln(f)}}\right)}{4\sqrt{i f-c\ln(f)}}-\frac{\sqrt{\pi}\, f^{a} e^{\frac{4 i d \ln(f) c-4 d f+e^{2}}{4 i f+4 c\ln(f)}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-i f}\, x+\frac{i e}{2\sqrt{-c\ln(f)-i f}}\right.}{4\sqrt{-c\ln(f)-i f}}$

input
```
int(f^(c*x^2+a)*cos(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c+4*d*f-e^2)/(c*ln(f)-I*f))/(I*f-c*
ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2)+1/2*I*e/(I*f-c*ln(f))^(1/2))-1/4*Pi
^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c-4*d*f+e^2)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)
^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*I*e/(-c*ln(f)-I*f)^(1/2))
```

3.122.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(135) = 270.

Time = 0.26 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.64

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}(c\log(f)-i f)\sqrt{-c\log(f)-i f}\operatorname{erf}\left(\frac{\left(2c^2x\log(f)^2+2f^2x+i ce\log(f)+ef\right)\sqrt{-c\log(f)-i f}}{2\left(c^2\log(f)^2+f^2\right)}\right)e^{\left(\frac{4ac^2\log(f)^3+4i c^2 d\log\ldots}{\ldots}\right.}}{\ldots}$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*(c*log(f) - I*f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*c^2*x*log
(f)^2 + 2*f^2*x + I*c*e*log(f) + e*f)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2
+ f^2))*e^(1/4*(4*a*c^2*log(f)^3 + 4*I*c^2*d*log(f)^2 - I*e^2*f + 4*I*d*f^
2 + (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(c*log(f) +
 I*f)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x - I*c*e*lo
g(f) + e*f)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*c^2*lo
g(f)^3 - 4*I*c^2*d*log(f)^2 + I*e^2*f - 4*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(
f))/(c^2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

3.122.6 Sympy [F]

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*cos(f*x**2+e*x+d),x)
```

output
```
Integral(f**(a + c*x**2)*cos(d + e*x + f*x**2), x)
```

3.122.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 761 vs. 2(135) = 270.

Time = 0.25 (sec) , antiderivative size = 761, normalized size of antiderivative = 4.16

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+2\,f^2}\left(\left(i\,f^{\frac{ce^2}{4\left(c^2\log(f)^2+f^2\right)}}f^a\cos\left(\frac{4\,c^2d\log(f)^2-e^2f+4\,df^2}{4\left(c^2\log(f)^2+f^2\right)}\right)+f^{\frac{ce^2}{4\left(c^2\log(f)^2+f^2\right)}}f^a\sin\left(\frac{4\,c^2d\log(f)}{4\left(c^2\log\right.}\right.\right.\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((I*f^(1/4*c*e^2/(c^2*log(f)^2
+ f^2))*f^a*cos(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f
^2)) + f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(1/4*(4*c^2*d*log(f)^2 -
e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x - I*
e)/sqrt(-c*log(f) + I*f)) + (-I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos
(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)) + f^(1/4*c
*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2
)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + I*e)/sqrt(-c*log(
f) - I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) - sqrt(pi)*sqrt(2*c^
2*log(f)^2 + 2*f^2)*((f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(1/4*(4*c^
2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)) - I*f^(1/4*c*e^2/(c^
2*log(f)^2 + f^2))*f^a*sin(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*l
og(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x - I*e)/sqrt(-c*log(f) + I*f
)) + (f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(1/4*(4*c^2*d*log(f)^2 - e
^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)) + I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2
))*f^a*sin(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)))
*erf(1/2*(2*(c*log(f) + I*f)*x + I*e)/sqrt(-c*log(f) - I*f)))*sqrt(-c*log(
f) + sqrt(c^2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

3.122.8 Giac [F]

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)\,dx=\int f^{cx^2+a}\cos\left(fx^2+ex+d\right)\,dx$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*cos(f*x^2 + e*x + d), x)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+a}\cos\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int(f^(a + c*x^2)*cos(d + e*x + f*x^2),x)
```

output
```
int(f^(a + c*x^2)*cos(d + e*x + f*x^2), x)
```

3.123 $\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx$

3.123.1 Optimal result

Integrand size = 23, antiderivative size = 211

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie+x(2if-c\log(f))}{\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}+\frac{e^{2id+\frac{e^2}{2if+c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+x(2if+c\log(f))}{\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}$$

output
```
1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(-
2*I*d-e^2/(2*I*f-c*ln(f)))*f^a*erf((I*e+x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))
^(1/2))*Pi^(1/2)/(2*I*f-c*ln(f))^(1/2)+1/8*exp(2*I*d+e^2/(2*I*f+c*ln(f)))*
f^a*erfi((I*e+x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*
ln(f))^(1/2)
```

3.123.2 Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.19

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{\sqrt{c}\sqrt{\log(f)}}\right.$$
$$+\frac{\sqrt[4]{-1}\left(-e^{\frac{e^2}{-2if+c\log(f)}}\text{erfi}\left(\frac{(-1)^{3/4}(e+2fx+icx\log(f))}{\sqrt{2f+ic\log(f)}}\right)(2f-ic\log(f))\sqrt{2f+ic\log(f)}(\cos(2d)-i\sin(2d))+\right.}{4f^2+c^2\log}$$

input
```
Integrate[f^(a + c*x^2)*Cos[d + e*x + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*((2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + (
(-1)^(1/4)*(-(E^(e^2/((-2*I)*f + c*Log[f]))*Erfi[((-1)^(3/4)*(e + 2*f*x +
I*c*x*Log[f]))/Sqrt[2*f + I*c*Log[f]]]*(2*f - I*c*Log[f])*Sqrt[2*f + I*c*L
og[f]]*(Cos[2*d] - I*Sin[2*d])) + E^(e^2/((2*I)*f + c*Log[f]))*Erfi[((-1)^
(1/4)*(e + 2*f*x - I*c*x*Log[f]))/Sqrt[2*f - I*c*Log[f]]]*Sqrt[2*f - I*c*L
og[f]]*(2*f + I*c*Log[f])*((-I)*Cos[2*d] + Sin[2*d])))/(4*f^2 + c^2*Log[f]
^2)))/8
```

3.123.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx$$
$$\Big\downarrow \text{4976}$$
$$\int\left(\frac{1}{4}f^{a+cx^2}e^{-2id-2iex-2ifx^2}+\frac{1}{4}f^{a+cx^2}e^{2id+2iex+2ifx^2}+\frac{1}{2}f^{a+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{-c\log(f)+2if}-2id}\operatorname{erf}\left(\frac{x(-c\log(f)+2if)+ie}{\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}f^a e^{\frac{e^2}{c\log(f)+2if}+2id}\operatorname{erfi}\left(\frac{x(c\log(f)+2if)+ie}{\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}f^a\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Cos[d + e*x + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^
((-2*I)*d - e^2/((2*I)*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + x*((2*I)*f -
 c*Log[f]))/Sqrt[(2*I)*f - c*Log[f]]])/(8*Sqrt[(2*I)*f - c*Log[f]]) + (E^(
(2*I)*d + e^2/((2*I)*f + c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + x*((2*I)*f +
c*Log[f]))/Sqrt[(2*I)*f + c*Log[f]]])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

3.123.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.123.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.91

method	result
risch	$\frac{\sqrt{\pi}\, f^a \mathrm{e}^{-\frac{2id\ln(f)c+4df-e^2}{c\ln(f)-2if}}\operatorname{erf}\left(x\sqrt{2if-c\ln(f)}+\frac{ie}{\sqrt{2if-c\ln(f)}}\right)}{8\sqrt{2if-c\ln(f)}}-\frac{\sqrt{\pi}\, f^a \mathrm{e}^{\frac{2id\ln(f)c-4df+e^2}{2if+c\ln(f)}}\operatorname{erf}\left(-\sqrt{-c\ln(f)-2if}\,x+\frac{ie}{\sqrt{-c\ln(f)-2}}\right.}{8\sqrt{-c\ln(f)-2if}}$

input
```
int(f^(c*x^2+a)*cos(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*Pi^(1/2)*f^a*exp(-(2*I*d*ln(f)*c+4*d*f-e^2)/(c*ln(f)-2*I*f))/(2*I*f-c*
ln(f))^(1/2)*erf(x*(2*I*f-c*ln(f))^(1/2)+I*e/(2*I*f-c*ln(f))^(1/2))-1/8*Pi
^(1/2)*f^a*exp((2*I*d*ln(f)*c-4*d*f+e^2)/(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f)
^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+I*e/(-c*ln(f)-2*I*f)^(1/2))+1/4*f^a*P
i^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

3.123.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(155) = 310$.

Time = 0.27 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.71

$$\int f^{a+cx^2} \cos^2\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{2\sqrt{\pi}\left(c^2\log\left(f\right)^2+4f^2\right)\sqrt{-c\log\left(f\right)}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x\right)+\sqrt{\pi}\left(c^2\log\left(f\right)^2-2icf\log\left(f\right)\right)\sqrt{-c\log(}}{}$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*lo
g(f))*x) + sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f
)*erf((c^2*x*log(f)^2 + 4*f^2*x + I*c*e*log(f) + 2*e*f)*sqrt(-c*log(f) - 2
*I*f)/(c^2*log(f)^2 + 4*f^2))*e^((a*c^2*log(f)^3 + 2*I*c^2*d*log(f)^2 - 2*
I*e^2*f + 8*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) +
sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf((c^2*
x*log(f)^2 + 4*f^2*x - I*c*e*log(f) + 2*e*f)*sqrt(-c*log(f) + 2*I*f)/(c^2*
log(f)^2 + 4*f^2))*e^((a*c^2*log(f)^3 - 2*I*c^2*d*log(f)^2 + 2*I*e^2*f - 8
*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)))/(c^3*log(f)^
3 + 4*c*f^2*log(f))
```

3.123.6 Sympy [F]

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*cos(f*x**2+e*x+d)**2,x)
```

output
```
Integral(f**(a + c*x**2)*cos(d + e*x + f*x**2)**2, x)
```

3.123.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 863, normalized size of antiderivative = 4.09

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^(c*e^2/(c^2*log(f)^2 + 4
*f^2))*f^a*cos(2*(c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2)
) + f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*sin(2*(c^2*d*log(f)^2 - e^2*f + 4
*d*f^2)/(c^2*log(f)^2 + 4*f^2)))*erf(((c*log(f) - 2*I*f)*x - I*e)/sqrt(-c*
log(f) + 2*I*f)) + (-I*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*cos(2*(c^2*d*l
og(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2)) + f^(c*e^2/(c^2*log(f)^
2 + 4*f^2))*f^a*sin(2*(c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4
*f^2)))*erf(((c*log(f) + 2*I*f)*x + I*e)/sqrt(-c*log(f) - 2*I*f)))*sqrt(c*
log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2
*log(f)^2 + 8*f^2)*((f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*cos(2*(c^2*d*log
(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2)) - I*f^(c*e^2/(c^2*log(f)^
2 + 4*f^2))*f^a*sin(2*(c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4
*f^2)))*erf(((c*log(f) - 2*I*f)*x - I*e)/sqrt(-c*log(f) + 2*I*f)) + (f^(c*
e^2/(c^2*log(f)^2 + 4*f^2))*f^a*cos(2*(c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(
c^2*log(f)^2 + 4*f^2)) + I*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*sin(2*(c^2
*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2)))*erf(((c*log(f) + 2
*I*f)*x + I*e)/sqrt(-c*log(f) - 2*I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^
2 + 4*f^2))*sqrt(-c*log(f)) + 2*sqrt(pi)*((c^2*f^a*log(f)^2 + 4*f^(a + 2))
*erf(x*conjugate(sqrt(-c*log(f)))) + (c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(
sqrt(-c*log(f))*x)))/((c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f)))
```

3.123.8 Giac [F]

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\int f^{cx^2+a}\cos\left(fx^2+ex+d\right)^2\,dx$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*cos(f*x^2 + e*x + d)^2, x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+a}\cos\left(f\,x^2+e\,x+d\right)^2\,dx$$

input
```
int(f^(a + c*x^2)*cos(d + e*x + f*x^2)^2,x)
```

output
```
int(f^(a + c*x^2)*cos(d + e*x + f*x^2)^2, x)
```

3.124 $\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx$

3.124.1 Optimal result

Integrand size = 23, antiderivative size = 369

$$\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx = \frac{3e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}} + \frac{e^{-3id-\frac{9e^2}{4(3if-c\log(f))}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{3ie+2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}} + \frac{3e^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}} + \frac{e^{3id+\frac{9e^2}{4(3if+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3ie+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

output
```
3/16*exp(-I*d-e^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e+2*x*(I*f-c*ln(f)))/(
I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/16*exp(-3*I*d-9/4*e^2/(
3*I*f-c*ln(f)))*f^a*erf(1/2*(3*I*e+2*x*(3*I*f-c*ln(f)))/(3*I*f-c*ln(f))^(1
/2))*Pi^(1/2)/(3*I*f-c*ln(f))^(1/2)+3/16*exp(I*d+e^2/(4*I*f+4*c*ln(f)))*f^
a*erfi(1/2*(I*e+2*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln
(f))^(1/2)+1/16*exp(3*I*d+9/4*e^2/(3*I*f+c*ln(f)))*f^a*erfi(1/2*(3*I*e+2*x
*(3*I*f+c*ln(f)))/(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(f))^(1/2)
```

3.124.2 Mathematica [A] (verified)

Time = 4.96 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.33

$$\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt[4]{-1}f^a\sqrt{\pi}\left(3e^{\frac{e^2}{4if+4c\log(f)}}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(e+2fx-2icx\log(f))}{2\sqrt{f-ic\log(f)}}\right)\sqrt{f-ic\log(f)}\left(9f^3+9icf^2\log(f)+c^2f\log^2(f)+\right.\right.}{}$$

input
```
Integrate[f^(a + c*x^2)*Cos[d + e*x + f*x^2]^3,x]
```

output
```
((-1)^(1/4)*f^a*Sqrt[Pi]*(3*E^(e^2/((4*I)*f + 4*c*Log[f]))*Erfi[((-1)^(1/4
)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*L
og[f]]*(9*f^3 + (9*I)*c*f^2*Log[f] + c^2*f*Log[f]^2 + I*c^3*Log[f]^3)*((-I
)*Cos[d] + Sin[d]) + (f - I*c*Log[f])*(-((3*f - I*c*Log[f])*(3*E^(e^2/((-4
*I)*f + 4*c*Log[f]))*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*S
qrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]]*(3*f + I*c*Log[f])*(Cos[d] - I*
Sin[d]) + E^((9*e^2)/(4*((-3*I)*f + c*Log[f])))*Erfi[((-1)^(3/4)*(3*e + 6*
f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*(f + I*c*Log[f])*Sqrt
[3*f + I*c*Log[f]]*(Cos[3*d] - I*Sin[3*d]))) + E^((9*e^2)/(4*((3*I)*f + c*
Log[f])))*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f -
 I*c*Log[f]])]*Sqrt[3*f - I*c*Log[f]]*(3*f^2 + (4*I)*c*f*Log[f] - c^2*Log[
f]^2)*((-I)*Cos[3*d] + Sin[3*d]))))/(16*(9*f^4 + 10*c^2*f^2*Log[f]^2 + c^4
*Log[f]^4))
```

3.124.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx$$
$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{3}{8}f^{a+cx^2}\exp\left(-3i(d+ex+fx^2)+2id+2iex+2ifx^2\right)+\frac{3}{8}f^{a+cx^2}\exp\left(-3i(d+ex+fx^2)+4id+4iex+4\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^a\exp\left(-\frac{9e^2}{4(-c\log(f)+3if)}-3id\right)\operatorname{erf}\left(\frac{2x(-c\log(f)+3if)+3ie}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{3\sqrt{\pi}f^ae^{-\frac{e^2}{-4c\log(f)+4if}-id}\operatorname{erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}+\frac{3\sqrt{\pi}f^ae^{\frac{e^2}{4c\log(f)+4if}+id}\operatorname{erfi}\left(\frac{2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}}+\frac{\sqrt{\pi}f^ae^{\frac{9e^2}{4(c\log(f)+3if)}+3id}\operatorname{erfi}\left(\frac{2x(c\log(f)+3if)+3ie}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}$$

input
```
Int[f^(a + c*x^2)*Cos[d + e*x + f*x^2]^3,x]
```

output
```
(3*E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + 2*x*(I*
f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(16*Sqrt[I*f - c*Log[f]]) + (E^(
(-3*I)*d - (9*e^2)/(4*((3*I)*f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[((3*I)*e + 2
*x*((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])])/(16*Sqrt[(3*I)*f -
 c*Log[f]]) + (3*E^(I*d + e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I
*e + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/(16*Sqrt[I*f + c*Log
[f]]) + (E^((3*I)*d + (9*e^2)/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[
((3*I)*e + 2*x*((3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f + c*Log[f]])])/(16*Sq
rt[(3*I)*f + c*Log[f]])
```

3.124.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.124.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.91

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4 i d \ln(f) c+12 d f-3 e^{2}\right)}{4(c\ln(f)-3 i f)}} \operatorname{erf}\left(x\sqrt{3 i f-c\ln(f)}+\frac{3 i e}{2\sqrt{3 i f-c\ln(f)}}\right)}{16\sqrt{3 i f-c\ln(f)}}+\frac{3\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{4 i d\ln(f) c+4 d f-e^{2}}{4(c\ln(f)-i f)}} \operatorname{erf}\left(x\sqrt{i f-c\ln(f)}+\frac{i e}{2\sqrt{i f-c}}\right.}{16\sqrt{i f-c\ln(f)}}$

input
```
int(f^(c*x^2+a)*cos(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*Pi^(1/2)*f^a*exp(-3/4*(4*I*d*ln(f)*c+12*d*f-3*e^2)/(c*ln(f)-3*I*f))/(
3*I*f-c*ln(f))^(1/2)*erf(x*(3*I*f-c*ln(f))^(1/2)+3/2*I*e/(3*I*f-c*ln(f))^(
1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c+4*d*f-e^2)/(c*ln(f)-I*f))/
(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2)+1/2*I*e/(I*f-c*ln(f))^(1/2))
-3/16*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c-4*d*f+e^2)/(I*f+c*ln(f)))/(-c*ln
(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*I*e/(-c*ln(f)-I*f)^(1/2))-1
/16*Pi^(1/2)*f^a*exp(3/4*(4*I*d*ln(f)*c-12*d*f+3*e^2)/(3*I*f+c*ln(f)))/(-c
*ln(f)-3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1/2)*x+3/2*I*e/(-c*ln(f)-3*I*f)
^(1/2))
```

3.124.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 707 vs. 2(269) = 538.

Time = 0.31 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.92

$$\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\left(c^3\log(f)^3-3i\,c^2f\log(f)^2+cf^2\log(f)-3i\,f^3\right)\sqrt{-c\log(f)-3i\,f}\,\operatorname{erf}\left(\frac{\left(2\,c^2x\log(f)^2+18\,f^2x+3i\,ce\log(\right.}{2\left(c^2\log(f)^2\right.}\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(pi)*(c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + c*f^2*log(f) - 3*I*f^
3)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 18*f^2*x + 3*I*c*e*
log(f) + 9*e*f)*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(4*
a*c^2*log(f)^3 + 12*I*c^2*d*log(f)^2 - 27*I*e^2*f + 108*I*d*f^2 + 9*(c*e^2
 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(c^3*log(f)^3 + 3*I
*c^2*f*log(f)^2 + c*f^2*log(f) + 3*I*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*
(2*c^2*x*log(f)^2 + 18*f^2*x - 3*I*c*e*log(f) + 9*e*f)*sqrt(-c*log(f) + 3*
I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(4*a*c^2*log(f)^3 - 12*I*c^2*d*log(f)^
2 + 27*I*e^2*f - 108*I*d*f^2 + 9*(c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 +
 9*f^2)) + 3*sqrt(pi)*(c^3*log(f)^3 - I*c^2*f*log(f)^2 + 9*c*f^2*log(f) -
9*I*f^3)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x + I*c*e
*log(f) + e*f)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*c^2
*log(f)^3 + 4*I*c^2*d*log(f)^2 - I*e^2*f + 4*I*d*f^2 + (c*e^2 + 4*a*f^2)*l
og(f))/(c^2*log(f)^2 + f^2)) + 3*sqrt(pi)*(c^3*log(f)^3 + I*c^2*f*log(f)^2
 + 9*c*f^2*log(f) + 9*I*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*c^2*x*log(f)
^2 + 2*f^2*x - I*c*e*log(f) + e*f)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f
^2))*e^(1/4*(4*a*c^2*log(f)^3 - 4*I*c^2*d*log(f)^2 + I*e^2*f - 4*I*d*f^2 +
 (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f
^2*log(f)^2 + 9*f^4)
```

3.124.6 Sympy [F]

$$\int f^{a+cx^2} \cos^3\left(d+ex+fx^2\right)\,dx = \int f^{a+cx^2} \cos^3\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*cos(f*x**2+e*x+d)**3,x)
```

output
```
Integral(f**(a + c*x**2)*cos(d + e*x + f*x**2)**3, x)
```

3.124.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2180 vs. 2(269) = 538.

Time = 0.30 (sec) , antiderivative size = 2180, normalized size of antiderivative = 5.91

$$\int f^{a+cx^2} \cos^3\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((I*c^2*f^(9/4*c*e^2/(c^2*lo
g(f)^2 + 9*f^2))*f^a*log(f)^2 + I*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^(
a + 2))*cos(3/4*(4*c^2*d*log(f)^2 - 9*e^2*f + 36*d*f^2)/(c^2*log(f)^2 + 9*
f^2)) + (c^2*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^a*log(f)^2 + f^(9/4*c*
e^2/(c^2*log(f)^2 + 9*f^2))*f^(a + 2))*sin(3/4*(4*c^2*d*log(f)^2 - 9*e^2*f
 + 36*d*f^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) - 3*I*f)*x - 3*
I*e)/sqrt(-c*log(f) + 3*I*f)) + ((-I*c^2*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^
2))*f^a*log(f)^2 - I*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^(a + 2))*cos(3
/4*(4*c^2*d*log(f)^2 - 9*e^2*f + 36*d*f^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*
f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^a*log(f)^2 + f^(9/4*c*e^2/(c^2*log(
f)^2 + 9*f^2))*f^(a + 2))*sin(3/4*(4*c^2*d*log(f)^2 - 9*e^2*f + 36*d*f^2)/
(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) + 3*I*f)*x + 3*I*e)/sqrt(-c*
log(f) - 3*I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)
*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((-I*c^2*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))
*f^a*log(f)^2 - 9*I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^(a + 2))*cos(1/4*
(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)) - (c^2*f^(1/4*c
*e^2/(c^2*log(f)^2 + f^2))*f^a*log(f)^2 + 9*f^(1/4*c*e^2/(c^2*log(f)^2 + f
^2))*f^(a + 2))*sin(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2
 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x - I*e)/sqrt(-c*log(f) + I*f)) + ((
I*c^2*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*log(f)^2 + 9*I*f^(1/4*c*e^...
```

3.124.8 Giac [F]

$$\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx=\int f^{cx^2+a}\cos\left(fx^2+ex+d\right)^3\,dx$$

input
```
integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + a)*cos(f*x^2 + e*x + d)^3, x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+a}\cos\left(f\,x^2+e\,x+d\right)^3\,dx$$

input
```
int(f^(a + c*x^2)*cos(d + e*x + f*x^2)^3,x)
```

output
```
int(f^(a + c*x^2)*cos(d + e*x + f*x^2)^3, x)
```

3.125 $\int f^{a+bx+cx^2}\cos(d+ex)\,dx$

3.125.1 Optimal result

Integrand size = 19, antiderivative size = 172

$$\int f^{a+bx+cx^2}\cos(d+ex)\,dx = -\frac{e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/4*exp(-I*d+1/4*(e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-I*e+b*ln(f)+2*c*x
*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(I*d+1/4*
(e-I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln
(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

3.125.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int f^{a+bx+cx^2}\cos(d+ex)\,dx$$
$$= \frac{e^{\frac{e(e-2ib\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(e^{\frac{ibe}{c}}\operatorname{erfi}\left(\frac{-ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)-i\sin(d))+\operatorname{erfi}\left(\frac{ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d\right.}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x],x]
```

output
```
(E^((e*(e - (2*I)*b*Log[f]))/(4*c*Log[f]))*f^(a - b^2/(4*c))*Sqrt[Pi]*(E^(
(I*b*e)/c)*Erfi[((-I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(C
os[d] - I*Sin[d]) + Erfi[(I*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]
])]*(Cos[d] + I*Sin[d])))/(4*Sqrt[c]*Sqrt[Log[f]])
```

3.125.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(d+ex)f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{1}{2}e^{-id-iex}f^{a+bx+cx^2}+\frac{1}{2}e^{id+iex}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+id}\text{erfi}\left(\frac{b\log(f)+2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\text{erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cos[d + e*x],x]
```

output
```
-1/4*(E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e
- b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]
]) + (E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + b
*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]
)
```

3.125.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.125.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} {\mathrm e}^{\frac{2 i \ln(f) b e -4 i d \ln(f) c +e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x +\frac{b \ln(f)-i e}{2 \sqrt{-c \ln(f)}}\right)}{4 \sqrt{-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} {\mathrm e}^{-\frac{2 i \ln(f) b e -4 i d \ln(f) c -e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f}\right.}{4 \sqrt{-c \ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*cos(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(1/4*(2*I*ln(f)*b*e-4*I*d*ln(f)*c+e^2)
/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-I*e)/(-c*l
n(f))^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-1/4*(2*I*ln(f)*b*e-4*I*d
*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(I*e+b
*ln(f))/(-c*ln(f))^(1/2))
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.02

$$\int f^{a+bx+cx^2}\cos(d+ex)\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\frac{\left((2\,cx+b)\log(f)-i\,e\right)\sqrt{-c\log(f)}}{2\,c\log(f)}\right)e^{\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-e^2+2\,(2i\,cd-i\,be)\log(f)}{4\,c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{er}}{4\,c\log\left(f\right)}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(e*x+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - I*e)*sqrt(-c*
log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - e^2 + 2*(2*I*c*d - I
*b*e)*log(f))/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*
log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2
- e^2 + 2*(-2*I*c*d + I*b*e)*log(f))/(c*log(f))))/(c*log(f))
```

3.125.6 Sympy [F]

$$\int f^{a+bx+cx^2}\cos(d+ex)\,dx = \int f^{a+bx+cx^2}\cos(d+ex)\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cos(e*x+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cos(d + e*x), x)
```

3.125.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.06

$$\int f^{a+bx+cx^2}\cos(d+ex)\,dx =$$

$$-\frac{\sqrt{\pi}\left(f^a\left(\cos\left(-\frac{2\,cd-be}{2\,c}\right)-i\,\sin\left(-\frac{2\,cd-be}{2\,c}\right)\right)\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}-\frac{1}{2}\left(b\log(f)+i\,e\right)\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^2}{4\,c\log(f)}\right)}+}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(e*x+d),x, algorithm="maxima")
```

output
```
-1/8*sqrt(pi)*(f^a*(cos(-1/2*(2*c*d - b*e)/c) - I*sin(-1/2*(2*c*d - b*e)/c
))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + I*e)*conjugate(1/sqr
t(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(cos(-1/2*(2*c*d - b*e)/c) + I
*sin(-1/2*(2*c*d - b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(
f) - I*e)*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(cos(
-1/2*(2*c*d - b*e)/c) - I*sin(-1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f)
 + b*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))) + f^
a*(cos(-1/2*(2*c*d - b*e)/c) + I*sin(-1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x
*log(f) + b*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f)
)))*sqrt(-c*log(f))/(c*f^(1/4*b^2/c)*log(f))
```

3.125.8 Giac [F]

$$\int f^{a+bx+cx^2}\cos(d+ex)\,dx=\int f^{cx^2+bx+a}\cos(ex+d)\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(e*x+d),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*cos(e*x + d), x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos(d+ex)\,dx=\int f^{c\,x^2+b\,x+a}\cos(d+e\,x)\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cos(d + e*x),x)
```

output
```
int(f^(a + b*x + c*x^2)*cos(d + e*x), x)
```

3.126 $\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx$

3.126.1 Optimal result

Integrand size = 21, antiderivative size = 231

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id+\frac{(2e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{2ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{e^{2id-\frac{(2ie+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{2ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/8*exp(-2*I*d+1/4*(2*e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-2*I*e+b*ln(f)
+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*
I*d-1/4*(2*I*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(2*I*e+b*ln(f)+2*c*x*ln(f)
)/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^(a-1/4*b^2/c)*er
fi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.88

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx$$
$$= \frac{e^{-\frac{ibe}{c}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(2e^{\frac{ibe}{c}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)+e^{\frac{e(e+2ib\log(f))}{c\log(f)}}\operatorname{erfi}\left(\frac{-2ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(2d)-i\sin(2d))+e^{\frac{e^2}{c\log(\ldots}}\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*(2*E^((I*b*e)/c)*Erfi[((b + 2*c*x)*Sqrt[Log[f]
])/(2*Sqrt[c])] + E^((e*(e + (2*I)*b*Log[f]))/(c*Log[f]))*Erfi[((-2*I)*e +
 (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] - I*Sin[2*d]) + E
^(e^2/(c*Log[f]))*Erfi[((2*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[
f]])]*(Cos[2*d] + I*Sin[2*d])))/(8*Sqrt[c]*E^((I*b*e)/c)*Sqrt[Log[f]])
```

3.126.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(d+ex)f^{a+bx+cx^2}\,dx$$
$$\Big\downarrow \; 4976$$
$$\int \left(\frac{1}{4}e^{-2id-2iex}f^{a+bx+cx^2}+\frac{1}{4}e^{2id+2iex}f^{a+bx+cx^2}+\frac{1}{2}f^{a+bx+cx^2}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{(2e+ib\log(f))^2}{4c\log(f)}-2id}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+$$
$$\frac{\sqrt{\pi}f^a e^{2id-\frac{(b\log(f)+2ie)^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cos[d + e*x]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) - (E^((-2*I)*d + (2*e + I*b*Log[f])^2/(4*c*Log[f]
))*f^a*Sqrt[Pi]*Erfi[((2*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[L
og[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^((2*I)*d - ((2*I)*e + b*Log[f])^2/
(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqr
t[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

3.126.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.126.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{\frac{i \ln(f) b e-2 i d \ln(f) c+e^{2}}{\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{b\ln(f)-2ie}{2\sqrt{-c\ln(f)}}\right)}{8\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{-\frac{i \ln(f) b e-2 i d \ln(f) c-e^{2}}{\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\right.}{8\sqrt{-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*cos(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp((I*ln(f)*b*e-2*I*d*ln(f)*c+e^2)/ln(f)
/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*I*e)/(-c*ln(f)
)^(1/2))-1/8*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-(I*ln(f)*b*e-2*I*d*ln(f)*c-e
^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*I*e+b*ln(f))/
(-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^(-1/4*b^2/c)*f^a/(-c*ln(f))^(1/2)*erf(-(-
c*ln(f))^(1/2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.97

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-c\log(f)}\,\operatorname{erf}\left(\frac{((2\,cx+b)\log(f)-2i\,e)\sqrt{-c\log(f)}}{2\,c\log(f)}\right)e^{\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-4\,e^2+4\,(2i\,cd-i\,be)\log(f)}{4\,c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log(f)}\,e}{8\,c\log}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - 2*I*e)*sqrt(-
c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*e^2 + 4*(2*I*c*d
 - I*b*e)*log(f))/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x +
 b)*log(f) + 2*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log
(f)^2 - 4*e^2 + 4*(-2*I*c*d + I*b*e)*log(f))/(c*log(f))) + 2*sqrt(pi)*sqrt
(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c)
)/(c*log(f))
```

3.126.6 Sympy [F]

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx = \int f^{a+bx+cx^2}\cos^2(d+ex)\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cos(e*x+d)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cos(d + e*x)**2, x)
```

3.126.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.73

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx$$
$$=\frac{\sqrt{\pi}\left(f^a\left(\cos\left(-\frac{2\,cd-be}{c}\right)-i\,\sin\left(-\frac{2\,cd-be}{c}\right)\right)\operatorname{erf}\left(x\overline{\sqrt{-c\log\left(f\right)}}-\frac{1}{2}\left(b\log\left(f\right)+2i\,e\right)\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^2}{c\log(f)}\right)}+f\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*(f^a*(cos(-(2*c*d - b*e)/c) - I*sin(-(2*c*d - b*e)/c))*erf(x
*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + 2*I*e)*conjugate(1/sqrt(-c*l
og(f))))*e^(e^2/(c*log(f))) + f^a*(cos(-(2*c*d - b*e)/c) + I*sin(-(2*c*d -
 b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) - 2*I*e)*conjug
ate(1/sqrt(-c*log(f))))*e^(e^2/(c*log(f))) + f^a*(cos(-(2*c*d - b*e)/c) -
I*sin(-(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f) + 2*I*e)*sqrt(-c
*log(f))/(c*log(f)))*e^(e^2/(c*log(f))) + f^a*(cos(-(2*c*d - b*e)/c) + I*s
in(-(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f) - 2*I*e)*sqrt(-c*lo
g(f))/(c*log(f)))*e^(e^2/(c*log(f))) + 2*f^a*erf(-1/2*b*conjugate(1/sqrt(-
c*log(f)))*log(f) + x*conjugate(sqrt(-c*log(f)))) - 2*f^a*erf(1/2*(2*c*x*l
og(f) + b*log(f))/sqrt(-c*log(f))))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

3.126.8 Giac [F]

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx=\int f^{cx^2+bx+a}\cos\left(ex+d\right)^2\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*cos(e*x + d)^2, x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx = \int f^{c\,x^2+b\,x+a}\cos\left(d+e\,x\right)^2\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cos(d + e*x)^2,x)
```

output
```
int(f^(a + b*x + c*x^2)*cos(d + e*x)^2, x)
```

3.127 $\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx$

3.127.1 Optimal result

Integrand size = 21, antiderivative size = 346

$$\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx=-\frac{3e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-3id+\frac{(3e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{e^{3id-\frac{(3ie+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

output
```
3/16*exp(-I*d+1/4*(e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-I*e+b*ln(f)+2*c*
x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(-3*I*d
+1/4*(3*e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-3*I*e+b*ln(f)+2*c*x*ln(f))/
c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+3/16*exp(I*d+1/4*(e-I*b*
ln(f))^2/c/ln(f))*f^a*erfi(1/2*(I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/
2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(3*I*d-1/4*(3*I*e+b*ln(f))^2/c/ln
(f))*f^a*erfi(1/2*(3*I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2
)/c^(1/2)/ln(f)^(1/2)
```

3.127.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.12

$$\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx$$
$$= \frac{e^{\frac{e(e-6ib\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(e^{\frac{e(2e+3ib\log(f))}{c\log(f)}}\cos(3d)\text{erfi}\left(\frac{-3ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+e^{\frac{2e^2}{c\log(f)}}\cos(3d)\text{erfi}\left(\frac{3ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+\right.}{}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x]^3,x]
```

output
```
(E^((e*(e - (6*I)*b*Log[f]))/(4*c*Log[f]))*f^(a - b^2/(4*c))*Sqrt[Pi]*(E^(
(e*(2*e + (3*I)*b*Log[f]))/(c*Log[f]))*Cos[3*d]*Erfi[((-3*I)*e + (b + 2*c*
x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + E^((2*e^2)/(c*Log[f]))*Cos[3*d]*Erf
i[((3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^(((2*I)*b
*e)/c)*Erfi[((-I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[d
] - I*Sin[d]) + 3*E^((I*b*e)/c)*Erfi[(I*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])]*(Cos[d] + I*Sin[d]) - I*E^((e*(2*e + (3*I)*b*Log[f]))/(c*L
og[f]))*Erfi[((-3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*Sin
[3*d] + I*E^((2*e^2)/(c*Log[f]))*Erfi[((3*I)*e + (b + 2*c*x)*Log[f])/(2*Sq
rt[c]*Sqrt[Log[f]])]*Sin[3*d]))/(16*Sqrt[c]*Sqrt[Log[f]])
```

3.127.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(d+ex)f^{a+bx+cx^2}\,dx$$
$$\Big\downarrow \text{4976}$$
$$\int \left(\frac{3}{8}e^{-id-iex}f^{a+bx+cx^2}+\frac{3}{8}e^{id+iex}f^{a+bx+cx^2}+\frac{1}{8}e^{-3id-3iex}f^{a+bx+cx^2}+\frac{1}{8}e^{3id+3iex}f^{a+bx+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$
-\frac{3\sqrt{\pi}f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\text{erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{\sqrt{\pi}f^a e^{\frac{(3e+ib\log(f))^2}{4c\log(f)}-3id}\text{erfi}\left(\frac{-b\log(f)-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{3\sqrt{\pi}f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+id}\text{erfi}\left(\frac{b\log(f)+2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi}f^a e^{3id-\frac{(b\log(f)+3ie)^2}{4c\log(f)}}\text{erfi}\left(\frac{b\log(f)+2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}
$$

input
```
Int[f^(a + b*x + c*x^2)*Cos[d + e*x]^3,x]
```

output
```
(-3*E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e -
b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f
]]) - (E^((-3*I)*d + (3*e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[
((3*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]
*Sqrt[Log[f]]) + (3*E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]
*Erfi[(I*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[
c]*Sqrt[Log[f]]) + (E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4*c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[((3*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])
])/(16*Sqrt[c]*Sqrt[Log[f]])
```

3.127.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.127.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{\frac{\frac{3i\ln(f)be}{2}-3id\ln(f)c+\frac{9e^{2}}{4}}{c\ln(f)}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{b\ln(f)-3ie}{2\sqrt{-c\ln(f)}}\right)}{16\sqrt{-c\ln(f)}}-\frac{3\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{\frac{2i\ln(f)be-4id\ln(f)c+e^{2}}{4\ln(f)c}} \operatorname{erf}\left(-\sqrt{-c\ln\right.}{16\sqrt{-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*cos(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(3/4*(2*I*ln(f)*b*e-4*I*d*ln(f)*c+3*e
^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-3*I*e)/
(-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(1/4*(2*I*ln(f)*b*e-
4*I*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(
b*ln(f)-I*e)/(-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-1/4*(
2*I*ln(f)*b*e-4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))
^(1/2)*x+1/2*(I*e+b*ln(f))/(-c*ln(f))^(1/2))-1/16*Pi^(1/2)*f^a*f^(-1/4*b^2
/c)*exp(-3/4*(2*I*ln(f)*b*e-4*I*d*ln(f)*c-3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)
*erf(-(-c*ln(f))^(1/2)*x+1/2*(3*I*e+b*ln(f))/(-c*ln(f))^(1/2))
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.99

$$\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx =$$

$$-\frac{3\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{((2\,cx+b)\log(f)-i\,e)\sqrt{-c\log(f)}}{2\,c\log(f)}\right)e^{\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-e^2+2\,(2i\,cd-i\,be)\log(f)}{4\,c\log(f)}\right)}+3\sqrt{\pi}\sqrt{-c\log(f}}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*(3*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - I*e)*sqrt(
-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - e^2 + 2*(2*I*c*d
- I*b*e)*log(f))/(c*log(f))) + 3*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x
+ b)*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(
f)^2 - e^2 + 2*(-2*I*c*d + I*b*e)*log(f))/(c*log(f))) + sqrt(pi)*sqrt(-c*l
og(f))*erf(1/2*((2*c*x + b)*log(f) - 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^
(-1/4*((b^2 - 4*a*c)*log(f)^2 - 9*e^2 + 6*(2*I*c*d - I*b*e)*log(f))/(c*log
(f))) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) + 3*I*e)*sqrt
(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - 9*e^2 + 6*(-2*I*
c*d + I*b*e)*log(f))/(c*log(f))))/(c*log(f))
```

3.127.6 Sympy [F]

$$\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx=\int f^{a+bx+cx^2}\cos^3{(d+ex)}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cos(e*x+d)**3,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cos(d + e*x)**3, x)
```

3.127.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.29 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.97

$$\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^3,x, algorithm="maxima")
```

output
```
-1/32*sqrt(pi)*(f^a*(cos(-3/2*(2*c*d - b*e)/c) - I*sin(-3/2*(2*c*d - b*e)/
c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + 3*I*e)*conjugate(1/
sqrt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*(cos(-3/2*(2*c*d - b*e)/c)
+ I*sin(-3/2*(2*c*d - b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*l
og(f) - 3*I*e)*conjugate(1/sqrt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*
(cos(-3/2*(2*c*d - b*e)/c) - I*sin(-3/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*l
og(f) + b*log(f) + 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(9/4*e^2/(c*log(f)
)) + f^a*(cos(-3/2*(2*c*d - b*e)/c) + I*sin(-3/2*(2*c*d - b*e)/c))*erf(1/2
*(2*c*x*log(f) + b*log(f) - 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(9/4*e^2/
(c*log(f))) + 3*f^a*(cos(-1/2*(2*c*d - b*e)/c) - I*sin(-1/2*(2*c*d - b*e)/
c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + I*e)*conjugate(1/sq
rt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + 3*f^a*(cos(-1/2*(2*c*d - b*e)/c)
+ I*sin(-1/2*(2*c*d - b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*l
og(f) - I*e)*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + 3*f^a*
(cos(-1/2*(2*c*d - b*e)/c) - I*sin(-1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*l
og(f) + b*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f)))
 + 3*f^a*(cos(-1/2*(2*c*d - b*e)/c) + I*sin(-1/2*(2*c*d - b*e)/c))*erf(1/2
*(2*c*x*log(f) + b*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c
*log(f))))*sqrt(-c*log(f))/(c*f^(1/4*b^2/c)*log(f))
```

3.127.8 Giac [F]

$$\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx=\int f^{cx^2+bx+a}\cos\left(ex+d\right)^3\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*cos(e*x + d)^3, x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx=\int f^{c\,x^2+b\,x+a}\cos\left(d+e\,x\right)^3\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cos(d + e*x)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)*cos(d + e*x)^3, x)
```

3.128 $\int f^{a+bx+cx^2} \cos(d+fx^2)\,dx$

3.128.1 Optimal result

Integrand size = 21, antiderivative size = 189

$$\int f^{a+bx+cx^2}\cos(d+fx^2)\,dx = -\frac{e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}} + \frac{e^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

output
```
-1/4*exp(-I*d+b^2*ln(f)^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(I*f
-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/4*exp(I*d-b
^2*ln(f)^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(I*f+c*ln(f)))/(I*
f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

3.128.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.22

$$\int f^{a+bx+cx^2}\cos(d+fx^2)\,dx =$$

$$-\frac{(-1)^{3/4}e^{\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\left(\operatorname{erfi}\left(\frac{(-1)^{3/4}(2fx+i(b+2cx)\log(f))}{2\sqrt{f+ic\log(f)}}\right)(f-ic\log(f))\sqrt{f+ic\log(f)}(-i\cos(d)-\sin(\right.}{4\left(f^2+c^2\right.}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cos[d + f*x^2],x]
```

output
```
-1/4*((-1)^(3/4)*E^((b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*(E
rfi[((-1)^(3/4)*(2*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*
(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*((-I)*Cos[d] - Sin[d]) + E^(((I/2)*b
^2*f*Log[f]^2)/(f^2 + c^2*Log[f]^2))*Erfi[((-1)^(1/4)*(2*f*x - I*(b + 2*c*
x)*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*(f + I*c*Log[f]
)*(Cos[d] + I*Sin[d])))/(f^2 + c^2*Log[f]^2)
```

3.128.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos\left(d+fx^2\right) f^{a+bx+cx^2} \, dx$$

$$\Big\downarrow \text{ 4976}$$

$$\int \left(\frac{1}{2} e^{-id-ifx^2} f^{a+bx+cx^2}+\frac{1}{2} e^{id+ifx^2} f^{a+bx+cx^2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\pi} f^a e^{id-\frac{b^2 \log^2(f)}{4c\log(f)+4if}} \text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}-\frac{\sqrt{\pi} f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id} \text{erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cos[d + f*x^2],x]
```

output
```
-1/4*(E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[
(b*Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f - c*
Log[f]] + (E^(I*d - (b^2*Log[f]^2)/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Er
fi[(b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/(4*Sqrt[I*
f + c*Log[f]])
```

3.128.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.128.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4 i d \ln(f) c+4 d f}{4(c \ln(f)-i f)}} \operatorname{erf}\left(-x \sqrt{i f-c \ln(f)}+\frac{\ln(f) b}{2 \sqrt{i f-c \ln(f)}}\right)}{4 \sqrt{i f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-4 i d \ln(f) c+4 d f}{4(i f+c \ln(f))}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-i f}\right.}{4 \sqrt{-c \ln(f)-i f}}$

input
```
int(f^(c*x^2+b*x+a)*cos(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*d*ln(f)*c+4*d*f)/(c*ln(f)-I*f)
)/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(I*f-c*ln(f))
^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*d*ln(f)*c+4*d*f)/(I*f+c
*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*ln(f)*b/(-c*
ln(f)-I*f)^(1/2))
```

3.128.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(145) = 290.

Time = 0.26 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.65

$$\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}(c\log(f)-i\,f)\sqrt{-c\log(f)-i\,f}\,\operatorname{erf}\left(\frac{\left(2\,f^2x-i\,bf\log(f)+\left(2\,c^2x+bc\right)\log(f)^2\right)\sqrt{-c\log(f)-i\,f}}{2\left(c^2\log(f)^2+f^2\right)}\right)e^{\left(\frac{4\,af^2\log(f)-\left(b^2c-\right.}{}\right.}}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*(c*log(f) - I*f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x - I
*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^
2 + f^2))*e^(1/4*(4*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2
+ (4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(c*log(
f) + I*f)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + I*b*f*log(f) + (2*c^2*x
 + b*c)*log(f)^2)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*
f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2 + (-4*I*c^2*d - I*b^2*
f)*log(f)^2)/(c^2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

3.128.6 Sympy [F]

$$\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cos(f*x**2+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cos(d + f*x**2), x)
```

3.128.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(145) = 290$.

Time = 0.25 (sec) , antiderivative size = 648, normalized size of antiderivative = 3.43

$$\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)\,dx$$

$$=\frac{\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+2\,f^2}\left(\left(i\,f^a\cos\left(\frac{4\,df^2+\left(4\,c^2d+b^2f\right)\log(f)^2}{4\left(c^2\log(f)^2+f^2\right)}\right)+f^a\sin\left(\frac{4\,df^2+\left(4\,c^2d+b^2f\right)\log(f)^2}{4\left(c^2\log(f)^2+f^2\right)}\right)\right)\operatorname{erf}\left(\frac{2\left(c\log(\cdots}{2\sqrt{\cdots}}\right.\cdots}{\cdots}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d),x, algorithm="maxima")
```

output
```
1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((I*f^a*cos(1/4*(4*d*f^2 + (4*c
^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + f^a*sin(1/4*(4*d*f^2 + (4*
c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f
)*x + b*log(f))/sqrt(-c*log(f) + I*f)) + (-I*f^a*cos(1/4*(4*d*f^2 + (4*c^2
*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + f^a*sin(1/4*(4*d*f^2 + (4*c^
2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*
x + b*log(f))/sqrt(-c*log(f) - I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 +
f^2)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((f^a*cos(1/4*(4*d*f^2 + (4*
c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) - I*f^a*sin(1/4*(4*d*f^2 +
(4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) -
I*f)*x + b*log(f))/sqrt(-c*log(f) + I*f)) + (f^a*cos(1/4*(4*d*f^2 + (4*c^2
*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + I*f^a*sin(1/4*(4*d*f^2 + (4*
c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f
)*x + b*log(f))/sqrt(-c*log(f) - I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2
 + f^2)))/(c^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + f^2*
e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))
```

3.128.8 Giac [F]

$$\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)\,dx=\int f^{cx^2+bx+a}\cos\left(fx^2+d\right)\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + d), x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\cos\left(f\,x^2+d\right)\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cos(d + f*x^2),x)
```

output
```
int(f^(a + b*x + c*x^2)*cos(d + f*x^2), x)
```

3.129 $\int f^{a+bx+cx^2} \cos^2(d+fx^2)\,dx$

3.129.1 Optimal result

Integrand size = 23, antiderivative size = 245

$$\int f^{a+bx+cx^2}\cos^2(d+fx^2)\,dx=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}+\frac{e^{2id-\frac{b^2\log^2(f)}{8if+4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(2if+c\log(f))}{2\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}$$

output
```
1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/
2)/ln(f)^(1/2)-1/8*exp(-2*I*d+b^2*ln(f)^2/(8*I*f-4*c*ln(f)))*f^a*erf(1/2*(
b*ln(f)-2*x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f-c*ln(f
))^(1/2)+1/8*exp(2*I*d-b^2*ln(f)^2/(8*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f
)+2*x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*ln(f))^(1/
2)
```

3.129.2 Mathematica [A] (warning: unable to verify)

Time = 2.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.23

$$\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx=\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2f^{-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{\sqrt{c}\sqrt{\log(f)}}\right.$$
$$+\frac{\sqrt[4]{-1}e^{\frac{b^2\log^2(f)}{8if-4c\log(f)}}\left(-\operatorname{erfi}\left(\frac{(-1)^{3/4}(4fx+i(b+2cx)\log(f))}{2\sqrt{2f+ic\log(f)}}\right)(2f-ic\log(f))\sqrt{2f+ic\log(f)}(\cos(2d)-i\sin(2d))\right.}{4f^2+c^2}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cos[d + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*((2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f
^(b^2/(4*c))*Sqrt[Log[f]]) + ((-1)^(1/4)*E^((b^2*Log[f]^2)/((8*I)*f - 4*c*
Log[f]))*(-(Erfi[((-1)^(3/4)*(4*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[2*f +
 I*c*Log[f]])]*(2*f - I*c*Log[f])*Sqrt[2*f + I*c*Log[f]]*(Cos[2*d] - I*Sin
[2*d])) + E^((I*b^2*f*Log[f]^2)/(4*f^2 + c^2*Log[f]^2))*Erfi[((-1)^(1/4)*(
4*f*x - I*(b + 2*c*x)*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Sqrt[2*f - I*c*
Log[f]]*(2*f + I*c*Log[f])*((-I)*Cos[2*d] + Sin[2*d])))/(4*f^2 + c^2*Log[f
]^2)))/8
```

3.129.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2\left(d+fx^2\right)f^{a+bx+cx^2}\,dx$$
$$\Big\downarrow \text{4976}$$
$$\int\left(\frac{1}{4}e^{-2id-2ifx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2id+2ifx^2}f^{a+bx+cx^2}+\frac{1}{2}f^{a+bx+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+8if}-2id}\operatorname{erf}\left(\frac{b\log(f)-2x(-c\log(f)+2if)}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}f^a e^{2id-\frac{b^2\log^2(f)}{4c\log(f)+8if}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2if)}{2\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cos[d + f*x^2]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) - (E^((-2*I)*d + (b^2*Log[f]^2)/((8*I)*f - 4*c*Lo
g[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*((2*I)*f - c*Log[f]))/(2*Sqrt[(2*I
)*f - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) + (E^((2*I)*d - (b^2*Log[f
]^2)/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((2*I)*f +
c*Log[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

3.129.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.129.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{-\frac{\ln(f)^{2} b^{2}+8 i d \ln(f) c+16 d f}{4(c\ln(f)-2 i f)}} \operatorname{erf}\left(-x\sqrt{2 i f-c\ln(f)}+\frac{\ln(f) b}{2\sqrt{2 i f-c\ln(f)}}\right)}{8\sqrt{2 i f-c\ln(f)}}-\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{-\frac{\ln(f)^{2} b^{2}-8 i d \ln(f) c+16 d f}{4(2 i f+c\ln(f))}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-\right.}{8\sqrt{-c\ln(f)-2 i f}}$

input
```
int(f^(c*x^2+b*x+a)*cos(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+8*I*d*ln(f)*c+16*d*f)/(c*ln(f)-2*I
*f))/(2*I*f-c*ln(f))^(1/2)*erf(-x*(2*I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*I*f
-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-8*I*d*ln(f)*c+16*d
*f)/(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+
1/2*ln(f)*b/(-c*ln(f)-2*I*f)^(1/2))-1/4*Pi^(1/2)*f^(-1/4*b^2/c)*f^a/(-c*ln
(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

3.129.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(185) = 370$.

Time = 0.27 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.64

$$\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\left(c^2\log\left(f\right)^2-2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-2i\,f}\,\operatorname{erf}\left(\frac{\left(8\,f^2x-2i\,bf\log(f)+\left(2\,c^2x+bc\right)\log(f)^2\right)\sqrt{-c\log(f)-2i\,f}}{2\left(c^2\log(f)^2+4\,f^2\right)}\right)e}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf
(1/2*(8*f^2*x - 2*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f)
- 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*(16*a*f^2*log(f) - (b^2*c - 4*a*c^
2)*log(f)^3 + 32*I*d*f^2 - 2*(-4*I*c^2*d - I*b^2*f)*log(f)^2)/(c^2*log(f)^
2 + 4*f^2)) + sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*
I*f)*erf(1/2*(8*f^2*x + 2*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c
*log(f) + 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*(16*a*f^2*log(f) - (b^2*c
- 4*a*c^2)*log(f)^3 - 32*I*d*f^2 - 2*(4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*
log(f)^2 + 4*f^2)) + 2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*erf
(1/2*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log(f)^3
 + 4*c*f^2*log(f))
```

3.129.6 Sympy [F]

$$\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cos(f*x**2+d)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cos(d + f*x**2)**2, x)
```

3.129.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 997, normalized size of antiderivative = 4.07

$$\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^a*f^(1/4*b^2/c)*cos(1/2*
(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)) + f^a*f^(1
/4*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 +
4*f^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*log(f))/sqrt(-c*log(f) + 2*I*
f)) + (-I*f^a*f^(1/4*b^2/c)*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2
)/(c^2*log(f)^2 + 4*f^2)) + f^a*f^(1/4*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d
 + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) + 2*I*f)
*x + b*log(f))/sqrt(-c*log(f) - 2*I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2
 + 4*f^2))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((f^a*f
^(1/4*b^2/c)*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2
 + 4*f^2)) - I*f^a*f^(1/4*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log
(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*log(f)
)/sqrt(-c*log(f) + 2*I*f)) + (f^a*f^(1/4*b^2/c)*cos(1/2*(16*d*f^2 + (4*c^2
*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)) + I*f^a*f^(1/4*b^2/c)*sin(1/
2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2
*(2*(c*log(f) + 2*I*f)*x + b*log(f))/sqrt(-c*log(f) - 2*I*f)))*sqrt(-c*log
(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)) + 2*sqrt(pi)*((c^2*f^a*e
^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4
*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2)))*erf(-1/2*b*conjugate(1/sqrt(-c*lo
g(f)))*log(f) + x*conjugate(sqrt(-c*log(f)))) - (c^2*f^a*e^(1/4*b^2*c*l...
```

3.129.8 Giac [F]

$$\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx=\int f^{cx^2+bx+a}\cos\left(fx^2+d\right)^2\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + d)^2, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\cos\left(f\,x^2+d\right)^2\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cos(d + f*x^2)^2,x)
```

output
```
int(f^(a + b*x + c*x^2)*cos(d + f*x^2)^2, x)
```

3.130 $\int f^{a+bx+cx^2} \cos^3\left(d+fx^2\right)\,dx$

3.130.1 Optimal result

Integrand size = 23, antiderivative size = 378

$$\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)\,dx = -\frac{3e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}} - \frac{e^{-3id+\frac{b^2\log^2(f)}{12if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}} + \frac{3e^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}} + \frac{e^{3id-\frac{b^2\log^2(f)}{4(3if+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

output
```
-3/16*exp(-I*d+b^2*ln(f)^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(I*
f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/16*exp(-3*
I*d+b^2*ln(f)^2/(12*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(3*I*f-c*ln(f
)))/(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f-c*ln(f))^(1/2)+3/16*exp(I*d-b^2
*ln(f)^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(I*f+c*ln(f)))/(I*f+
c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)+1/16*exp(3*I*d-1/4*b^2*ln(f)^
2/(3*I*f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(3*I*f+c*ln(f)))/(3*I*f+c*ln(
f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(f))^(1/2)
```

3.130.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3285 vs. 2(378) = 756.

Time = 7.58 (sec) , antiderivative size = 3285, normalized size of antiderivative = 8.69

$$\int f^{a+bx+cx^2} \cos^3\left(d+fx^2\right)\,dx = \text{Result too large to show}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cos[d + f*x^2]^3,x]
```

output
```
(f^a*Sqrt[Pi]*(-27*(-1)^(3/4)*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f^
3*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt
[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]] + 27*(-1)^(1/4)*c*E^(((I/4)*b^2*Lo
g[f]^2)/(f - I*c*Log[f]))*f^2*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f]
- (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log[f]]
 - 3*(-1)^(3/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f*Cos[d]*Erf
i[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log
[f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]] + 3*(-1)^(1/4)*c^3*E^(((I/4)*b^2*Log[
f]^2)/(f - I*c*Log[f]))*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I
)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]] - 3
*(-1)^(3/4)*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f^3*Cos[3*d]*Erfi[
((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log
[f]])]*Sqrt[3*f - I*c*Log[f]] + (-1)^(1/4)*c*E^(((I/4)*b^2*Log[f]^2)/(3*f
- I*c*Log[f]))*f^2*Cos[3*d]*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c
*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]] - 3*
(-1)^(3/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f*Cos[3*d]*Erfi
[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Lo
g[f]])]*Log[f]^2*Sqrt[3*f - I*c*Log[f]] + (-1)^(1/4)*c^3*E^(((I/4)*b^2*Log
[f]^2)/(3*f - I*c*Log[f]))*Cos[3*d]*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] -
 (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c...
```

3.130.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3\left(d+fx^2\right) f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{3}{8}e^{-id-ifx^2} f^{a+bx+cx^2}+\frac{3}{8}e^{id+ifx^2} f^{a+bx+cx^2}+\frac{1}{8}e^{-3id-3ifx^2} f^{a+bx+cx^2}+\frac{1}{8}e^{3id+3ifx^2} f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\operatorname{erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}-\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+12if}-3id}\operatorname{erf}\left(\frac{b\log(f)-2x(-c\log(f)+3if)}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{\sqrt{\pi}f^a\exp\left(3id-\frac{b^2\log^2(f)}{4(c\log(f)+3if)}\right)\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+3if)}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}+\frac{3\sqrt{\pi}f^a e^{id-\frac{b^2\log^2(f)}{4c\log(f)+4if}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cos[d + f*x^2]^3,x]
```

output
```
(-3*E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b
*Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(16*Sqrt[I*f -
c*Log[f]]) - (E^((-3*I)*d + (b^2*Log[f]^2)/((12*I)*f - 4*c*Log[f]))*f^a*Sq
rt[Pi]*Erf[(b*Log[f] - 2*x*((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f - c*Log[f
]])])/(16*Sqrt[(3*I)*f - c*Log[f]]) + (3*E^(I*d - (b^2*Log[f]^2)/((4*I)*f
+ 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt
[I*f + c*Log[f]])])/(16*Sqrt[I*f + c*Log[f]]) + (E^((3*I)*d - (b^2*Log[f]^
2)/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((3*I)*f +
c*Log[f]))/(2*Sqrt[(3*I)*f + c*Log[f]])])/(16*Sqrt[(3*I)*f + c*Log[f]])
```

3.130. $\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)dx$

3.130.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.130.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+12 i d \ln(f) c+36 d f}{4(c\ln(f)-3 i f)}} \operatorname{erf}\left(-x\sqrt{3 i f-c\ln(f)}+\frac{\ln(f) b}{2\sqrt{3 i f-c\ln(f)}}\right)}{16\sqrt{3 i f-c\ln(f)}}-\frac{3\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4 i d \ln(f) c+4 d f}{4(c\ln(f)-i f)}} \operatorname{erf}\left(-x\sqrt{i f-c\ln\right.}{16\sqrt{i f-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*cos(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+12*I*d*ln(f)*c+36*d*f)/(c*ln(f)-3
*I*f))/(3*I*f-c*ln(f))^(1/2)*erf(-x*(3*I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(3*I
*f-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*d*ln(f)*c+4
*d*f)/(c*ln(f)-I*f))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*ln
(f)*b/(I*f-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*d*l
n(f)*c+4*d*f)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2
)*x+1/2*ln(f)*b/(-c*ln(f)-I*f)^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*
b^2-12*I*d*ln(f)*c+36*d*f)/(3*I*f+c*ln(f)))/(-c*ln(f)-3*I*f)^(1/2)*erf(-(-
c*ln(f)-3*I*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-3*I*f)^(1/2))
```

3.130.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(289) = 578$.

Time = 0.29 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.92

$$\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\left(c^3\log\left(f\right)^3-3i\,c^2f\log\left(f\right)^2+cf^2\log\left(f\right)-3i\,f^3\right)\sqrt{-c\log\left(f\right)-3i\,f}\,\mathrm{erf}\left(\frac{\left(18\,f^2x-3i\,bf\log(f)+\left(2\,c^2x+bc\right.\right.}{2\left(c^2\log(f)^2\right.}\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^3,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(pi)*(c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + c*f^2*log(f) - 3*I*f^
3)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(18*f^2*x - 3*I*b*f*log(f) + (2*c^2*x +
 b*c)*log(f)^2)*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(36
*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 108*I*d*f^2 - 3*(-4*I*c^2*d -
 I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(c^3*log(f)^3 + 3*I
*c^2*f*log(f)^2 + c*f^2*log(f) + 3*I*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*
(18*f^2*x + 3*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) + 3*
I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(36*a*f^2*log(f) - (b^2*c - 4*a*c^2)*l
og(f)^3 - 108*I*d*f^2 - 3*(4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2 +
9*f^2)) + 3*sqrt(pi)*(c^3*log(f)^3 - I*c^2*f*log(f)^2 + 9*c*f^2*log(f) - 9
*I*f^3)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x - I*b*f*log(f) + (2*c^2*x +
 b*c)*log(f)^2)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*f^
2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 + (4*I*c^2*d + I*b^2*f)*
log(f)^2)/(c^2*log(f)^2 + f^2)) + 3*sqrt(pi)*(c^3*log(f)^3 + I*c^2*f*log(f
)^2 + 9*c*f^2*log(f) + 9*I*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + I
*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^
2 + f^2))*e^(1/4*(4*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2
+ (-4*I*c^2*d - I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 +
10*c^2*f^2*log(f)^2 + 9*f^4)
```

3.130.6 Sympy [F]

$$\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cos(f*x**2+d)**3,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cos(d + f*x**2)**3, x)
```

3.130.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2456 vs. $2(289) = 578$.

Time = 0.28 (sec) , antiderivative size = 2456, normalized size of antiderivative = 6.50

$$\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((I*c^2*f^a*e^(1/4*b^2*c*log
(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(
c^2*log(f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2
*log(f)^2 + 9*f^2)) + (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))
*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*sin(3/4
*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*
(2*(c*log(f) - 3*I*f)*x + b*log(f))/sqrt(-c*log(f) + 3*I*f)) + ((-I*c^2*f^
a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 - I*f^(a + 2)*e^(1/
4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2
*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^
2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^
2 + f^2)))*sin(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 +
 9*f^2)))*erf(1/2*(2*(c*log(f) + 3*I*f)*x + b*log(f))/sqrt(-c*log(f) - 3*I
*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*
log(f)^2 + 2*f^2)*(((-I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^
2))*log(f)^2 - 9*I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))
)*cos(1/4*(4*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) - (
c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f^(a +
2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^
2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*...
```

3.130.8 Giac [F]

$$\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)\,dx=\int f^{cx^2+bx+a}\cos\left(fx^2+d\right)^3\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + d)^3, x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\cos\left(f\,x^2+d\right)^3\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cos(d + f*x^2)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)*cos(d + f*x^2)^3, x)
```

3.131 $\int f^{a+bx+cx^2} \cos(d+ex+fx^2)\, dx$

3.131.1 Optimal result

Integrand size = 24, antiderivative size = 208

$$\int f^{a+bx+cx^2} \cos\left(d+ex+fx^2\right)\, dx = \frac{e^{-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}} f^a \sqrt{\pi}\operatorname{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}} + \frac{e^{id+\frac{(e-ib\log(f))^2}{4if+4c\log(f)}} f^a \sqrt{\pi}\operatorname{erfi}\left(\frac{ie+b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

output
```
1/4*exp(-I*d-(e+I*b*ln(f))^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e-b*ln(f)+2
*x*(I*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/4*ex
p(I*d+(e-I*b*ln(f))^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(I*e+b*ln(f)+2*x*(I*
f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

3.131.2 Mathematica [A] (warning: unable to verify)

Time = 1.75 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.67

$$\int f^{a+bx+cx^2} \cos\left(d+ex+fx^2\right)\, dx$$
$$= \frac{\sqrt[4]{-1}e^{-\frac{1}{4}i\left(\frac{e^2}{f-ic\log(f)}+\frac{b^2\log^2(f)}{f+ic\log(f)}\right)} f^{\frac{f(-be+af)+ac^2\log^2(f)}{f^2+c^2\log^2(f)}} \sqrt{\pi}\left(-e^{\frac{ie^2f}{2\left(f^2+c^2\log^2(f)\right)}} f^{\frac{be}{2f-2ic\log(f)}}\operatorname{erfi}\left(\frac{(-1)^{3/4}(e+2fx+i(b+2cx)\log}{2\sqrt{f+ic\log(f)}}\right.\right.}{}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2],x]
```

output
```
((-1)^(1/4)*f^((f*(-(b*e) + a*f) + a*c^2*Log[f]^2)/(f^2 + c^2*Log[f]^2))*S
qrt[Pi]*(-(E^(((I/2)*e^2*f)/(f^2 + c^2*Log[f]^2))*f^((b*e)/(2*f - (2*I)*c*
Log[f]))*Erfi[((-1)^(3/4)*(e + 2*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f +
I*c*Log[f]])]*(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*(Cos[d] - I*Sin[d])) +
 E^(((I/2)*b^2*f*Log[f]^2)/(f^2 + c^2*Log[f]^2))*f^((b*e)/(2*f + (2*I)*c*L
og[f]))*Erfi[((-1)^(1/4)*(e + 2*f*x - I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f - I
*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*(f + I*c*Log[f])*((-I)*Cos[d] + Sin[d]))
)/(4*E^((I/4)*(e^2/(f - I*c*Log[f]) + (b^2*Log[f]^2)/(f + I*c*Log[f])))*(f
^2 + c^2*Log[f]^2))
```

3.131.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2}\cos\left(d+ex+fx^2\right)dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{1}{2}e^{-id-iex-ifx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{id+iex+ifx^2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^a\exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\text{erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}f^a\exp\left(\frac{(e-ib\log(f))^2}{4c\log(f)+4if}+id\right)\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2],x]
```

output
```
(E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(
I*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(4*Sqrt[
I*f - c*Log[f]]) + (E^(I*d + (e - I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]))*f^
a*Sqrt[Pi]*Erfi[(I*e + b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Lo
g[f]])])/(4*Sqrt[I*f + c*Log[f]])
```

3.131.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.131.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-2 i \ln(f) b e+4 i d \ln(f) c+4 d f-e^{2}}{4(c\ln(f)-i f)}} \operatorname{erf}\left(-x\sqrt{i f-c\ln(f)}+\frac{b\ln(f)-i e}{2\sqrt{i f-c\ln(f)}}\right)}{4\sqrt{i f-c\ln(f)}} - \frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+2 i \ln(f) b e-4 i d \ln(f) c+4d}{4(i f+c\ln(f))}}}{4\sqrt{-}}$

input
```
int(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*I*ln(f)*b*e+4*I*d*ln(f)*c+4*d*f-
e^2)/(c*ln(f)-I*f))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*(b*
ln(f)-I*e)/(I*f-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I
*ln(f)*b*e-4*I*d*ln(f)*c+4*d*f-e^2)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*er
f(-(-c*ln(f)-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-c*ln(f)-I*f)^(1/2))
```

3.131.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 377 vs. 2(155) = 310.

Time = 0.26 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.81

$$\int f^{a+bx+cx^2}\cos\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}(c\log(f)-i\,f)\sqrt{-c\log(f)-i\,f}\,\mathrm{erf}\left(\frac{\left(2\,f^2x+(2\,c^2x+bc)\log(f)^2+ef+(i\,ce-i\,bf)\log(f)\right)\sqrt{-c\log(f)-i\,f}}{2\left(c^2\log(f)^2+f^2\right)}\right)e^{\left(-\frac{(b^2c-}{}\right.}}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*(c*log(f) - I*f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x + (
2*c^2*x + b*c)*log(f)^2 + e*f + (I*c*e - I*b*f)*log(f))*sqrt(-c*log(f) - I
*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*e^2*f -
4*I*d*f^2 - (4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*log(f)^2 - (c*e^2 - 2*b*e*f
+ 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(c*log(f) + I*f)*sqrt(
-c*log(f) + I*f)*erf(1/2*(2*f^2*x + (2*c^2*x + b*c)*log(f)^2 + e*f + (-I*c
*e + I*b*f)*log(f))*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((
b^2*c - 4*a*c^2)*log(f)^3 - I*e^2*f + 4*I*d*f^2 - (-4*I*c^2*d + 2*I*b*c*e
- I*b^2*f)*log(f)^2 - (c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 +
f^2)))/(c^2*log(f)^2 + f^2)
```

3.131.6 Sympy [F]

$$\int f^{a+bx+cx^2}\cos\left(d+ex+fx^2\right)\,dx = \int f^{a+bx+cx^2}\cos\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cos(f*x**2+e*x+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cos(d + e*x + f*x**2), x)
```

3.131.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1008 vs. 2(155) = 310.

Time = 0.25 (sec) , antiderivative size = 1008, normalized size of antiderivative = 4.85

$$\int f^{a+bx+cx^2}\cos\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
-1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((I*f^(1/4*c*e^2/(c^2*log(f)^2
 + f^2))*f^a*cos(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f
)^2)/(c^2*log(f)^2 + f^2)) + f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(-1
/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2
+ f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f) - I*e)*sqrt(-c*log(f) +
I*f)/(c*log(f) - I*f)) + (-I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(-1
/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2
+ f^2)) + f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(-1/4*(e^2*f - 4*d*f^2
 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2
*(c*log(f) + I*f)*x + b*log(f) + I*e)*sqrt(-c*log(f) - I*f)/(c*log(f) + I*
f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) - sqrt(pi)*sqrt(2*c^2*log(f
)^2 + 2*f^2)*((f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(-1/4*(e^2*f - 4*
d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) - I*f^
(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d
- 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) -
 I*f)*x + b*log(f) - I*e)*sqrt(-c*log(f) + I*f)/(c*log(f) - I*f)) + (f^(1/
4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2
*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + I*f^(1/4*c*e^2/(c^2*log(
f)^2 + f^2))*f^a*sin(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*l
og(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f...
```

3.131.8 Giac [F]

$$\int f^{a+bx+cx^2}\cos\left(d+ex+fx^2\right)\,dx = \int f^{cx^2+bx+a}\cos\left(fx^2+ex+d\right)\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + e*x + d), x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\cos\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2),x)
```

output
```
int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2), x)
```

3.132 $\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx$

3.132.1 Optimal result

Integrand size = 26, antiderivative size = 268

$$\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id-\frac{(2e+ib\log(f))^2}{8if-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{2ie-b\log(f)+2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}+\frac{e^{2id+\frac{(2e-ib\log(f))^2}{8if+4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{2ie+b\log(f)+2x(2if+c\log(f))}{2\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}$$

output
```
1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/
2)/ln(f)^(1/2)+1/8*exp(-2*I*d-(2*e+I*b*ln(f))^2/(8*I*f-4*c*ln(f)))*f^a*erf
(1/2*(2*I*e-b*ln(f)+2*x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(
2*I*f-c*ln(f))^(1/2)+1/8*exp(2*I*d+(2*e-I*b*ln(f))^2/(8*I*f+4*c*ln(f)))*f^
a*erfi(1/2*(2*I*e+b*ln(f)+2*x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(
1/2)/(2*I*f+c*ln(f))^(1/2)
```

3.132.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1118 vs. 2(268) = 536.

Time = 6.58 (sec) , antiderivative size = 1118, normalized size of antiderivative = 4.17

$$\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)dx$$

$$= \frac{f^a\sqrt{\pi}\left(8\sqrt{c}f^{2-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\sqrt{\log(f)}+2c^{5/2}f^{-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\log^{\frac{5}{2}}(f)-2(-1)^{3/4}ce^{\frac{i\left(-4e^2+\right.}{}}\right.}{}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*(8*Sqrt[c]*f^(2 - b^2/(4*c))*Erfi[((b + 2*c*x)*Sqrt[Log[f]])
/(2*Sqrt[c])]*Sqrt[Log[f]] + (2*c^(5/2)*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2
*Sqrt[c])]*Log[f]^(5/2))/f^(b^2/(4*c)) - 2*(-1)^(3/4)*c*E^(((I/4)*(-4*e^2
+ (4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*f*Cos[2*d]*Erfi[((
-1)^(1/4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c
*Log[f]])]*Log[f]*Sqrt[2*f - I*c*Log[f]] + (-1)^(1/4)*c^2*E^(((I/4)*(-4*e^
2 + (4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*Cos[2*d]*Erfi[((
-1)^(1/4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c
*Log[f]])]*Log[f]^2*Sqrt[2*f - I*c*Log[f]] - (2*(-1)^(1/4)*c*f*Cos[2*d]*Er
fi[((-1)^(3/4)*(2*e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f
+ I*c*Log[f]])]*Log[f]*Sqrt[2*f + I*c*Log[f]])/E^(((I/4)*(-4*e^2 - (4*I)*b
*e*Log[f] + b^2*Log[f]^2))/(2*f + I*c*Log[f])) + ((-1)^(3/4)*c^2*Cos[2*d]*
Erfi[((-1)^(3/4)*(2*e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*
f + I*c*Log[f]])]*Log[f]^2*Sqrt[2*f + I*c*Log[f]])/E^(((I/4)*(-4*e^2 - (4*
I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f + I*c*Log[f])) + 2*(-1)^(1/4)*c*E^(((I
/4)*(-4*e^2 + (4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*f*Erfi
[((-1)^(1/4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f -
I*c*Log[f]])]*Log[f]*Sqrt[2*f - I*c*Log[f]]*Sin[2*d] + (-1)^(3/4)*c^2*E^((
(I/4)*(-4*e^2 + (4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*Erfi
[((-1)^(1/4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f...
```

3.132.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+bx+cx^2}+\frac{1}{2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a\exp\left(-\frac{(2e+ib\log(f))^2}{-4c\log(f)+8if}-2id\right)\text{erf}\left(\frac{-b\log(f)+2x(-c\log(f)+2if)+2ie}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}f^a\exp\left(\frac{(2e-ib\log(f))^2}{4c\log(f)+8if}+2id\right)\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2if)+2ie}{2\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) + (E^((-2*I)*d - (2*e + I*b*Log[f])^2/((8*I)*f -
4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[((2*I)*e - b*Log[f] + 2*x*((2*I)*f - c*Log[f
]))/(2*Sqrt[(2*I)*f - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) + (E^((2*I
)*d + (2*e - I*b*Log[f])^2/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I
)*e + b*Log[f] + 2*x*((2*I)*f + c*Log[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/
(8*Sqrt[(2*I)*f + c*Log[f]])
```

3.132.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.132.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-4 i \ln(f) b e+8 i d \ln(f) c+16 d f-4 e^{2}}{4(c \ln(f)-2 i f)}} \operatorname{erf}\left(-x\sqrt{2 i f-c\ln(f)}+\frac{b\ln(f)-2 i e}{2\sqrt{2 i f-c \ln(f)}}\right)}{8\sqrt{2 i f-c\ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4 i \ln(f) b e-8 i d \ln(f)}{4(2 i f+c\ln(f))}}}{}$

input
```
int(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*ln(f)*b*e+8*I*d*ln(f)*c+16*d*f
-4*e^2)/(c*ln(f)-2*I*f))/(2*I*f-c*ln(f))^(1/2)*erf(-x*(2*I*f-c*ln(f))^(1/2
)+1/2*(b*ln(f)-2*I*e)/(2*I*f-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln
(f)^2*b^2+4*I*ln(f)*b*e-8*I*d*ln(f)*c+16*d*f-4*e^2)/(2*I*f+c*ln(f)))/(-c*l
n(f)-2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-c*ln
(f)-2*I*f)^(1/2))-1/4*Pi^(1/2)*f^(-1/4*b^2/c)*f^a/(-c*ln(f))^(1/2)*erf(-(-
c*ln(f))^(1/2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

3.132.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(199) = 398.

Time = 0.26 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.75

$$\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\left(c^2\log(f)^2-2icf\log(f)\right)\sqrt{-c\log(f)-2if}\operatorname{erf}\left(\frac{\left(8f^2x+\left(2c^2x+bc\right)\log(f)^2+4ef-2\left(-ice+ibf\right)\log(f)\right)\sqrt{-c\log}}{2\left(c^2\log(f)^2+4f^2\right)}\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf
(1/2*(8*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 4*e*f - 2*(-I*c*e + I*b*f)*log(
f))*sqrt(-c*log(f) - 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(-1/4*((b^2*c - 4*a*
c^2)*log(f)^3 + 8*I*e^2*f - 32*I*d*f^2 + 2*(-4*I*c^2*d + 2*I*b*c*e - I*b^2
*f)*log(f)^2 - 4*(c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2
)) + sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf(
1/2*(8*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 4*e*f - 2*(I*c*e - I*b*f)*log(f)
)*sqrt(-c*log(f) + 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(-1/4*((b^2*c - 4*a*c^
2)*log(f)^3 - 8*I*e^2*f + 32*I*d*f^2 + 2*(4*I*c^2*d - 2*I*b*c*e + I*b^2*f)
*log(f)^2 - 4*(c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2))
+ 2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sq
rt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log(f)^3 + 4*c*f^2*log(f))
```

3.132.6 Sympy [F]

$$\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cos(f*x**2+e*x+d)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cos(d + e*x + f*x**2)**2, x)
```

3.132.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 1487, normalized size of antiderivative = 5.55

$$\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
-1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^a*cos(-1/2*(4*e^2*f - 1
6*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2))*e^
(c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c) + f^a*e^(c*e^2*lo
g(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c)*sin(-1/2*(4*e^2*f - 16*d*f
^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/
2*(2*(c*log(f) - 2*I*f)*x + b*log(f) - 2*I*e)*sqrt(-c*log(f) + 2*I*f)/(c*l
og(f) - 2*I*f)) + (-I*f^a*cos(-1/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*
e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2))*e^(c*e^2*log(f)/(c^2*log(f)^2
 + 4*f^2) + 1/4*b^2*log(f)/c) + f^a*e^(c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2)
 + 1/4*b^2*log(f)/c)*sin(-1/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e + b
^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) + 2*I*f)*x +
 b*log(f) + 2*I*e)*sqrt(-c*log(f) - 2*I*f)/(c*log(f) + 2*I*f)))*sqrt(c*log
(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2*lo
g(f)^2 + 8*f^2)*((f^a*cos(-1/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e +
b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2))*e^(c*e^2*log(f)/(c^2*log(f)^2 + 4
*f^2) + 1/4*b^2*log(f)/c) - I*f^a*e^(c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2) +
 1/4*b^2*log(f)/c)*sin(-1/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e + b^2
*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b
*log(f) - 2*I*e)*sqrt(-c*log(f) + 2*I*f)/(c*log(f) - 2*I*f)) + (f^a*cos(-1
/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log...
```

3.132.8 Giac [F]

$$\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\int f^{cx^2+bx+a}\cos\left(fx^2+ex+d\right)^2\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + e*x + d)^2, x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\cos\left(f\,x^2+e\,x+d\right)^2\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2)^2,x)
```

output
```
int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2)^2, x)
```

3.133 $\int f^{a+bx+cx^2} \cos^3(d+ex+fx^2)\,dx$

3.133.1 Optimal result

Integrand size = 26, antiderivative size = 422

$$\int f^{a+bx+cx^2}\cos^3\left(d+ex+fx^2\right)dx=\frac{3e^{-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}+\frac{e^{-3id-\frac{(3e+ib\log(f))^2}{4(3if-c\log(f))}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{3ie-b\log(f)+2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}}+\frac{3e^{id+\frac{(e-ib\log(f))^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie+b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}}+\frac{e^{3id-\frac{(3ie+b\log(f))^2}{4(3if+c\log(f))}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{3ie+b\log(f)+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

output
```
3/16*exp(-I*d-(e+I*b*ln(f))^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e-b*ln(f)+
2*x*(I*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/16*
exp(-3*I*d-1/4*(3*e+I*b*ln(f))^2/(3*I*f-c*ln(f)))*f^a*erf(1/2*(3*I*e-b*ln(
f)+2*x*(3*I*f-c*ln(f)))/(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f-c*ln(f))^(1
/2)+3/16*exp(I*d+(e-I*b*ln(f))^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(I*e+b*ln
(f)+2*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)+1
/16*exp(3*I*d-1/4*(3*I*e+b*ln(f))^2/(3*I*f+c*ln(f)))*f^a*erfi(1/2*(3*I*e+b
*ln(f)+2*x*(3*I*f+c*ln(f)))/(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(f)
)^(1/2)
```

3.133.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3829 vs. 2(422) = 844.

Time = 6.98 (sec) , antiderivative size = 3829, normalized size of antiderivative = 9.07

$$\int f^{a+bx+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx=\text{Result too large to show}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2]^3,x]
```

output
```
(f^a*Sqrt[Pi]*(-27*(-1)^(3/4)*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log
[f]^2))/(f - I*c*Log[f]))*f^3*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log
[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]] +
27*(-1)^(1/4)*c*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I
*c*Log[f]))*f^2*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*
x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log[f]] - 3*(-1)^
(3/4)*c^2*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log
[f]))*f*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]
))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]] + 3*(-1)^(1/4)*
c^3*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*
Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sq
rt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]] - 3*(-1)^(3/4)*E^(((I/4
)*(-9*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^3*Cos[
3*d]*Erfi[((-1)^(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sq
rt[3*f - I*c*Log[f]])]*Sqrt[3*f - I*c*Log[f]] + (-1)^(1/4)*c*E^(((I/4)*(-9
*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^2*Cos[3*d]*
Erfi[((-1)^(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*
f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]] - 3*(-1)^(3/4)*c^2*E^(((I/
4)*(-9*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f*Cos[3
*d]*Erfi[((-1)^(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*...
```

3.133.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{4976}$$

$$\int \left(\frac{3}{8}f^{a+bx+cx^2}\exp\left(-3i\left(d+ex+fx^2\right)+2id+2iex+2ifx^2\right)+\frac{3}{8}f^{a+bx+cx^2}\exp\left(-3i\left(d+ex+fx^2\right)+4id+4i\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\pi}f^a\exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\operatorname{erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}f^a\exp\left(-\frac{(3e+ib\log(f))^2}{4(-c\log(f)+3if)}-3id\right)\operatorname{erf}\left(\frac{-b\log(f)+2x(-c\log(f)+3if)+3ie}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{3\sqrt{\pi}f^a\exp\left(\frac{(e-ib\log(f))^2}{4c\log(f)+4if}+id\right)\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}}+\frac{\sqrt{\pi}f^a\exp\left(3id-\frac{(b\log(f)+3ie)^2}{4(c\log(f)+3if)}\right)\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+3if)+3ie}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2]^3,x]
```

output
```
(3*E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf
[(I*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(16*Sq
rt[I*f - c*Log[f]]) + (E^((-3*I)*d - (3*e + I*b*Log[f])^2/(4*((3*I)*f - c*
Log[f])))*f^a*Sqrt[Pi]*Erf[((3*I)*e - b*Log[f] + 2*x*((3*I)*f - c*Log[f]))
/(2*Sqrt[(3*I)*f - c*Log[f]])])/(16*Sqrt[(3*I)*f - c*Log[f]]) + (3*E^(I*d
+ (e - I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + b*Lo
g[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/(16*Sqrt[I*f + c*L
og[f]]) + (E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4*((3*I)*f + c*Log[f])))*f
^a*Sqrt[Pi]*Erfi[((3*I)*e + b*Log[f] + 2*x*((3*I)*f + c*Log[f]))/(2*Sqrt[(
3*I)*f + c*Log[f]])])/(16*Sqrt[(3*I)*f + c*Log[f]])
```

3.133. $\int f^{a+bx+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx$

3.133.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.133.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-6 i \ln(f) b e+12 i d \ln(f) c+36 d f-9 e^{2}}{4(c \ln(f)-3 i f)}} \operatorname{erf}\left(-x\sqrt{3 i f-c\ln(f)}+\frac{b\ln(f)-3 i e}{2\sqrt{3 i f-c\ln(f)}}\right)}{16\sqrt{3 i f-c\ln(f)}}-\frac{3\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-2 i \ln(f) b e+4 i d \ln(}{4(c\ln(f)-i f)}}}{1}$

input
```
int(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-6*I*ln(f)*b*e+12*I*d*ln(f)*c+36*d
*f-9*e^2)/(c*ln(f)-3*I*f))/(3*I*f-c*ln(f))^(1/2)*erf(-x*(3*I*f-c*ln(f))^(1
/2)+1/2*(b*ln(f)-3*I*e)/(3*I*f-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*
(ln(f)^2*b^2-2*I*ln(f)*b*e+4*I*d*ln(f)*c+4*d*f-e^2)/(c*ln(f)-I*f))/(I*f-c*
ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-I*e)/(I*f-c*ln(f))^(1
/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c+4
*d*f-e^2)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+
1/2*(I*e+b*ln(f))/(-c*ln(f)-I*f)^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^
2*b^2+6*I*ln(f)*b*e-12*I*d*ln(f)*c+36*d*f-9*e^2)/(3*I*f+c*ln(f)))/(-c*ln(f
)-3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1/2)*x+1/2*(3*I*e+b*ln(f))/(-c*ln(f)
-3*I*f)^(1/2))
```

3.133.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(312) = 624$.

Time = 0.30 (sec) , antiderivative size = 859, normalized size of antiderivative = 2.04

$$\int f^{a+bx+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(pi)*(c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + c*f^2*log(f) - 3*I*f^
3)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(18*f^2*x + (2*c^2*x + b*c)*log(f)^2 +
9*e*f - 3*(-I*c*e + I*b*f)*log(f))*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 +
 9*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + 27*I*e^2*f - 108*I*d*f^2 +
3*(-4*I*c^2*d + 2*I*b*c*e - I*b^2*f)*log(f)^2 - 9*(c*e^2 - 2*b*e*f + 4*a*f
^2)*log(f))/(c^2*log(f)^2 + 9*f^2)) + 3*sqrt(pi)*(c^3*log(f)^3 - I*c^2*f*l
og(f)^2 + 9*c*f^2*log(f) - 9*I*f^3)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x
 + (2*c^2*x + b*c)*log(f)^2 + e*f + (I*c*e - I*b*f)*log(f))*sqrt(-c*log(f)
 - I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*e^2*
f - 4*I*d*f^2 - (4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*log(f)^2 - (c*e^2 - 2*b*
e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + 3*sqrt(pi)*(c^3*log(f)^3 +
I*c^2*f*log(f)^2 + 9*c*f^2*log(f) + 9*I*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2
*(2*f^2*x + (2*c^2*x + b*c)*log(f)^2 + e*f + (-I*c*e + I*b*f)*log(f))*sqrt
(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^
3 - I*e^2*f + 4*I*d*f^2 - (-4*I*c^2*d + 2*I*b*c*e - I*b^2*f)*log(f)^2 - (c
*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(c^3*lo
g(f)^3 + 3*I*c^2*f*log(f)^2 + c*f^2*log(f) + 3*I*f^3)*sqrt(-c*log(f) + 3*I
*f)*erf(1/2*(18*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 9*e*f - 3*(I*c*e - I*b*
f)*log(f))*sqrt(-c*log(f) + 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(-1/4*((b^2*c
 - 4*a*c^2)*log(f)^3 - 27*I*e^2*f + 108*I*d*f^2 + 3*(4*I*c^2*d - 2*I*b*...
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx=\text{Timed out}$$

input
```
integrate(f**(c*x**2+b*x+a)*cos(f*x**2+e*x+d)**3,x)
```

output
```
Timed out
```

3.133.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4348 vs. 2(312) = 624.

Time = 0.31 (sec) , antiderivative size = 4348, normalized size of antiderivative = 10.30

$$\int f^{a+bx+cx^2} \cos^3\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
-1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((I*c^2*f^a*e^(1/4*b^2*c*lo
g(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/
2*b*e*f*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*l
og(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1
/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2)))*cos(-3/4*(9*e^2*f - 36*d*f^2 - (4*c
^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*f^a*e^(1/
4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9
*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4
*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*
f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2)))*sin(-3/4*(9*e^2*f - 36*d*f^
2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2
*(2*(c*log(f) - 3*I*f)*x + b*log(f) - 3*I*e)*sqrt(-c*log(f) + 3*I*f)/(c*lo
g(f) - 3*I*f)) + ((-I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) +
 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2
+ f^2))*log(f)^2 - I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)
+ 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2
 + f^2)))*cos(-3/4*(9*e^2*f - 36*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f
)^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^
2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2
*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f...
```

3.133.8 Giac [F]

$$\int f^{a+bx+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx=\int f^{cx^2+bx+a}\cos\left(fx^2+ex+d\right)^3\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + e*x + d)^3, x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\cos\left(f\,x^2+e\,x+d\right)^3\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2)^3, x)
```

3.134 $\int f^{a+bx+cx^2} \cos(a+bx+ex^2)\,dx$

3.134.1 Optimal result

Integrand size = 24, antiderivative size = 209

$$\int f^{a+bx+cx^2}\cos(a+bx+ex^2)\,dx = \frac{e^{-\left((i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)}\sqrt{\pi}\operatorname{erf}\left(\frac{b(i-\log(f))+2x(ie-c\log(f))}{2\sqrt{ie-c\log(f)}}\right)}{4\sqrt{ie-c\log(f)}} + \frac{e^{(i+\log(f))\left(a-\frac{b^2(i+\log(f))}{4ie+4c\log(f)}\right)}\sqrt{\pi}\operatorname{erfi}\left(\frac{b(i+\log(f))+2x(ie+c\log(f))}{2\sqrt{ie+c\log(f)}}\right)}{4\sqrt{ie+c\log(f)}}$$

output
```
-1/4*erf(1/2*(-b*(I-ln(f))-2*x*(I*e-c*ln(f)))/(I*e-c*ln(f))^(1/2))*Pi^(1/2
)/exp((I-ln(f))*(a-b^2*(I-ln(f))/(4*I*e-4*c*ln(f))))/(I*e-c*ln(f))^(1/2)+1
/4*exp((I+ln(f))*(a-b^2*(I+ln(f))/(4*I*e+4*c*ln(f))))*erfi(1/2*(b*(I+ln(f)
)+2*x*(I*e+c*ln(f)))/(I*e+c*ln(f))^(1/2))*Pi^(1/2)/(I*e+c*ln(f))^(1/2)
```

3.134.2 Mathematica [A] (warning: unable to verify)

Time = 1.64 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.56

$$\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)\,dx =$$

$$-\frac{ie^{-\frac{b^2c\log^3(f)}{2\left(e^2+c^2\log^2(f)\right)}}f^{a-\frac{b^2}{2(e-ic\log(f))}}\sqrt{\pi}\left(-e^{\frac{1}{4}b^2\left(\frac{1}{-ie+c\log(f)}+\frac{\log^2(f)}{ie+c\log(f)}\right)}f^{\frac{ib^2c\log(f)}{e^2+c^2\log^2(f)}}\operatorname{erfi}\left(\frac{-i(b+2ex)+(b+2cx)\log(f)}{2\sqrt{-ie+c\log(f)}}\right)(e-\right.}{}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cos[a + b*x + e*x^2],x]
```

output
```
((-1/4*I)*f^(a - b^2/(2*(e - I*c*Log[f])))*Sqrt[Pi]*(-(E^((b^2*(((-I)*e +
c*Log[f])^(-1) + Log[f]^2/(I*e + c*Log[f])))/4)*f^((I*b^2*c*Log[f])/(e^2 +
 c^2*Log[f]^2))*Erfi[((-I)*(b + 2*e*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[(-I)*
e + c*Log[f]])]*(e - I*c*Log[f])*Sqrt[(-I)*e + c*Log[f]]*(Cos[a] - I*Sin[a
])) + E^((b^2*(Log[f]^2/((-I)*e + c*Log[f]) + (I*e + c*Log[f])^(-1)))/4)*E
rfi[(I*(b + 2*e*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[I*e + c*Log[f]])]*(e + I*
c*Log[f])*Sqrt[I*e + c*Log[f]]*(Cos[a] + I*Sin[a])))/(E^((b^2*c*Log[f]^3)/
(2*(e^2 + c^2*Log[f]^2)))*(e^2 + c^2*Log[f]^2))
```

3.134.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4976, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)\,dx$$

$$\Big\downarrow \; 4976$$

$$\int \left(\frac{1}{2}e^{-ia-ibx-iex^2}f^{a+bx+cx^2}+\frac{1}{2}e^{ia+ibx+iex^2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{\pi}\exp\left(-\left((-\log(f)+i)\left(a-\frac{b^2(-\log(f)+i)}{-4c\log(f)+4ie}\right)\right)\right)\operatorname{erf}\left(\frac{b(-\log(f)+i)+2x(-c\log(f)+ie)}{2\sqrt{-c\log(f)+ie}}\right)}{4\sqrt{-c\log(f)+ie}}+\frac{\sqrt{\pi}\exp\left((\log(f)+i)\left(a-\frac{b^2(\log(f)+i)}{4c\log(f)+4ie}\right)\right)\operatorname{erfi}\left(\frac{b(\log(f)+i)+2x(c\log(f)+ie)}{2\sqrt{c\log(f)+ie}}\right)}{4\sqrt{c\log(f)+ie}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cos[a + b*x + e*x^2],x]
```

output
```
(Sqrt[Pi]*Erf[(b*(I - Log[f]) + 2*x*(I*e - c*Log[f]))/(2*Sqrt[I*e - c*Log[
f]])])/(4*E^((I - Log[f])*(a - (b^2*(I - Log[f]))/((4*I)*e - 4*c*Log[f])))
*Sqrt[I*e - c*Log[f]]) + (E^((I + Log[f])*(a - (b^2*(I + Log[f]))/((4*I)*e
 + 4*c*Log[f])))*Sqrt[Pi]*Erfi[(b*(I + Log[f]) + 2*x*(I*e + c*Log[f]))/(2*
Sqrt[I*e + c*Log[f]])])/(4*Sqrt[I*e + c*Log[f]])
```

3.134.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4976
```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.134.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{\ln(f)^{2}b^{2}+4i\ln(f)ac-2i\ln(f)b^{2}+4ae-b^{2}}{4(c\ln(f)-ie)}}\operatorname{erf}\left(-\sqrt{ie-c\ln(f)}\,x+\frac{b\ln(f)-ib}{2\sqrt{ie-c\ln(f)}}\right)}{4\sqrt{ie-c\ln(f)}}-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{-\ln(f)^{2}b^{2}+4i\ln(f)ac-2i\ln(f)b^{2}-4\cdots}{4ie+4c\ln(f)}}\cdots}{4\sqrt{-\cdots}}$

input
```
int(f^(c*x^2+b*x+a)*cos(e*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*ln(f)*a*c-2*I*ln(f)*b^2+4*a*e-
b^2)/(c*ln(f)-I*e))/(I*e-c*ln(f))^(1/2)*erf(-(I*e-c*ln(f))^(1/2)*x+1/2*(b*
ln(f)-I*b)/(I*e-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(1/4*(-ln(f)^2*b^2+4*I
*ln(f)*a*c-2*I*ln(f)*b^2-4*a*e+b^2)/(I*e+c*ln(f)))/(-c*ln(f)-I*e)^(1/2)*er
f(-(-c*ln(f)-I*e)^(1/2)*x+1/2*(b*ln(f)+I*b)/(-c*ln(f)-I*e)^(1/2))
```

3.134.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(153) = 306$.

Time = 0.27 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.82

$$\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}(c\log(f)-i\,e)\sqrt{-c\log(f)-i\,e}\,\mathrm{erf}\left(\frac{\left(2\,e^2x+(2\,c^2x+bc)\log(f)^2+be+(i\,bc-i\,be)\log(f)\right)\sqrt{-c\log(f)-i\,e}}{2\left(c^2\log(f)^2+e^2\right)}\right)e^{\left(-\frac{\left(b^2c-4\right.}{}\right.}}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(e*x^2+b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*(c*log(f) - I*e)*sqrt(-c*log(f) - I*e)*erf(1/2*(2*e^2*x + (
2*c^2*x + b*c)*log(f)^2 + b*e + (I*b*c - I*b*e)*log(f))*sqrt(-c*log(f) - I
*e)/(c^2*log(f)^2 + e^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*b^2*e -
4*I*a*e^2 - (-2*I*b^2*c + 4*I*a*c^2 + I*b^2*e)*log(f)^2 - (b^2*c - 2*b^2*e
 + 4*a*e^2)*log(f))/(c^2*log(f)^2 + e^2)) + sqrt(pi)*(c*log(f) + I*e)*sqrt
(-c*log(f) + I*e)*erf(1/2*(2*e^2*x + (2*c^2*x + b*c)*log(f)^2 + b*e + (-I*
b*c + I*b*e)*log(f))*sqrt(-c*log(f) + I*e)/(c^2*log(f)^2 + e^2))*e^(-1/4*(
(b^2*c - 4*a*c^2)*log(f)^3 - I*b^2*e + 4*I*a*e^2 - (2*I*b^2*c - 4*I*a*c^2
- I*b^2*e)*log(f)^2 - (b^2*c - 2*b^2*e + 4*a*e^2)*log(f))/(c^2*log(f)^2 +
e^2)))/(c^2*log(f)^2 + e^2)
```

3.134.6 Sympy [F]

$$\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)\,dx=\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cos(e*x**2+b*x+a),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cos(a + b*x + e*x**2), x)
```

3.134.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 1058, normalized size of antiderivative = 5.06

$$\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(e*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*((f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*(cos(1/2*arctan2(e,
-c*log(f))) + I*sin(1/2*arctan2(e, -c*log(f))))*cos(-1/4*(b^2*e - 4*a*e^2
+ (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2)/(c^2*log(f)^2 + e^2)) - f^(1/4*b^2
*c/(c^2*log(f)^2 + e^2))*f^a*(-I*cos(1/2*arctan2(e, -c*log(f))) + sin(1/2*
arctan2(e, -c*log(f))))*sin(-1/4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b
^2*e)*log(f)^2)/(c^2*log(f)^2 + e^2)))*erf(x*conjugate(sqrt(-c*log(f) + I*
e)) - 1/2*(b*log(f) + I*b)*conjugate(1/sqrt(-c*log(f) + I*e))) + (f^(1/4*b
^2*c/(c^2*log(f)^2 + e^2))*f^a*(cos(1/2*arctan2(e, -c*log(f))) - I*sin(1/2
*arctan2(e, -c*log(f))))*cos(-1/4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 -
b^2*e)*log(f)^2)/(c^2*log(f)^2 + e^2)) - f^(1/4*b^2*c/(c^2*log(f)^2 + e^2)
)*f^a*(I*cos(1/2*arctan2(e, -c*log(f))) + sin(1/2*arctan2(e, -c*log(f))))*
sin(-1/4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2)/(c^2*log
(f)^2 + e^2)))*erf(x*conjugate(sqrt(-c*log(f) - I*e)) - 1/2*(b*log(f) - I*
b)*conjugate(1/sqrt(-c*log(f) - I*e))) + (f^(1/4*b^2*c/(c^2*log(f)^2 + e^2
))*f^a*(cos(1/2*arctan2(e, -c*log(f))) - I*sin(1/2*arctan2(e, -c*log(f))))
*cos(-1/4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2)/(c^2*lo
g(f)^2 + e^2)) - f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*(I*cos(1/2*arctan2
(e, -c*log(f))) + sin(1/2*arctan2(e, -c*log(f))))*sin(-1/4*(b^2*e - 4*a*e^
2 + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2)/(c^2*log(f)^2 + e^2)))*erf(1/2*(
2*(c*log(f) - I*e)*x + b*log(f) - I*b)*sqrt(-c*log(f) + I*e)/(c*log(f) ...
```

3.134.8 Giac [F]

$$\int f^{a+bx+cx^2} \cos \left(a+bx+ex^2\right) \, dx = \int f^{cx^2+bx+a} \cos \left(ex^2+bx+a\right) \, dx$$

input
```
integrate(f^(c*x^2+b*x+a)*cos(e*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)*cos(e*x^2 + b*x + a), x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \cos \left(a+bx+ex^2\right) \, dx = \int f^{c\,x^2+b\,x+a} \cos \left(e\,x^2+b\,x+a\right) \, dx$$

input
```
int(f^(a + b*x + c*x^2)*cos(a + b*x + e*x^2),x)
```

output
```
int(f^(a + b*x + c*x^2)*cos(a + b*x + e*x^2), x)
```

3.135 $\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx$

3.135.1 Optimal result

Integrand size = 22, antiderivative size = 245

$$\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx=\frac{f^2F^{ac+bcx}}{bc\log(F)}-\frac{2ef^2F^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)}+\frac{2bcf^2F^{ac+bcx}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}-\frac{2ef^2F^{ac+bcx}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)}+\frac{bcf^2F^{ac+bcx}\log(F)\sin^2(d+ex)}{4e^2+b^2c^2\log^2(F)}$$

output
```
f^2*F^(b*c*x+a*c)/b/c/ln(F)-2*e*f^2*F^(b*c*x+a*c)*cos(e*x+d)/(e^2+b^2*c^2*
ln(F)^2)+2*e^2*f^2*F^(b*c*x+a*c)/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)+2*b*c*f
^2*F^(b*c*x+a*c)*ln(F)*sin(e*x+d)/(e^2+b^2*c^2*ln(F)^2)-2*e*f^2*F^(b*c*x+a
*c)*cos(e*x+d)*sin(e*x+d)/(4*e^2+b^2*c^2*ln(F)^2)+b*c*f^2*F^(b*c*x+a*c)*ln
(F)*sin(e*x+d)^2/(4*e^2+b^2*c^2*ln(F)^2)
```

3.135.2 Mathematica [A] (verified)

Time = 7.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.73

$$\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx$$
$$=\frac{f^2F^{c(a+bx)}(1+\sin(d+ex))^2\left(\frac{3}{bc\log(F)}-\frac{4e\cos(d+ex)}{e^2+b^2c^2\log^2(F)}-\frac{bc\cos(2(d+ex))\log(F)}{4e^2+b^2c^2\log^2(F)}+\frac{4bc\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}-\frac{2e\sin(2(d+e}{4e^2+b^2c^2\log^2}\right.}{2\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^4}$$

input
```
Integrate[F^(c*(a + b*x))*(f + f*Sin[d + e*x])^2,x]
```

output
```
(f^2*F^(c*(a + b*x))*(1 + Sin[d + e*x])^2*(3/(b*c*Log[F]) - (4*e*Cos[d + e
*x])/(e^2 + b^2*c^2*Log[F]^2) - (b*c*Cos[2*(d + e*x)]*Log[F])/(4*e^2 + b^2
*c^2*Log[F]^2) + (4*b*c*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2) - (2
*e*Sin[2*(d + e*x)])/(4*e^2 + b^2*c^2*Log[F]^2)))/(2*(Cos[(d + e*x)/2] + S
in[(d + e*x)/2])^4)
```

3.135.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\sin(d+ex)+f)^2\,dx$$

$$\Big\downarrow \; 7292$$

$$\int f^2(\sin(d+ex)+1)^2F^{ac+bcx}dx$$

$$\Big\downarrow \; 27$$

$$f^2\int F^{ac+bxc}(\sin(d+ex)+1)^2dx$$

$$\Big\downarrow \; 7293$$

$$f^2\int \left(\sin^2(d+ex)F^{ac+bxc}+2\sin(d+ex)F^{ac+bxc}+F^{ac+bxc}\right)dx$$

$$\downarrow 2009$$

$$f^2\left(\frac{bc\log(F)\sin^2(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+4e^2}+\frac{2bc\log(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{2e\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{2e\sin(d+ex)\cos\ldots}{b^2c^2\log^2(\ldots}\right.$$

input
```
Int[F^(c*(a + b*x))*(f + f*Sin[d + e*x])^2,x]
```

output
```
f^2*(F^(a*c + b*c*x)/(b*c*Log[F]) - (2*e*F^(a*c + b*c*x)*Cos[d + e*x])/(e^
2 + b^2*c^2*Log[F]^2) + (2*e^2*F^(a*c + b*c*x))/(b*c*Log[F]*(4*e^2 + b^2*c
^2*Log[F]^2)) + (2*b*c*F^(a*c + b*c*x)*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2
*Log[F]^2) - (2*e*F^(a*c + b*c*x)*Cos[d + e*x]*Sin[d + e*x])/(4*e^2 + b^2*
c^2*Log[F]^2) + (b*c*F^(a*c + b*c*x)*Log[F]*Sin[d + e*x]^2)/(4*e^2 + b^2*c
^2*Log[F]^2))
```

3.135.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.135.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.79

method	result
risch	$\frac{3f^2F^{c(xb+a)}}{2bc\ln(F)}-\frac{2F^{c(xb+a)}ef^2\cos(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{2F^{c(xb+a)}\ln(F)bcf^2\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}-\frac{\ln(F)cbf^2F^{c(xb+a)}\cos(2ex+2d)}{2\left(4e^2+b^2c^2\ln(F)^2\right)}-\frac{ef^2}{}$
parallelrisch	$-\frac{2F^{c(xb+a)}f^2\left(\frac{b^2c^2\ln(F)^2\left(e^2+b^2c^2\ln(F)^2\right)\cos(2ex+2d)}{4}+\frac{cbe\ln(F)\left(e^2+b^2c^2\ln(F)^2\right)\sin(2ex+2d)}{2}+\left(4e^2+b^2c^2\ln(F)^2\right)\left(-\sin(ex\right.\right.}{bc\ln(F)\left(e^2+b^2c^2\ln(F)^2\right)\left(4e^2+b^2c^2\ln(F)^2\right)}$
default	$\frac{F^{ac}f^2\left(\frac{4F^{bcx}}{bc\ln(F)}+\frac{-\frac{8e\,e^{bcx\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{4e^2+b^2c^2\ln(F)^2}+\frac{8e\,e^{bcx\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{4e^2+b^2c^2\ln(F)^2}-\frac{2\left(b^2c^2\ln(F)^2+2e^2\right)e^{bcx\ln(F)}}{bc\ln(F)\left(4e^2+b^2c^2\ln(F)^2\right)}+\frac{4\left(b^2c^2\ln(F)^2-2e^2\right)e^{bcx}}{bc\ln(F)\left(4e^2+b^2\right.}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)^2}\right.}{}$
parts	$\frac{f^2F^{c(xb+a)}}{bc\ln(F)}+\frac{-\frac{4e\,f^2e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{4e^2+b^2c^2\ln(F)^2}+\frac{4e\,f^2e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{4e^2+b^2c^2\ln(F)^2}+\frac{2e^2f^2e^{c(xb+a)\ln(F)}}{bc\ln(F)\left(4e^2+b^2c^2\ln(F)^2\right)}+\frac{2e^2f^2e^{c(xb+a)}}{bc\ln(F)\left(4e\right.}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)^2}$
norman	$\frac{\left(b^4c^4\ln(F)^4-2\ln(F)^3b^3c^3e+7b^2c^2e^2\ln(F)^2-8e^3bc\ln(F)+6e^4\right)f^2e^{c(xb+a)\ln(F)}}{bc\ln(F)\left(b^4c^4\ln(F)^4+5b^2c^2e^2\ln(F)^2+4e^4\right)}+\frac{f^2\left(b^4c^4\ln(F)^4+2\ln(F)^3b^3c^3e+7b^2c^2e^2\ln(F)^2+8e^3\right.}{\left(b^4c^4\ln(F)^4+5b^2c^2e^2\ln(F\right.}$

input
```
int(F^(c*(b*x+a))*(f+f*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
3/2/b/c/ln(F)*f^2*F^(c*(b*x+a))-2*F^(c*(b*x+a))*e*f^2/(e^2+b^2*c^2*ln(F)^2
)*cos(e*x+d)+2*F^(c*(b*x+a))*ln(F)*b*c*f^2/(e^2+b^2*c^2*ln(F)^2)*sin(e*x+d
)-1/2/(4*e^2+b^2*c^2*ln(F)^2)*ln(F)*c*b*f^2*F^(c*(b*x+a))*cos(2*e*x+2*d)-e
*f^2*F^(c*(b*x+a))/(4*e^2+b^2*c^2*ln(F)^2)*sin(2*e*x+2*d)
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.04

$$\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx =$$
$$-\frac{\left(2\,b^3c^3ef^2\cos\left(ex+d\right)\log\left(F\right)^3+8\,bce^3f^2\cos\left(ex+d\right)\log\left(F\right)-6\,e^4f^2+\left(b^4c^4f^2\cos\left(ex+d\right)^2-2\,b^4c^4\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d))^2,x, algorithm="fricas")
```

output
```
-(2*b^3*c^3*e*f^2*cos(e*x + d)*log(F)^3 + 8*b*c*e^3*f^2*cos(e*x + d)*log(F
) - 6*e^4*f^2 + (b^4*c^4*f^2*cos(e*x + d)^2 - 2*b^4*c^4*f^2)*log(F)^4 + (b
^2*c^2*e^2*f^2*cos(e*x + d)^2 - 8*b^2*c^2*e^2*f^2)*log(F)^2 - 2*(b^4*c^4*f
^2*log(F)^4 - b^3*c^3*e*f^2*cos(e*x + d)*log(F)^3 + 4*b^2*c^2*e^2*f^2*log(
F)^2 - b*c*e^3*f^2*cos(e*x + d)*log(F))*sin(e*x + d))*F^(b*c*x + a*c)/(b^5
*c^5*log(F)^5 + 5*b^3*c^3*e^2*log(F)^3 + 4*b*c*e^4*log(F))
```

3.135.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.75 (sec) , antiderivative size = 2465, normalized size of antiderivative = 10.06

$$\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*(f+f*sin(e*x+d))**2,x)
```

output
```
Piecewise((x*(f*sin(d) + f)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0))
, (f**2*x*sin(d + e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f**2*x - f**2*sin
(d + e*x)*cos(d + e*x)/(2*e) - 2*f**2*cos(d + e*x)/e, Eq(F, 1)), (F**(a*c)
*(f**2*x*sin(d + e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f**2*x - f**2*sin(
d + e*x)*cos(d + e*x)/(2*e) - 2*f**2*cos(d + e*x)/e), Eq(b, 0)), (f**2*x*s
in(d + e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f**2*x - f**2*sin(d + e*x)*c
os(d + e*x)/(2*e) - 2*f**2*cos(d + e*x)/e, Eq(c, 0)), (-F**(a*c + b*c*x)*f
**2*x*sin(I*b*c*x*log(F) - d) + I*F**(a*c + b*c*x)*f**2*x*cos(I*b*c*x*log(
F) - d) + F**(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F) - d)**2/(3*b*c*log(F))
+ 2*I*F**(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F) - d)*cos(I*b*c*x*log(F) - d
)/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F) - d)/(b*c*log(
F)) + 2*F**(a*c + b*c*x)*f**2*cos(I*b*c*x*log(F) - d)**2/(3*b*c*log(F)) -
2*I*F**(a*c + b*c*x)*f**2*cos(I*b*c*x*log(F) - d)/(b*c*log(F)) + F**(a*c +
 b*c*x)*f**2/(b*c*log(F)), Eq(e, -I*b*c*log(F))), (F**(a*c + b*c*x)*f**2*x
*sin(I*b*c*x*log(F)/2 - d)**2/4 - I*F**(a*c + b*c*x)*f**2*x*sin(I*b*c*x*lo
g(F)/2 - d)*cos(I*b*c*x*log(F)/2 - d)/2 - F**(a*c + b*c*x)*f**2*x*cos(I*b*
c*x*log(F)/2 - d)**2/4 + 3*I*F**(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F)/2 -
d)*cos(I*b*c*x*log(F)/2 - d)/(2*b*c*log(F)) - 8*F**(a*c + b*c*x)*f**2*sin(
I*b*c*x*log(F)/2 - d)/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2*cos(I*b*c*x*l
og(F)/2 - d)**2/(b*c*log(F)) + 4*I*F**(a*c + b*c*x)*f**2*cos(I*b*c*x*lo...
```

3.135.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(245) = 490$.

Time = 0.26 (sec) , antiderivative size = 581, normalized size of antiderivative = 2.37

$$\int F^{c(a+bx)}(f + f\sin(d + ex))^2\,dx =$$

$$-\frac{\left(\left(F^{ac}b^2c^2\cos\left(2\,d\right)\log\left(F\right)^2 + 2\,F^{ac}bce\log\left(F\right)\sin\left(2\,d\right)\right)F^{bcx}\cos\left(2\,ex\right) + \left(F^{ac}b^2c^2\cos\left(2\,d\right)\log\left(F\right)^2 - 2\right.}{}$$

$$-\frac{\left(\left(F^{ac}bc\log\left(F\right)\sin\left(d\right) + F^{ac}e\cos\left(d\right)\right)F^{bcx}\cos\left(ex + 2\,d\right) - \left(F^{ac}bc\log\left(F\right)\sin\left(d\right) - F^{ac}e\cos\left(d\right)\right)F^{bcx}\cos\right.}{b^2c^2\cos\left(d\right)^2\log\left(F\right)^2 + b^2c^2}$$

$$+\frac{F^{bcx+ac}f^2}{bc\log\left(F\right)}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d))^2,x, algorithm="maxima")
```

output
```
-1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d)
)*F^(b*c*x)*cos(2*e*x) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*
c*e*log(F)*sin(2*d))*F^(b*c*x)*cos(2*e*x + 4*d) - (F^(a*c)*b^2*c^2*log(F)^
2*sin(2*d) - 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2*e*x) + (F^(a
*c)*b^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)
*sin(2*e*x + 4*d) - 2*(F^(a*c)*b^2*c^2*cos(2*d)^2*log(F)^2 + F^(a*c)*b^2*c
^2*log(F)^2*sin(2*d)^2 + 4*(F^(a*c)*cos(2*d)^2 + F^(a*c)*sin(2*d)^2)*e^2)*
F^(b*c*x))*f^2/(b^3*c^3*cos(2*d)^2*log(F)^3 + b^3*c^3*log(F)^3*sin(2*d)^2
+ 4*(b*c*cos(2*d)^2*log(F) + b*c*log(F)*sin(2*d)^2)*e^2) - ((F^(a*c)*b*c*l
og(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x + 2*d) - (F^(a*c)*b*c*l
og(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x) - (F^(a*c)*b*c*cos(d)*
log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^(a*c)*b*c*cos(d)*
log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x))*f^2/(b^2*c^2*cos(d)^2*log(F
)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2) + F^(b*c*x +
a*c)*f^2/(b*c*log(F))
```

3.135.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 1738, normalized size of antiderivative = 7.09

$$\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d))^2,x, algorithm="giac")
```

output
```
-1/2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F)
 - 1/2*pi*a*c + 2*e*x + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*
c*sgn(F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*f^2*sin(1/2*p
i*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*e*x + 2
*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x
*log(abs(F)) + a*c*log(abs(F))) - 1/2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) -
 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)*log(abs(F))/
(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2) + (pi*b*c*sgn
(F) - pi*b*c - 4*e)*f^2*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*
c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sg
n(F) - pi*b*c - 4*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 3*(2*b*
c*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi
*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) -
 (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) -
pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 2*(2*b*c*f^2*log(abs
(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a
*c + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2)
 - (pi*b*c*sgn(F) - pi*b*c + 2*e)*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c
*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F))^2...
```

3.135.9 Mupad [B] (verification not implemented)

Time = 27.57 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx =$$

$$-\frac{F^{a\,c+b\,c\,x}\,f^2\,\left(\frac{b^4\,c^4\,\ln(F)^4\,\cos(2\,d+2\,e\,x)}{2}-\frac{3\,b^4\,c^4\,\ln(F)^4}{2}-2\,b^4\,c^4\,\sin\left(d+e\,x\right)\,\ln\left(F\right)^4-6\,e^4-\frac{15\,b^2\,c^2\,e^2\,\ln(F)^2}{2}+b^3 \cdots\right.}{\cdots}$$

input
```
int(F^(c*(a + b*x))*(f + f*sin(d + e*x))^2,x)
```

output
```
-(F^(a*c + b*c*x)*f^2*((b^4*c^4*log(F)^4*cos(2*d + 2*e*x))/2 - (3*b^4*c^4*
log(F)^4)/2 - 2*b^4*c^4*sin(d + e*x)*log(F)^4 - 6*e^4 - (15*b^2*c^2*e^2*lo
g(F)^2)/2 + b^3*c^3*e*log(F)^3*sin(2*d + 2*e*x) - 8*b^2*c^2*e^2*sin(d + e*
x)*log(F)^2 + b*c*e^3*log(F)*sin(2*d + 2*e*x) + (b^2*c^2*e^2*log(F)^2*cos(
2*d + 2*e*x))/2 + 2*b^3*c^3*e*cos(d + e*x)*log(F)^3 + 8*b*c*e^3*cos(d + e*
x)*log(F)))/(b*c*log(F)*(4*e^4 + b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2
))
```

3.135. $\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx$

3.136 $\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx$

3.136.1 Optimal result

Integrand size = 20, antiderivative size = 99

$$\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx=\frac{fF^{ac+bcx}}{bc\log(F)}-\frac{efF^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{bcfF^{ac+bcx}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

output
```
f*F^(b*c*x+a*c)/b/c/ln(F)-e*f*F^(b*c*x+a*c)*cos(e*x+d)/(e^2+b^2*c^2*ln(F)^
2)+b*c*f*F^(b*c*x+a*c)*ln(F)*sin(e*x+d)/(e^2+b^2*c^2*ln(F)^2)
```

3.136.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

$$\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx=\frac{fF^{c(a+bx)}\left(e^2-bce\cos(d+ex)\log(F)+b^2c^2\log^2(F)+b^2c^2\log^2(F)\sin(d+ex)\right)}{bc\log(F)\left(e^2+b^2c^2\log^2(F)\right)}$$

input
```
Integrate[F^(c*(a + b*x))*(f + f*Sin[d + e*x]),x]
```

output
```
(f*F^(c*(a + b*x))*(e^2 - b*c*e*Cos[d + e*x]*Log[F] + b^2*c^2*Log[F]^2 + b
^2*c^2*Log[F]^2*Sin[d + e*x]))/(b*c*Log[F]*(e^2 + b^2*c^2*Log[F]^2))
```

3.136.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\sin(d+ex)+f)\,dx$$

$$\Big\downarrow \text{7292}$$

$$\int f(\sin(d+ex)+1)F^{ac+bcx}dx$$

$$\Big\downarrow \text{27}$$

$$f\int F^{ac+bxc}(\sin(d+ex)+1)dx$$

$$\Big\downarrow \text{7293}$$

$$f\int \left(\sin(d+ex)F^{ac+bxc}+F^{ac+bxc}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$f\left(\frac{bc\log(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{e\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{F^{ac+bcx}}{bc\log(F)}\right)$$

input
```
Int[F^(c*(a + b*x))*(f + f*Sin[d + e*x]),x]
```

output
```
f*(F^(a*c + b*c*x)/(b*c*Log[F]) - (e*F^(a*c + b*c*x)*Cos[d + e*x])/(e^2 +
b^2*c^2*Log[F]^2) + (b*c*F^(a*c + b*c*x)*Log[F]*Sin[d + e*x])/(e^2 + b^2*c
^2*Log[F]^2))
```

3.136.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.136.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

method	result
parallelrisch	$\frac{f\,F^{c(xb+a)}\left(\sin(ex+d)b^2c^2\ln(F)^2+b^2c^2\ln(F)^2-bc\ln(F)e\cos(ex+d)+e^2\right)}{\left(e^2+b^2c^2\ln(F)^2\right)bc\ln(F)}$
risch	$\frac{f\,F^{c(xb+a)}}{bc\ln(F)}+\frac{e\,F^{c(xb+a)}f\cos(ex+d)}{-e^2-b^2c^2\ln(F)^2}-\frac{\ln(F)cb\,F^{c(xb+a)}f\sin(ex+d)}{-e^2-b^2c^2\ln(F)^2}$
parts	$\frac{f\,F^{c(xb+a)}}{bc\ln(F)}+\frac{\frac{ef\,e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}-\frac{ef\,e^{c(xb+a)\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{2fbc\ln(F)e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$
norman	$\frac{\frac{f\left(b^2c^2\ln(F)^2-\ln(F)bce+e^2\right)e^{c(xb+a)\ln(F)}}{\left(e^2+b^2c^2\ln(F)^2\right)bc\ln(F)}+\frac{f\left(b^2c^2\ln(F)^2+\ln(F)bce+e^2\right)e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{\left(e^2+b^2c^2\ln(F)^2\right)bc\ln(F)}+\frac{2fbc\ln(F)e^{c(xb+a)\ln(F)}\tan}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$

input
```
int(F^(c*(b*x+a))*(f+f*sin(e*x+d)),x,method=_RETURNVERBOSE)
```

output
```
f*F^(c*(b*x+a))*(sin(e*x+d)*b^2*c^2*ln(F)^2+b^2*c^2*ln(F)^2-b*c*ln(F)*e*co
s(e*x+d)+e^2)/(e^2+b^2*c^2*ln(F)^2)/b/c/ln(F)
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

$$\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx = \frac{\left(b^2c^2f\log\left(F\right)^2\sin\left(ex+d\right)+b^2c^2f\log\left(F\right)^2-bcef\cos\left(ex+d\right)\log\left(F\right)+e^2f\right)F^{bcx+ac}}{b^3c^3\log\left(F\right)^3+bce^2\log\left(F\right)}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d)),x, algorithm="fricas")
```

output
```
(b^2*c^2*f*log(F)^2*sin(e*x + d) + b^2*c^2*f*log(F)^2 - b*c*e*f*cos(e*x +
d)*log(F) + e^2*f)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + b*c*e^2*log(F))
```

3.136.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 542, normalized size of antiderivative = 5.47

$$\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx$$

$$= \begin{cases} x(f\sin\left(d\right)+f) \\ fx-\frac{f\cos\left(d+ex\right)}{e} \\ F^{ac}\left(fx-\frac{f\cos\left(d+ex\right)}{e}\right) \\ fx-\frac{f\cos\left(d+ex\right)}{e} \\ -\frac{F^{ac+bcx}fx\sin\left(ibcx\log\left(F\right)-d\right)}{2}+\frac{iF^{ac+bcx}fx\cos\left(ibcx\log\left(F\right)-d\right)}{2}+\frac{F^{ac+bcx}f\sin\left(ibcx\log\left(F\right)-d\right)}{2bc\log\left(F\right)}-\frac{iF^{ac+bcx}f\cos\left(ibcx\log\left(F\right)-d\right)}{bc\log\left(F\right)} \\ \frac{F^{ac+bcx}fx\sin\left(ibcx\log\left(F\right)+d\right)}{2}-\frac{iF^{ac+bcx}fx\cos\left(ibcx\log\left(F\right)+d\right)}{2}-\frac{F^{ac+bcx}f\sin\left(ibcx\log\left(F\right)+d\right)}{2bc\log\left(F\right)}+\frac{iF^{ac+bcx}f\cos\left(ibcx\log\left(F\right)+d\right)}{bc\log\left(F\right)}+ \\ \frac{F^{ac+bcx}b^2c^2f\log\left(F\right)^2\sin\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+bce^2\log\left(F\right)}+\frac{F^{ac+bcx}b^2c^2f\log\left(F\right)^2}{b^3c^3\log\left(F\right)^3+bce^2\log\left(F\right)}-\frac{F^{ac+bcx}bcef\log\left(F\right)\cos\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+bce^2\log\left(F\right)}+\frac{F^{ac+bcx}e^2f}{b^3c^3\log\left(F\right)^3+bce^2\log\left(F\right)} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*(f+f*sin(e*x+d)),x)
```

output
```
Piecewise((x*(f*sin(d) + f), Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (
f*x - f*cos(d + e*x)/e, Eq(F, 1)), (F**(a*c)*(f*x - f*cos(d + e*x)/e), Eq(
b, 0)), (f*x - f*cos(d + e*x)/e, Eq(c, 0)), (-F**(a*c + b*c*x)*f*x*sin(I*b
*c*x*log(F) - d)/2 + I*F**(a*c + b*c*x)*f*x*cos(I*b*c*x*log(F) - d)/2 + F*
*(a*c + b*c*x)*f*sin(I*b*c*x*log(F) - d)/(2*b*c*log(F)) - I*F**(a*c + b*c*
x)*f*cos(I*b*c*x*log(F) - d)/(b*c*log(F)) + F**(a*c + b*c*x)*f/(b*c*log(F)
), Eq(e, -I*b*c*log(F))), (F**(a*c + b*c*x)*f*x*sin(I*b*c*x*log(F) + d)/2
- I*F**(a*c + b*c*x)*f*x*cos(I*b*c*x*log(F) + d)/2 - F**(a*c + b*c*x)*f*si
n(I*b*c*x*log(F) + d)/(2*b*c*log(F)) + I*F**(a*c + b*c*x)*f*cos(I*b*c*x*lo
g(F) + d)/(b*c*log(F)) + F**(a*c + b*c*x)*f/(b*c*log(F)), Eq(e, I*b*c*log(
F))), (F**(a*c + b*c*x)*b**2*c**2*f*log(F)**2*sin(d + e*x)/(b**3*c**3*log(
F)**3 + b*c*e**2*log(F)) + F**(a*c + b*c*x)*b**2*c**2*f*log(F)**2/(b**3*c*
*3*log(F)**3 + b*c*e**2*log(F)) - F**(a*c + b*c*x)*b*c*e*f*log(F)*cos(d +
e*x)/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c + b*c*x)*e**2*f/(b*
*3*c**3*log(F)**3 + b*c*e**2*log(F)), True))
```

3.136.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(99) = 198$.

Time = 0.22 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.20

$$\int F^{c(a+bx)}(f + f\sin(d+ex))\,dx =$$
$$-\frac{((F^{ac}bc\log(F)\sin(d) + F^{ac}e\cos(d))F^{bcx}\cos(ex+2d) - (F^{ac}bc\log(F)\sin(d) - F^{ac}e\cos(d))F^{bcx}\cos \cdots}{2\left(b^2c^2\cos(d)^2\log(F)^2 + b^2c \cdots\right.}$$
$$+\frac{F^{bcx+ac}f}{bc\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d)),x, algorithm="maxima")
```

output
```
-1/2*((F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x + 2
*d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x) -
(F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x + 2*d) -
(F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x))*f/(b^2*
c^2*cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*
e^2) + F^(b*c*x + a*c)*f/(b*c*log(F))
```

3.136.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 923, normalized size of antiderivative = 9.32

$$\int F^{c(a+bx)}(f + f\sin(d+ex))\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d)),x, algorithm="giac")
```

output
```
2*(2*b*c*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1
/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)
^2) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F)
 - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + (2*b*c*f*log(abs(
F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*
c + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2)
- (pi*b*c*sgn(F) - pi*b*c + 2*e)*f*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x
+ 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi
*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) -
(2*b*c*f*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*s
gn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) -
pi*b*c - 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c - 2*e)*f*cos(1/2*pi*b*c*x*sgn(F
) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*lo
g(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*
c*log(abs(F))) - (-I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*p
i*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c
 + 4*b*c*log(abs(F)) + 4*I*e) - I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b
*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-2*I*pi*b*c*sgn(
F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) + a*...
```

3.136.9 Mupad [B] (verification not implemented)

Time = 25.80 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int F^{c(a+bx)}(f + f\sin(d+ex))\,dx$$
$$= \frac{F^{a\,c+b\,c\,x}\,f\,\left(e^2+b^2\,c^2\ln\left(F\right)^2+b^2\,c^2\,\sin\left(d+e\,x\right)\,\ln\left(F\right)^2-b\,c\,e\,\cos\left(d+e\,x\right)\,\ln\left(F\right)\right)}{b\,c\,\ln\left(F\right)\,\left(b^2\,c^2\ln\left(F\right)^2+e^2\right)}$$

input
```
int(F^(c*(a + b*x))*(f + f*sin(d + e*x)),x)
```

output
```
(F^(a*c + b*c*x)*f*(e^2 + b^2*c^2*log(F)^2 + b^2*c^2*sin(d + e*x)*log(F)^2
 - b*c*e*cos(d + e*x)*log(F)))/(b*c*log(F)*(e^2 + b^2*c^2*log(F)^2))
```

3.136. $\int F^{c(a+bx)}(f + f\sin(d + ex))\, dx$

3.137 $\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx$

3.137.1 Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx = -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},ie^{i(d+ex)}\right)}{f(e-ibc\log(F))}$$

output
```
-2*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-I*b*c*ln(F)/e],[2-I*b*c*ln
(F)/e],I*exp(I*(e*x+d)))/f/(e-I*b*c*ln(F))
```

3.137.2 Mathematica [A] (verified)

Time = 3.87 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.60

$$\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx = \frac{2F^{c(a+bx)}\left(-i\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{e},1-\frac{ibc\log(F)}{e},i\cos(d+ex)-\sin(d+ex)\right)-\frac{1}{\cos(d)+i(1+\sin(}\right.}{ef}$$

input
```
Integrate[F^(c*(a + b*x))/(f + f*Sin[d + e*x]),x]
```

output
```
(2*F^(c*(a + b*x))*((-I)*Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*
b*c*Log[F])/e, I*Cos[d + e*x] - Sin[d + e*x]] - (Cos[d] + I*(1 + Sin[d]))^
(-1) + Sin[(e*x)/2]/((Cos[d/2] + Sin[d/2])*(Cos[(d + e*x)/2] + Sin[(d + e*
x)/2]))))/(e*f)
```

3.137.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4956, 4952}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{f\sin(d+ex)+f}\,dx$$

$$\Big\downarrow \text{4956}$$

$$\frac{\int F^{c(a+bx)}\csc^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)dx}{2f}$$

$$\Big\downarrow \text{4952}$$

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},ie^{i(d+ex)}\right)}{f(e-ibc\log(F))}$$

input
```
Int[F^(c*(a + b*x))/(f + f*Sin[d + e*x]),x]
```

output
```
(-2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log[F]
)/e, 2 - (I*b*c*Log[F])/e, I*E^(I*(d + e*x))])/(f*(e - I*b*c*Log[F]))
```

3.137.3.1 Defintions of rubi rules used

rule 4952
```
Int[Csc[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_
))), x_Symbol] :> Simp[(-2*I)^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*(F^(c*(a + b
*x))/(I*e*n + b*c*Log[F]))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)),
 1 + n/2 - I*b*c*(Log[F]/(2*e)), E^(2*I*k*Pi)*E^(2*I*(d + e*x))], x] /; Fre
eQ[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

rule 4956
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Simp[2^n*f^n   Int[F^(c*(a + b*x))*Cos[d/2 - f*(Pi/(4
*g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[
f^2 - g^2, 0] && ILtQ[n, 0]
```

3.137.4 Maple [F]

$$\int \frac{F^{c(xb+a)}}{f+f\sin(ex+d)}dx$$

input
```
int(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x)
```

output
```
int(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x)
```

3.137.5 Fricas [F]

$$\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{f\sin(ex+d)+f}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)/(f*sin(e*x + d) + f), x)
```

3.137.6 Sympy [F]

$$\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx = \frac{\int \frac{F^{ac+bcx}}{\sin(d+ex)+1}\,dx}{f}$$

input
```
integrate(F**(c*(b*x+a))/(f+f*sin(e*x+d)),x)
```

output
```
Integral(F**(a*c + b*c*x)/(sin(d + e*x) + 1), x)/f
```

3.137.7 Maxima [F]

$$\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{f\sin(ex+d)+f}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x, algorithm="maxima")
```

output
```
2*(6*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F) + 2*(F^(a*c)*b^3*c^3*log(F)^3 + 4*F^
(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(e*x + d)^2 + 2*(F^(a*c)*b^3*c^3*log(F)
^3 + 4*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(e*x + d)^2 + (5*F^(a*c)*b^2*c
^2*e*log(F)^2 - 4*F^(a*c)*e^3)*F^(b*c*x)*cos(e*x + d) + (F^(a*c)*b^3*c^3*l
og(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(e*x + d) - (6*F^(b*c*x)
*F^(a*c)*b*c*e^2*log(F) + (F^(a*c)*b^2*c^2*e*log(F)^2 + 4*F^(a*c)*e^3)*F^(
b*c*x)*cos(e*x + d) + (F^(a*c)*b^3*c^3*log(F)^3 + 4*F^(a*c)*b*c*e^2*log(F)
)*F^(b*c*x)*sin(e*x + d))*cos(2*e*x + 2*d) - 2*((F^(a*c)*b^5*c^5*e*log(F)^
5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f*cos(2*e*x
 + 2*d)^2 + 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3
 + 4*F^(a*c)*b*c*e^5*log(F))*f*cos(e*x + d)^2 + 4*(F^(a*c)*b^5*c^5*e*log(F
)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f*cos(e*x
 + d)*sin(2*e*x + 2*d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e
^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f*sin(2*e*x + 2*d)^2 + 4*(F^(a*c)*
b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*lo
g(F))*f*sin(e*x + d)^2 + 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3
*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f*sin(e*x + d) + (F^(a*c)*b^5*c^
5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*
f - 2*(2*(F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*
F^(a*c)*b*c*e^5*log(F))*f*sin(e*x + d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + ...
```

3.137.8 Giac [F]

$$\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{f\sin(ex+d)+f}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)/(f*sin(e*x + d) + f), x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx = \int \frac{F^{c\,(a+b\,x)}}{f+f\,\sin\,(d+e\,x)}\,dx$$

input
```
int(F^(c*(a + b*x))/(f + f*sin(d + e*x)),x)
```

output
```
int(F^(c*(a + b*x))/(f + f*sin(d + e*x)), x)
```

3.138 $\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx$

3.138.1 Optimal result

Integrand size = 22, antiderivative size = 184

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx = -\frac{F^{c(a+bx)}\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)}{6ef^2}-\frac{bcF^{c(a+bx)}\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\log(F)}{6e^2f^2}-\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},ie^{i(d+ex)}\right)(e+ibc\log(F))}{3e^2f^2}$$

output
```
-1/6*F^(c*(b*x+a))*cot(1/2*d+1/4*Pi+1/2*e*x)*csc(1/2*d+1/4*Pi+1/2*e*x)^2/e
/f^2-1/6*b*c*F^(c*(b*x+a))*csc(1/2*d+1/4*Pi+1/2*e*x)^2*ln(F)/e^2/f^2-2/3*e
xp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-I*b*c*ln(F)/e],[2-I*b*c*ln(F)/
e],I*exp(I*(e*x+d)))*(e+I*b*c*ln(F))/e^2/f^2
```

3.138.2 Mathematica [A] (verified)

Time = 3.21 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.30

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx = \frac{F^{c(a+bx)}\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)\left(2e^2\sin\left(\frac{1}{2}(d+ex)\right)-e(e+bc\log(F))\left(\cos\left(\frac{1}{2}(d+ex)\right)+\text{si}\right.\right.}{}$$

input
```
Integrate[F^(c*(a + b*x))/(f + f*Sin[d + e*x])^2,x]
```

output
```
(F^(c*(a + b*x))*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*(2*e^2*Sin[(d + e*x
)/2] - e*(e + b*c*Log[F])*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2]) + 2*(e^2 +
 b^2*c^2*Log[F]^2)*Sin[(d + e*x)/2]*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^
2 - (1 - I)*(1 - (1 - I)*Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*
b*c*Log[F])/e, I*Cos[d + e*x] - Sin[d + e*x]])*(e^2 + b^2*c^2*Log[F]^2)*(C
os[(d + e*x)/2] + Sin[(d + e*x)/2])^3))/(3*e^3*f^2*(1 + Sin[d + e*x])^2)
```

3.138.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4956, 4949, 4952}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{(f\sin(d+ex)+f)^2}\,dx$$

$$\Big\downarrow \text{4956}$$

$$\frac{\int F^{c(a+bx)}\csc^4\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)dx}{4f^2}$$

$$\Big\downarrow \text{4949}$$

$$\frac{\frac{2}{3}\left(\frac{b^2c^2\log^2(F)}{e^2}+1\right)\int F^{c(a+bx)}\csc^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)dx-\frac{2bc\log(F)\csc^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)F^{c(a+bx)}}{3e^2}-\frac{2\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)F^{c(a+bx)}}{3e}}{4f^2}$$

$$\Big\downarrow \text{4952}$$

$$\frac{-\frac{8e^{i(d+ex)}F^{c(a+bx)}\left(\frac{b^2c^2\log^2(F)}{e^2}+1\right)\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},ie^{i(d+ex)}\right)}{3(e-ibc\log(F))}-\frac{2bc\log(F)\csc^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)F^{c(a+bx)}}{3e^2}-\ldots}{4f^2}$$

input
```
Int[F^(c*(a + b*x))/(f + f*Sin[d + e*x])^2,x]
```

output
```
((-2*F^(c*(a + b*x))*Cot[d/2 + Pi/4 + (e*x)/2]*Csc[d/2 + Pi/4 + (e*x)/2]^2
)/(3*e) - (2*b*c*F^(c*(a + b*x))*Csc[d/2 + Pi/4 + (e*x)/2]^2*Log[F])/(3*e^
2) - (8*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Lo
g[F])/e, 2 - (I*b*c*Log[F])/e, I*E^(I*(d + e*x))]*(1 + (b^2*c^2*Log[F]^2)/
e^2))/(3*(e - I*b*c*Log[F])))/(4*f^2)
```

3.138.3.1 Defintions of rubi rules used

rule 4949
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (-Simp[F^(c*(a + b*x))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

rule 4952
```
Int[Csc[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_
))), x_Symbol] :> Simp[(-2*I)^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*(F^(c*(a + b
*x))/(I*e*n + b*c*Log[F]))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)),
 1 + n/2 - I*b*c*(Log[F]/(2*e)), E^(2*I*k*Pi)*E^(2*I*(d + e*x))], x] /; Fre
eQ[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

rule 4956
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Simp[2^n*f^n   Int[F^(c*(a + b*x))*Cos[d/2 - f*(Pi/(4
*g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[
f^2 - g^2, 0] && ILtQ[n, 0]
```

3.138.4 Maple [F]

$$\int \frac{F^{c(xb+a)}}{\left(f + f\sin\left(ex+d\right)\right)^2}dx$$

input
```
int(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x)
```

output
```
int(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x)
```

3.138.5 Fricas [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(f\sin(ex+d)+f)^2}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x, algorithm="fricas")
```

output
```
integral(-F^(b*c*x + a*c)/(f^2*cos(e*x + d)^2 - 2*f^2*sin(e*x + d) - 2*f^2
), x)
```

3.138.6 Sympy [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx = \frac{\int \frac{F^{ac+bcx}}{\sin^2(d+ex)+2\sin(d+ex)+1}\,dx}{f^2}$$

input
```
integrate(F**(c*(b*x+a))/(f+f*sin(e*x+d))**2,x)
```

output
```
Integral(F**(a*c + b*c*x)/(sin(d + e*x)**2 + 2*sin(d + e*x) + 1), x)/f**2
```

3.138.7 Maxima [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(f\sin(ex+d)+f)^2}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x, algorithm="maxima")
```

output
```
4*(6*(F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(
a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d)^2 + 80*(F^(a*c)*b^3*c^3*e^
2*log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(e*x + d)^2 + 6*(F^(a
*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e
^4*log(F))*F^(b*c*x)*sin(2*e*x + 2*d)^2 + 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3
 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(e*x + d)^2 - 20*(F^(a*c)*b^4*c
^4*e*log(F)^4 - 26*F^(a*c)*b^2*c^2*e^3*log(F)^2)*F^(b*c*x)*cos(e*x + d) -
140*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 8*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*si
n(e*x + d) - 40*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 5*F^(a*c)*b*c*e^4*log(F))*
F^(b*c*x) - ((F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 +
 144*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) + 4*(F^(a*c)*b^4*c
^4*e*log(F)^4 + 10*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 96*F^(a*c)*e^5)*F^(b*c*x
)*cos(e*x + d) - 2*(F^(a*c)*b^4*c^4*e*log(F)^4 + 25*F^(a*c)*b^2*c^2*e^3*lo
g(F)^2 + 144*F^(a*c)*e^5)*F^(b*c*x)*sin(2*e*x + 2*d) - 20*(F^(a*c)*b^3*c^3
*e^2*log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(e*x + d) + 40*(F^
(a*c)*b^3*c^3*e^2*log(F)^3 - 5*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x))*cos(4*e*
x + 4*d) - 4*(2*(F^(a*c)*b^4*c^4*e*log(F)^4 + 25*F^(a*c)*b^2*c^2*e^3*log(F
)^2 + 144*F^(a*c)*e^5)*F^(b*c*x)*cos(2*e*x + 2*d) + 20*(F^(a*c)*b^3*c^3*e^
2*log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(e*x + d) + (F^(a*c)*
b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e^...
```

3.138.8 Giac [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(f\sin{(ex+d)}+f)^2}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)/(f*sin(e*x + d) + f)^2, x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx = \int \frac{F^{c\,(a+b\,x)}}{\left(f+f\,\sin\left(d+e\,x\right)\right)^2}\,dx$$

input

```
int(F^(c*(a + b*x))/(f + f*sin(d + e*x))^2,x)
```

output

```
int(F^(c*(a + b*x))/(f + f*sin(d + e*x))^2, x)
```

3.139 $\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx$

3.139.1 Optimal result

Integrand size = 22, antiderivative size = 245

$$\begin{aligned}\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx &= \frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2bcf^2F^{ac+bcx}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)} \\ &+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)} \\ &+\frac{bcf^2F^{ac+bcx}\cos^2(d+ex)\log(F)}{4e^2+b^2c^2\log^2(F)} \\ &+\frac{2ef^2F^{ac+bcx}\sin(d+ex)}{e^2+b^2c^2\log^2(F)} \\ &+\frac{2ef^2F^{ac+bcx}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)}\end{aligned}$$

output
```
f^2*F^(b*c*x+a*c)/b/c/ln(F)+2*b*c*f^2*F^(b*c*x+a*c)*cos(e*x+d)*ln(F)/(e^2+
b^2*c^2*ln(F)^2)+2*e^2*f^2*F^(b*c*x+a*c)/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)
+b*c*f^2*F^(b*c*x+a*c)*cos(e*x+d)^2*ln(F)/(4*e^2+b^2*c^2*ln(F)^2)+2*e*f^2*
F^(b*c*x+a*c)*sin(e*x+d)/(e^2+b^2*c^2*ln(F)^2)+2*e*f^2*F^(b*c*x+a*c)*cos(e
*x+d)*sin(e*x+d)/(4*e^2+b^2*c^2*ln(F)^2)
```

3.139.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.93

$$\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx$$
$$=\frac{f^2F^{c(a+bx)}\left(12e^4+15b^2c^2e^2\log^2(F)+3b^4c^4\log^4(F)+b^2c^2\cos(2(d+ex))\log^2(F)\left(e^2+b^2c^2\log^2(F)\right)+4\right.}{}$$

input
```
Integrate[F^(c*(a + b*x))*(f + f*Cos[d + e*x])^2,x]
```

output
```
(f^2*F^(c*(a + b*x))*(12*e^4 + 15*b^2*c^2*e^2*Log[F]^2 + 3*b^4*c^4*Log[F]^
4 + b^2*c^2*Cos[2*(d + e*x)]*Log[F]^2*(e^2 + b^2*c^2*Log[F]^2) + 4*b^2*c^2
*Cos[d + e*x]*Log[F]^2*(4*e^2 + b^2*c^2*Log[F]^2) + 16*b*c*e^3*Log[F]*Sin[
d + e*x] + 4*b^3*c^3*e*Log[F]^3*Sin[d + e*x] + 2*b*c*e^3*Log[F]*Sin[2*(d +
 e*x)] + 2*b^3*c^3*e*Log[F]^3*Sin[2*(d + e*x)]))/(2*(4*b*c*e^4*Log[F] + 5*
b^3*c^3*e^2*Log[F]^3 + b^5*c^5*Log[F]^5))
```

3.139.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\cos(d+ex)+f)^2\,dx$$
$$\Big\downarrow \; 7292$$
$$\int f^2(\cos(d+ex)+1)^2F^{ac+bcx}dx$$
$$\Big\downarrow \; 27$$
$$f^2\int F^{ac+bxc}(\cos(d+ex)+1)^2dx$$
$$\Big\downarrow \; 7293$$
$$f^2\int \left(\cos^2(d+ex)F^{ac+bxc}+2\cos(d+ex)F^{ac+bxc}+F^{ac+bxc}\right)dx$$

$\downarrow$ 2009

$$f^2\left(\frac{2e\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{bc\log(F)\cos^2(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+4e^2}+\frac{2bc\log(F)\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{2e\sin(d+ex)\cos}{b^2c^2\log^2(}\right.$$

input
```
Int[F^(c*(a + b*x))*(f + f*Cos[d + e*x])^2,x]
```

output
```
f^2*(F^(a*c + b*c*x)/(b*c*Log[F]) + (2*b*c*F^(a*c + b*c*x)*Cos[d + e*x]*Lo
g[F])/(e^2 + b^2*c^2*Log[F]^2) + (2*e^2*F^(a*c + b*c*x))/(b*c*Log[F]*(4*e^
2 + b^2*c^2*Log[F]^2)) + (b*c*F^(a*c + b*c*x)*Cos[d + e*x]^2*Log[F])/(4*e^
2 + b^2*c^2*Log[F]^2) + (2*e*F^(a*c + b*c*x)*Sin[d + e*x])/(e^2 + b^2*c^2*
Log[F]^2) + (2*e*F^(a*c + b*c*x)*Cos[d + e*x]*Sin[d + e*x])/(4*e^2 + b^2*c
^2*Log[F]^2))
```

3.139.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.139.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.79

method	result
risch	$\frac{3f^2F^{c(xb+a)}}{2bc\ln(F)}+\frac{2\ln(F)cbf^2F^{c(xb+a)}\cos(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{2F^{c(xb+a)}ef^2\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{\ln(F)cbf^2F^{c(xb+a)}\cos(2ex+2d)}{2b^2c^2\ln(F)^2+8e^2}+ef^2$
parallelrisch	$\frac{2F^{c(xb+a)}f^2\left(\frac{b^2c^2\ln(F)^2\left(e^2+b^2c^2\ln(F)^2\right)\cos(2ex+2d)}{4}+\frac{cbe\ln(F)\left(e^2+b^2c^2\ln(F)^2\right)\sin(2ex+2d)}{2}+\left(4e^2+b^2c^2\ln(F)^2\right)\left(\cos(ex+d)\right.\right.}{bc\ln(F)\left(e^2+b^2c^2\ln(F)^2\right)\left(4e^2+b^2c^2\ln(F)^2\right)}$
norman	$\frac{\frac{12e^3f^2e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{b^4c^4\ln(F)^4+5b^2c^2e^2\ln(F)^2+4e^4}+\frac{4\left(2b^2c^2\ln(F)^2+5e^2\right)ef^2e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{b^4c^4\ln(F)^4+5b^2c^2e^2\ln(F)^2+4e^4}+\frac{2f^2\left(2b^4c^4\ln(F)^4+8b^2c^2e^2\ln(F)^2+3e^4\right)e^{c}}{bc\ln(F)\left(b^4c^4\ln(F)^4+5b^2c^2e^2\ln(F)^2\right.}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)^2}$
default	$F^{ac}f^2\left(\frac{2F^{bcx}}{bc\ln(F)}+\frac{\frac{8e\,e^{bcx\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}+\frac{4bc\ln(F)e^{bcx\ln(F)}}{e^2+b^2c^2\ln(F)^2}-\frac{4bc\ln(F)e^{bcx\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}+\frac{\frac{8e\,e^{bcx\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{4e^2+b^2c^2\ln(F)^2}-8e}{}\right.$
parts	$\frac{f^2F^{c(xb+a)}}{bc\ln(F)}+\frac{\frac{\left(b^2c^2\ln(F)^2+2e^2\right)f^2e^{c(xb+a)\ln(F)}}{bc\ln(F)\left(4e^2+b^2c^2\ln(F)^2\right)}+\frac{\left(b^2c^2\ln(F)^2+2e^2\right)f^2e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4}{bc\ln(F)\left(4e^2+b^2c^2\ln(F)^2\right)}+\frac{4ef^2e^{c(xb+a)\ln(F)}\tan\left(\frac{e}{2}\right.}{4e^2+b^2c^2\ln(F)^2}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)}$

input
```
int(F^(c*(b*x+a))*(f+f*cos(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
3/2/b/c/ln(F)*f^2*F^(c*(b*x+a))+2*ln(F)*c*b*f^2*F^(c*(b*x+a))/(e^2+b^2*c^2
*ln(F)^2)*cos(e*x+d)+2*F^(c*(b*x+a))*e*f^2/(e^2+b^2*c^2*ln(F)^2)*sin(e*x+d
)+1/2/(4*e^2+b^2*c^2*ln(F)^2)*ln(F)*c*b*f^2*F^(c*(b*x+a))*cos(2*e*x+2*d)+e
*f^2*F^(c*(b*x+a))/(4*e^2+b^2*c^2*ln(F)^2)*sin(2*e*x+2*d)
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.99

$$\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx$$
$$=\frac{\left(6\,e^4f^2+\left(b^4c^4f^2\cos\left(ex+d\right)^2+2\,b^4c^4f^2\cos\left(ex+d\right)+b^4c^4f^2\right)\log\left(F\right)^4+\left(b^2c^2e^2f^2\cos\left(ex+d\right)^2+8\,b^2\right.}{b}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d))^2,x, algorithm="fricas")
```

output
```
(6*e^4*f^2 + (b^4*c^4*f^2*cos(e*x + d)^2 + 2*b^4*c^4*f^2*cos(e*x + d) + b^
4*c^4*f^2)*log(F)^4 + (b^2*c^2*e^2*f^2*cos(e*x + d)^2 + 8*b^2*c^2*e^2*f^2*
cos(e*x + d) + 7*b^2*c^2*e^2*f^2)*log(F)^2 + 2*((b^3*c^3*e*f^2*cos(e*x + d
) + b^3*c^3*e*f^2)*log(F)^3 + (b*c*e^3*f^2*cos(e*x + d) + 4*b*c*e^3*f^2)*l
og(F))*sin(e*x + d))*F^(b*c*x + a*c)/(b^5*c^5*log(F)^5 + 5*b^3*c^3*e^2*log
(F)^3 + 4*b*c*e^4*log(F))
```

3.139.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.97 (sec) , antiderivative size = 2394, normalized size of antiderivative = 9.77

$$\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*(f+f*cos(e*x+d))**2,x)
```

output
```
Piecewise((x*(f*cos(d) + f)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0))
, (f**2*x*sin(d + e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f**2*x + f**2*sin
(d + e*x)*cos(d + e*x)/(2*e) + 2*f**2*sin(d + e*x)/e, Eq(F, 1)), (F**(a*c)
*(f**2*x*sin(d + e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f**2*x + f**2*sin(
d + e*x)*cos(d + e*x)/(2*e) + 2*f**2*sin(d + e*x)/e), Eq(b, 0)), (f**2*x*s
in(d + e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f**2*x + f**2*sin(d + e*x)*c
os(d + e*x)/(2*e) + 2*f**2*sin(d + e*x)/e, Eq(c, 0)), (I*F**(a*c + b*c*x)*
f**2*x*sin(I*b*c*x*log(F) - d) + F**(a*c + b*c*x)*f**2*x*cos(I*b*c*x*log(F
) - d) + 2*F**(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F) - d)**2/(3*b*c*log(F))
 - 2*I*F**(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F) - d)*cos(I*b*c*x*log(F) -
d)/(3*b*c*log(F)) - I*F**(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F) - d)/(b*c*l
og(F)) + F**(a*c + b*c*x)*f**2*cos(I*b*c*x*log(F) - d)**2/(3*b*c*log(F)) +
 F**(a*c + b*c*x)*f**2/(b*c*log(F)), Eq(e, -I*b*c*log(F))), (-F**(a*c + b*
c*x)*f**2*x*sin(I*b*c*x*log(F)/2 - d)**2/4 + I*F**(a*c + b*c*x)*f**2*x*sin
(I*b*c*x*log(F)/2 - d)*cos(I*b*c*x*log(F)/2 - d)/2 + F**(a*c + b*c*x)*f**2
*x*cos(I*b*c*x*log(F)/2 - d)**2/4 + I*F**(a*c + b*c*x)*f**2*sin(I*b*c*x*lo
g(F)/2 - d)*cos(I*b*c*x*log(F)/2 - d)/(2*b*c*log(F)) + 4*I*F**(a*c + b*c*x
)*f**2*sin(I*b*c*x*log(F)/2 - d)/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2*co
s(I*b*c*x*log(F)/2 - d)**2/(b*c*log(F)) + 8*F**(a*c + b*c*x)*f**2*cos(I*b*
c*x*log(F)/2 - d)/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2/(b*c*log(F)), ...
```

3.139.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(245) = 490$.

Time = 0.24 (sec) , antiderivative size = 578, normalized size of antiderivative = 2.36

$$\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx$$
$$= \frac{\left(\left(F^{ac}b^2c^2\cos(2d)\log(F)^2 + 2F^{ac}bce\log(F)\sin(2d)\right)F^{bcx}\cos(2ex) + \left(F^{ac}b^2c^2\cos(2d)\log(F)^2 - 2F^{a\cdots}\right.}{\cdots}$$
$$+ \frac{\left((F^{ac}bc\cos(d)\log(F) - F^{ac}e\sin(d))F^{bcx}\cos(ex+2d) + (F^{ac}bc\cos(d)\log(F) + F^{ac}e\sin(d))F^{bcx}\cos\cdots\right.}{b^2c^2\cos(d)^2\log(F)^2 + b^2c^{\cdots}}$$
$$+ \frac{F^{bcx+ac}f^2}{bc\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d))^2,x, algorithm="maxima")
```

output
```
1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d))
*F^(b*c*x)*cos(2*e*x) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*c
*e*log(F)*sin(2*d))*F^(b*c*x)*cos(2*e*x + 4*d) - (F^(a*c)*b^2*c^2*log(F)^2
*sin(2*d) - 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2*e*x) + (F^(a*
c)*b^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*
sin(2*e*x + 4*d) + 2*(F^(a*c)*b^2*c^2*cos(2*d)^2*log(F)^2 + F^(a*c)*b^2*c^
2*log(F)^2*sin(2*d)^2 + 4*(F^(a*c)*cos(2*d)^2 + F^(a*c)*sin(2*d)^2)*e^2)*F
^(b*c*x))*f^2/(b^3*c^3*cos(2*d)^2*log(F)^3 + b^3*c^3*log(F)^3*sin(2*d)^2 +
 4*(b*c*cos(2*d)^2*log(F) + b*c*log(F)*sin(2*d)^2)*e^2) + ((F^(a*c)*b*c*co
s(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x + 2*d) + (F^(a*c)*b*c*co
s(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x) + (F^(a*c)*b*c*log(F)*s
in(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^(a*c)*b*c*log(F)*s
in(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x))*f^2/(b^2*c^2*cos(d)^2*log(F)
^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2) + F^(b*c*x + a
*c)*f^2/(b*c*log(F))
```

3.139.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 1736, normalized size of antiderivative = 7.09

$$\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d))^2,x, algorithm="giac")
```

output
```
1/2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F)
- 1/2*pi*a*c + 2*e*x + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c
*sgn(F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*f^2*sin(1/2*pi
*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*e*x + 2*
d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*
log(abs(F)) + a*c*log(abs(F))) + 2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/
2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)*log(abs(F))/(4*b^2*
c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) + (pi*b*c*sgn(F) - p
i*b*c + 2*e)*f^2*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F
) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b
*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 2*(2*b*c*f^2*cos(1
/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x -
 d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^
2) + (pi*b*c*sgn(F) - pi*b*c - 2*e)*f^2*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b
*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2
+ (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)
)) + 1/2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sg
n(F) - 1/2*pi*a*c - 2*e*x - 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (p
i*b*c*sgn(F) - pi*b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 4*e)*f^2*sin(1
/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*...
```

3.139.9 Mupad [B] (verification not implemented)

Time = 27.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx$$
$$= \frac{F^{a\,c+b\,c\,x}\,f^2\,\left(6\,e^4+\frac{3\,b^4\,c^4\,\ln(F)^4}{2}+2\,b^4\,c^4\,\cos\left(d+e\,x\right)\,\ln\left(F\right)^4+\frac{b^4\,c^4\,\ln(F)^4\,\cos(2\,d+2\,e\,x)}{2}+\frac{15\,b^2\,c^2\,e^2\,\ln(F)^2}{2}+8\,b\,c\right.}{}$$

input
```
int(F^(c*(a + b*x))*(f + f*cos(d + e*x))^2,x)
```

output
```
(F^(a*c + b*c*x)*f^2*(6*e^4 + (3*b^4*c^4*log(F)^4)/2 + 2*b^4*c^4*cos(d + e
*x)*log(F)^4 + (b^4*c^4*log(F)^4*cos(2*d + 2*e*x))/2 + (15*b^2*c^2*e^2*log
(F)^2)/2 + 8*b*c*e^3*sin(d + e*x)*log(F) + 8*b^2*c^2*e^2*cos(d + e*x)*log(
F)^2 + b^3*c^3*e*log(F)^3*sin(2*d + 2*e*x) + b*c*e^3*log(F)*sin(2*d + 2*e*
x) + (b^2*c^2*e^2*log(F)^2*cos(2*d + 2*e*x))/2 + 2*b^3*c^3*e*sin(d + e*x)*
log(F)^3))/(b*c*log(F)*(4*e^4 + b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2)
)
```

3.139. $\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx$

3.140 $\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx$

3.140.1 Optimal result

Integrand size = 20, antiderivative size = 98

$$\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx=\frac{fF^{ac+bcx}}{bc\log(F)}+\frac{bcfF^{ac+bcx}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{efF^{ac+bcx}\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

output
```
f*F^(b*c*x+a*c)/b/c/ln(F)+b*c*f*F^(b*c*x+a*c)*cos(e*x+d)*ln(F)/(e^2+b^2*c^
2*ln(F)^2)+e*f*F^(b*c*x+a*c)*sin(e*x+d)/(e^2+b^2*c^2*ln(F)^2)
```

3.140.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

$$\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx=\frac{fF^{c(a+bx)}\left(e^2+b^2c^2\log^2(F)+b^2c^2\cos(d+ex)\log^2(F)+bce\log(F)\sin(d+ex)\right)}{bc\log(F)\left(e^2+b^2c^2\log^2(F)\right)}$$

input
```
Integrate[F^(c*(a + b*x))*(f + f*Cos[d + e*x]),x]
```

output
```
(f*F^(c*(a + b*x))*(e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cos[d + e*x]*Log[F]^2
 + b*c*e*Log[F]*Sin[d + e*x]))/(b*c*Log[F]*(e^2 + b^2*c^2*Log[F]^2))
```

3.140.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\cos(d+ex)+f)\,dx$$

$$\Big\downarrow \text{7292}$$

$$\int f(\cos(d+ex)+1)F^{ac+bcx}dx$$

$$\Big\downarrow \text{27}$$

$$f\int F^{ac+bxc}(\cos(d+ex)+1)dx$$

$$\Big\downarrow \text{7293}$$

$$f\int \left(\cos(d+ex)F^{ac+bxc}+F^{ac+bxc}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$f\left(\frac{e\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{bc\log(F)\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{F^{ac+bcx}}{bc\log(F)}\right)$$

input
```
Int[F^(c*(a + b*x))*(f + f*Cos[d + e*x]),x]
```

output
```
f*(F^(a*c + b*c*x)/(b*c*Log[F]) + (b*c*F^(a*c + b*c*x)*Cos[d + e*x]*Log[F]
)/(e^2 + b^2*c^2*Log[F]^2) + (e*F^(a*c + b*c*x)*Sin[d + e*x])/(e^2 + b^2*c
^2*Log[F]^2))
```

3.140.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.140.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$\frac{f\,F^{c(xb+a)}\left(\cos(ex+d)b^{2}c^{2}\ln(F)^{2}+b^{2}c^{2}\ln(F)^{2}+e\sin(ex+d)\ln(F)bc+e^{2}\right)}{bc\ln(F)\left(e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}$	83
risch	$\frac{f\,F^{c(xb+a)}}{bc\ln(F)}+\frac{\ln(F)cbf\,F^{c(xb+a)}\cos(ex+d)}{e^{2}+b^{2}c^{2}\ln(F)^{2}}+\frac{ef\,F^{c(xb+a)}\sin(ex+d)}{e^{2}+b^{2}c^{2}\ln(F)^{2}}$	96
parts	$\frac{f\,F^{c(xb+a)}}{bc\ln(F)}+\frac{\frac{fbc\ln(F)\mathrm{e}^{c(xb+a)\ln(F)}}{e^{2}+b^{2}c^{2}\ln(F)^{2}}+\frac{2ef\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^{2}+b^{2}c^{2}\ln(F)^{2}}-\frac{fbc\ln(F)\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{e^{2}+b^{2}c^{2}\ln(F)^{2}}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}$	158
norman	$\frac{\frac{\left(2b^{2}c^{2}\ln(F)^{2}+e^{2}\right)f\,\mathrm{e}^{c(xb+a)\ln(F)}}{bc\ln(F)\left(e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}+\frac{e^{2}f\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{\left(e^{2}+b^{2}c^{2}\ln(F)^{2}\right)bc\ln(F)}+\frac{2ef\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^{2}+b^{2}c^{2}\ln(F)^{2}}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}$	166

input
```
int(F^(c*(b*x+a))*(f+f*cos(e*x+d)),x,method=_RETURNVERBOSE)
```

output
```
f*F^(c*(b*x+a))*(cos(e*x+d)*b^2*c^2*ln(F)^2+b^2*c^2*ln(F)^2+e*sin(e*x+d)*l
n(F)*b*c+e^2)/b/c/ln(F)/(e^2+b^2*c^2*ln(F)^2)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx = \frac{\left(bcef\log(F)\sin(ex+d)+e^2f+\left(b^2c^2f\cos(ex+d)+b^2c^2f\right)\log(F)^2\right)F^{bcx+ac}}{b^3c^3\log(F)^3+bce^2\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d)),x, algorithm="fricas")
```

output
```
(b*c*e*f*log(F)*sin(e*x + d) + e^2*f + (b^2*c^2*f*cos(e*x + d) + b^2*c^2*f
)*log(F)^2)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + b*c*e^2*log(F))
```

3.140.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 478, normalized size of antiderivative = 4.88

$$\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx = \begin{cases} x(f\cos(d)+f) & \text{for } F \\ fx+\frac{f\sin(d+ex)}{e} & \text{for } F \\ F^{ac}\left(fx+\frac{f\sin(d+ex)}{e}\right) & \text{for } b \\ fx+\frac{f\sin(d+ex)}{e} & \text{for } c \\ \frac{iF^{ac+bcx}fx\sin(ibcx\log(F)-d)}{2}+\frac{F^{ac+bcx}fx\cos(ibcx\log(F)-d)}{2}-\frac{iF^{ac+bcx}f\sin(ibcx\log(F)-d)}{2bc\log(F)}+\frac{F^{ac+bcx}f}{bc\log(F)} & \text{for } e \\ \frac{iF^{ac+bcx}fx\sin(ibcx\log(F)+d)}{2}+\frac{F^{ac+bcx}fx\cos(ibcx\log(F)+d)}{2}-\frac{iF^{ac+bcx}f\sin(ibcx\log(F)+d)}{2bc\log(F)}+\frac{F^{ac+bcx}f}{bc\log(F)} & \text{for } e \\ \frac{F^{ac+bcx}b^2c^2f\log(F)^2\cos(d+ex)}{b^3c^3\log(F)^3+bce^2\log(F)}+\frac{F^{ac+bcx}b^2c^2f\log(F)^2}{b^3c^3\log(F)^3+bce^2\log(F)}+\frac{F^{ac+bcx}bcef\log(F)\sin(d+ex)}{b^3c^3\log(F)^3+bce^2\log(F)}+\frac{F^{ac+bcx}e^2f}{b^3c^3\log(F)^3+bce^2\log(F)} & \text{othe} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*(f+f*cos(e*x+d)),x)
```

output
```
Piecewise((x*(f*cos(d) + f), Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (
f*x + f*sin(d + e*x)/e, Eq(F, 1)), (F**(a*c)*(f*x + f*sin(d + e*x)/e), Eq(
b, 0)), (f*x + f*sin(d + e*x)/e, Eq(c, 0)), (I*F**(a*c + b*c*x)*f*x*sin(I*
b*c*x*log(F) - d)/2 + F**(a*c + b*c*x)*f*x*cos(I*b*c*x*log(F) - d)/2 - I*F
**(a*c + b*c*x)*f*sin(I*b*c*x*log(F) - d)/(2*b*c*log(F)) + F**(a*c + b*c*x
)*f/(b*c*log(F)), Eq(e, -I*b*c*log(F))), (I*F**(a*c + b*c*x)*f*x*sin(I*b*c
*x*log(F) + d)/2 + F**(a*c + b*c*x)*f*x*cos(I*b*c*x*log(F) + d)/2 - I*F**(
a*c + b*c*x)*f*sin(I*b*c*x*log(F) + d)/(2*b*c*log(F)) + F**(a*c + b*c*x)*f
/(b*c*log(F)), Eq(e, I*b*c*log(F))), (F**(a*c + b*c*x)*b**2*c**2*f*log(F)*
*2*cos(d + e*x)/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c + b*c*x)
*b**2*c**2*f*log(F)**2/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c +
 b*c*x)*b*c*e*f*log(F)*sin(d + e*x)/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)
) + F**(a*c + b*c*x)*e**2*f/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)), True)
)
```

3.140.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(98) = 196$.

Time = 0.22 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.20

$$\int F^{c(a+bx)}(f + f\cos(d + ex))\,dx$$
$$= \frac{((F^{ac}bc\cos(d)\log(F) - F^{ac}e\sin(d))F^{bcx}\cos(ex + 2d) + (F^{ac}bc\cos(d)\log(F) + F^{ac}e\sin(d))F^{bcx}\cos(e}{2\left(b^2c^2\cos(d)^2\log(F)^2 + b^2c^2\,\mathrm{l}\right.}$$
$$+ \frac{F^{bcx+ac}f}{bc\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d)),x, algorithm="maxima")
```

output
```
1/2*((F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x + 2*
d) + (F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x) + (
F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x + 2*d) - (
F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x))*f/(b^2*c
^2*cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e
^2) + F^(b*c*x + a*c)*f/(b*c*log(F))
```

3.140.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 920, normalized size of antiderivative = 9.39

$$\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d)),x, algorithm="giac")
```

output
```
(2*b*c*f*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*
pi*a*c + e*x + d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) -
pi*b*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 2*e)*f*sin(1/2*pi*b*c*x*sgn(F
) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*lo
g(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*
c*log(abs(F))) + (2*b*c*f*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*
a*c*sgn(F) - 1/2*pi*a*c - e*x - d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 +
(pi*b*c*sgn(F) - pi*b*c - 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 2*e)*f*sin(1
/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x -
 d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x
*log(abs(F)) + a*c*log(abs(F))) + 2*(2*b*c*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/
2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(ab
s(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/
2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*
c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*
c*log(abs(F))) + I*(I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*
pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*
c + 4*b*c*log(abs(F)) + 4*I*e) - I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*
b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-2*I*pi*b*c*sgn
(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) + a...
```

3.140.9 Mupad [B] (verification not implemented)

Time = 26.70 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx$$
$$= \frac{F^{a\,c+b\,c\,x}\,f\,\left(e^2+b^2\,c^2\ln\left(F\right)^2+b^2\,c^2\,\cos\left(d+e\,x\right)\,\ln\left(F\right)^2+b\,c\,e\,\sin\left(d+e\,x\right)\,\ln\left(F\right)\right)}{b\,c\,\ln\left(F\right)\,\left(b^2\,c^2\ln\left(F\right)^2+e^2\right)}$$

input
```
int(F^(c*(a + b*x))*(f + f*cos(d + e*x)),x)
```

output
```
(F^(a*c + b*c*x)*f*(e^2 + b^2*c^2*log(F)^2 + b^2*c^2*cos(d + e*x)*log(F)^2
 + b*c*e*sin(d + e*x)*log(F)))/(b*c*log(F)*(e^2 + b^2*c^2*log(F)^2))
```

3.140. $\int F^{c(a+bx)}(f + f\cos(d+ex))\,dx$

3.141 $\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx$

3.141.1 Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx = \frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},-e^{i(d+ex)}\right)}{f(ie+bc\log(F))}$$

output
```
2*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-I*b*c*ln(F)/e],[2-I*b*c*ln(
F)/e],-exp(I*(e*x+d)))/f/(b*c*ln(F)+I*e)
```

3.141.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx = -\frac{2ie^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},-e^{i(d+ex)}\right)}{f(e-ibc\log(F))}$$

input
```
Integrate[F^(c*(a + b*x))/(f + f*Cos[d + e*x]),x]
```

output
```
((-2*I)*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Lo
g[F])/e, 2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))])/(f*(e - I*b*c*Log[F]))
```

3.141.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4957, 4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{f\cos(d+ex)+f}\,dx$$

$$\Big\downarrow \ 4957$$

$$\frac{\int F^{c(a+bx)}\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)dx}{2f}$$

$$\Big\downarrow \ 4951$$

$$\frac{2e^{i(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},-e^{i(d+ex)}\right)}{f(bc\log(F)+ie)}$$

input
```
Int[F^(c*(a + b*x))/(f + f*Cos[d + e*x]),x]
```

output
```
(2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])
/e, 2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))])/(f*(I*e + b*c*Log[F]))
```

3.141.3.1 Defintions of rubi rules used

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

rule 4957
```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Simp[2^n*f^n   Int[F^(c*(a + b*x))*Cos[d/2 + e*(x/2)]
^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && IL
tQ[n, 0]
```

3.141.4 Maple [F]

$$\int \frac{F^{c(xb+a)}}{f+f\cos\left(ex+d\right)}dx$$

input
```
int(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x)
```

output
```
int(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x)
```

3.141.5 Fricas [F]

$$\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{f\cos\left(ex+d\right)+f}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)/(f*cos(e*x + d) + f), x)
```

3.141.6 Sympy [F]

$$\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx = \frac{\int \frac{F^{ac+bcx}}{\cos\left(d+ex\right)+1}\,dx}{f}$$

input
```
integrate(F**(c*(b*x+a))/(f+f*cos(e*x+d)),x)
```

output
```
Integral(F**(a*c + b*c*x)/(cos(d + e*x) + 1), x)/f
```

3.141.7 Maxima [F]

$$\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx=\int \frac{F^{(bx+a)c}}{f\cos(ex+d)+f}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x, algorithm="maxima")
```

output
```
2*(6*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F) + 2*(F^(a*c)*b^3*c^3*log(F)^3 + 4*F^
(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(e*x + d)^2 + 2*(F^(a*c)*b^3*c^3*log(F)
^3 + 4*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(e*x + d)^2 + (F^(a*c)*b^3*c^3
*log(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(e*x + d) - (5*F^(a*c)
*b^2*c^2*e*log(F)^2 - 4*F^(a*c)*e^3)*F^(b*c*x)*sin(e*x + d) + (6*F^(b*c*x)
*F^(a*c)*b*c*e^2*log(F) + (F^(a*c)*b^3*c^3*log(F)^3 + 4*F^(a*c)*b*c*e^2*lo
g(F))*F^(b*c*x)*cos(e*x + d) - (F^(a*c)*b^2*c^2*e*log(F)^2 + 4*F^(a*c)*e^3
)*F^(b*c*x)*sin(e*x + d))*cos(2*e*x + 2*d) - 2*((F^(a*c)*b^5*c^5*e*log(F)^
5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f*cos(2*e*x
 + 2*d)^2 + 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3
 + 4*F^(a*c)*b*c*e^5*log(F))*f*cos(e*x + d)^2 + (F^(a*c)*b^5*c^5*e*log(F)^
5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f*sin(2*e*x
 + 2*d)^2 + 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3
 + 4*F^(a*c)*b*c*e^5*log(F))*f*sin(2*e*x + 2*d)*sin(e*x + d) + 4*(F^(a*c)*
b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*lo
g(F))*f*sin(e*x + d)^2 + 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3
*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f*cos(e*x + d) + (F^(a*c)*b^5*c^
5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*
f + 2*(2*(F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*
F^(a*c)*b*c*e^5*log(F))*f*cos(e*x + d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + ...
```

3.141.8 Giac [F]

$$\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx=\int \frac{F^{(bx+a)c}}{f\cos(ex+d)+f}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)/(f*cos(e*x + d) + f), x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx = \int \frac{F^{c\,(a+b\,x)}}{f+f\,\cos\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))/(f + f*cos(d + e*x)),x)
```

output
```
int(F^(c*(a + b*x))/(f + f*cos(d + e*x)), x)
```

3.142 $\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx$

3.142.1 Optimal result

Integrand size = 22, antiderivative size = 169

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx = -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},-e^{i(d+ex)}\right)(ie-bc\log(F))}{3e^2f^2} - \frac{bcF^{c(a+bx)}\log(F)\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{6e^2f^2} + \frac{F^{c(a+bx)}\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{6ef^2}$$

output
```
-2/3*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-I*b*c*ln(F)/e],[2-I*b*c*
ln(F)/e],-exp(I*(e*x+d)))*(I*e-b*c*ln(F))/e^2/f^2-1/6*b*c*F^(c*(b*x+a))*ln
(F)*sec(1/2*e*x+1/2*d)^2/e^2/f^2+1/6*F^(c*(b*x+a))*sec(1/2*e*x+1/2*d)^2*ta
n(1/2*e*x+1/2*d)/e/f^2
```

3.142.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx = \frac{2F^{c(a+bx)}\cos\left(\frac{1}{2}(d+ex)\right)\left(-bc\cos\left(\frac{1}{2}(d+ex)\right)\log(F)+4e^{i(d+ex)}\cos^3\left(\frac{1}{2}(d+ex)\right)\text{Hypergeometric2F1}\left(2\right.\right.}{3e^2f^2(1+\cos(d+ex))^2}$$

input
```
Integrate[F^(c*(a + b*x))/(f + f*Cos[d + e*x])^2,x]
```

output
```
(2*F^(c*(a + b*x))*Cos[(d + e*x)/2]*(-(b*c*Cos[(d + e*x)/2]*Log[F]) + 4*E^
(I*(d + e*x))*Cos[(d + e*x)/2]^3*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])/e
, 2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))]*((-I)*e + b*c*Log[F]) + e*Sin[(d
 + e*x)/2]))/(3*e^2*f^2*(1 + Cos[d + e*x])^2)
```

3.142.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4957, 4948, 4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{(f\cos(d+ex)+f)^2}dx$$

$$\Big\downarrow \text{4957}$$

$$\frac{\int F^{c(a+bx)}\sec^4\left(\frac{d}{2}+\frac{ex}{2}\right)dx}{4f^2}$$

$$\Big\downarrow \text{4948}$$

$$\frac{\frac{2}{3}\left(\frac{b^2c^2\log^2(F)}{e^2}+1\right)\int F^{c(a+bx)}\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)dx-\frac{2bc\log(F)\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{3e^2}+\frac{2\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{3e}}{4f^2}$$

$$\Big\downarrow \text{4951}$$

$$\frac{\frac{8e^{i(d+ex)}F^{c(a+bx)}\left(\frac{b^2c^2\log^2(F)}{e^2}+1\right)\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},-e^{i(d+ex)}\right)}{3(bc\log(F)+ie)}-\frac{2bc\log(F)\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{3e^2}+\frac{2\tan}{}}{4f^2}$$

input
```
Int[F^(c*(a + b*x))/(f + f*Cos[d + e*x])^2,x]
```

output
```
((8*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log[F]
)/e, 2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))]*(1 + (b^2*c^2*Log[F]^2)/e^2))
/(3*(I*e + b*c*Log[F])) - (2*b*c*F^(c*(a + b*x))*Log[F]*Sec[d/2 + (e*x)/2]
^2)/(3*e^2) + (2*F^(c*(a + b*x))*Sec[d/2 + (e*x)/2]^2*Tan[d/2 + (e*x)/2])/
(3*e))/(4*f^2)
```

3.142.3.1 Defintions of rubi rules used

rule 4948
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(
e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2))   Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b,
 c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && N
eQ[n, 2]
```

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

rule 4957
```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Simp[2^n*f^n   Int[F^(c*(a + b*x))*Cos[d/2 + e*(x/2)]
^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && IL
tQ[n, 0]
```

3.142.4 Maple [F]

$$\int \frac{F^{c(xb+a)}}{(f + f\cos(ex+d))^2} dx$$

input
```
int(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x)
```

output
```
int(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x)
```

3.142.5 Fricas [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx=\int \frac{F^{(bx+a)c}}{(f\cos{(ex+d)}+f)^2}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)/(f^2*cos(e*x + d)^2 + 2*f^2*cos(e*x + d) + f^2),
x)
```

3.142.6 Sympy [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx=\frac{\int \frac{F^{ac+bcx}}{\cos^2{(d+ex)}+2\cos{(d+ex)}+1}\,dx}{f^2}$$

input
```
integrate(F**(c*(b*x+a))/(f+f*cos(e*x+d))**2,x)
```

output
```
Integral(F**(a*c + b*c*x)/(cos(d + e*x)**2 + 2*cos(d + e*x) + 1), x)/f**2
```

3.142.7 Maxima [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx=\int \frac{F^{(bx+a)c}}{(f\cos{(ex+d)}+f)^2}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x, algorithm="maxima")
```

output
```
4*(6*(F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(
a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d)^2 + 80*(F^(a*c)*b^3*c^3*e^
2*log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(e*x + d)^2 + 6*(F^(a
*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e
^4*log(F))*F^(b*c*x)*sin(2*e*x + 2*d)^2 + 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3
 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(e*x + d)^2 - 140*(F^(a*c)*b^3*
c^3*e^2*log(F)^3 - 8*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(e*x + d) + 20*(
F^(a*c)*b^4*c^4*e*log(F)^4 - 26*F^(a*c)*b^2*c^2*e^3*log(F)^2)*F^(b*c*x)*si
n(e*x + d) - 40*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 5*F^(a*c)*b*c*e^4*log(F))*
F^(b*c*x) + ((F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 +
 144*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) + 20*(F^(a*c)*b^3*
c^3*e^2*log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(e*x + d) - 2*(
F^(a*c)*b^4*c^4*e*log(F)^4 + 25*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 144*F^(a*c)
*e^5)*F^(b*c*x)*sin(2*e*x + 2*d) + 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 10*F^(a
*c)*b^2*c^2*e^3*log(F)^2 - 96*F^(a*c)*e^5)*F^(b*c*x)*sin(e*x + d) - 40*(F^
(a*c)*b^3*c^3*e^2*log(F)^3 - 5*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x))*cos(4*e*
x + 4*d) + 4*((F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3
+ 144*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) + 20*(F^(a*c)*b^3
*c^3*e^2*log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(e*x + d) - 2*
(F^(a*c)*b^4*c^4*e*log(F)^4 + 25*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 144*F^(...
```

3.142.8 Giac [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(f\cos(ex+d)+f)^2}\,dx$$

input
```
integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)/(f*cos(e*x + d) + f)^2, x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx = \int \frac{F^{c\,(a+b\,x)}}{\left(f+f\,\cos\left(d+e\,x\right)\right)^2}\,dx$$

input
```
int(F^(c*(a + b*x))/(f + f*cos(d + e*x))^2,x)
```

output
```
int(F^(c*(a + b*x))/(f + f*cos(d + e*x))^2, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```